10. The Hydrogen Bomb

1. Introduction

On July 16, 1945, the United States detonated the world's first, large yield, atomic device at the trinity site near Alamogordo, New Mexico. With a yield of 20 kilotons of TNT, the bomb changed forever man's view of warfare and opened the possibility of destroying any city or markedly altering any physical feature on the earth's surface with one bomb. The threat of nuclear proliferation and with it the increased possibility that small bands of terrorists could obtain and use such weapons, threaten all civilized men to this day. It is therefore with a profound sense of the gravity of the situation that the author gives an explanation of how the bomb works with out using $E=mc^2$.

2. Potential Energy

An atom of the bomb material attracts a particle of mass m_p at a distance r from the center of mass of the atom with potential energy PE(r). As the particle goes from r_o to r, $\Delta PE(r) \equiv PE(r) - PE(r_o)$, $r_{pa} \le r \le r_o$. r_o is the radius of the atom, r_{pa} is the radius of the particle where $r_{pa} << r_o$. PE(r) is given by 10.1 below, all quantities measured from an inertial frame at rest w.r.t. the center of mass of m_{At}

10.1 $$PE(r) = -m_p m_{At} \frac{H}{r_o}\left[\frac{1}{(p+2)}\left[p+3 - \left(\frac{r}{r_o}\right)^{p+2}\right]\right], \; r_{pa} \le r \le r_o, \; -3 < p \le 0$$

An atom of the bomb material adsorbs particle m_p (Whose origin is discussed in sec. 4) and the particle is attracted to the center of the atom by the atomic force with $\Delta PE(r) = PE(r) - PE(r_o)$ given by:

10.2 $$\Delta PE(r) = -m_p m_{At} \frac{H}{r_o(p+2)}\left[1 - \left(\frac{r}{r_o}\right)^{p+2}\right], \; r_{pa} \le r \le r_o$$

The translational kinetic energy of the particle increases by ΔKE_{pa} as given by 10.3

10.3 $$-\Delta PE(r) = m_p m_{At} \frac{H}{r_o(p+2)}\left[1 - \left(\frac{r}{r_o}\right)^{p+2}\right] = \Delta KE_{pa}, \; r_{pa} \le r \le r_o$$

10.3 is numerically evaluated for specific values of r and r_o in sec 4.

For future use.

From 3.17, $\frac{1}{2}U^2_{rms}(r) + \Psi(r) = C_1$ and the binding energy of the atom is:

BE=T+V=$mC_1=\frac{1}{2}mU^2_{rms}(r_o)+m\Psi(r_o)$, where $T+V=2\pi\int_0^{r_o} r^2\rho(r)U^2_{rms}(r)dr+4\pi\int_0^{r_o} r^2\Psi(r)\rho(r)dr$

and using 3.8, $\Psi(r)=-\frac{mH}{r_o}\frac{(p+3)}{(p+2)}[1-\frac{1}{(p+3)}(\frac{r}{r_o})^{p+2}]$, with $\Psi(r_o)=-\frac{mH}{r_o}$. The binding energy becomes: $BE=\frac{1}{2}mU^2_{rms}(r_o)-\frac{m^2H}{r_o}$, with density $\rho(r)=\frac{(p+3)m}{4\pi r_o^3}\left(\frac{r}{r_o}\right)^p$. By direct computation (See appendix 10A):

$V=-\frac{m^2H}{h_o}\frac{(p+3)}{(p+2.5)}$, $T=\frac{m^2H}{h_o}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU^2_{rms}(r_o)$, $-2.5^+\leq p\leq 0$, $T+V=\frac{1}{2}mU^2_{rms}(r_o)-\frac{m^2H}{r_o}<0$

$V=-\infty$, $T=+\infty$, $-3<p\leq -2.5^-$, $T+V=\frac{1}{2}mU^2_{rms}(r_o)-\frac{m^2H}{r_o}<0$

A physical process to create bomb material is discussed in the following section.

3. Creation of Bomb Material

As explained below: In order to create bomb material, atoms must be created with $p=-3+10^{-n}$, $n\gtrsim 2$. In order to create atoms with $p=-3+10^{-n}$, $n\gtrsim 2$, consider the following scenario.

A. Using 4.6 and table 4.1, the average pressure two adjacent atoms exert on one another is $P_o=2r_onK\frac{T_i}{\overline{R}_i}$ and with Fe as an example at $T_i=293^oK$ and $\overline{R}_i=4.6\cdot 10^{-11}$cm, P_o becomes: $P_o=1.7\cdot 10^{12}\frac{dy}{cm^2}$.

B. Place the bomb material between the poles of a high voltage source. This will increase the radius of the atoms by Δr_o and decrease $\overline{R}_i$ from $\overline{R}_i$ to $\overline{R}_i-2\Delta r_o$. P becomes $P_{\Delta r_o}=\frac{KT_i}{(2r_o+\overline{R}_i)^2\overline{R}_i(1-\frac{2\Delta r_o}{\overline{R}_i})}$ and $P_{\Delta r_o}$ becomes: $P_{\Delta r_o}=\frac{P_o}{1-\frac{2\Delta r_o}{\overline{R}_i}}$. An arbitrarily large value of $P_{\Delta r_o}$ occurs as $1-2\frac{\Delta r_o}{\overline{R}_i}\to 0$ and with $1-2\frac{\Delta r_o}{\overline{R}_i}=0$, Δr_o becomes: $\Delta r_o=2.3\cdot 10^{-11}$cm.

C. The change in binding energy of an atom is $\Delta BE=\frac{1}{2}m\Delta U^2_{rms}(r_o)+\frac{m^2H}{r_o^2}\Delta r_o$ and assuming $\Delta U^2_{rms}(r_o)=0$, ΔBE becomes $\Delta BE=\frac{m^2H}{r_o^2}\Delta r_o$ and evaluating for Fe yields $\Delta BE=68\Delta r_o$erg.

Evaluating ΔBE for $\Delta r_o=2.3\cdot10^{-11}$cm. yields $\Delta BE=1.6\cdot10^{-9}$erg $=1.0\cdot10^{3}$ev.

D. The vibrating surface of the atom A_1 with average radius $r_o+\Delta r_o$, Δr_o positive or negative, is represented by $\mathfrak{h}_1=r(\hat{r})\cdot\hat{r}+\chi_1(r(\hat{r}),t)\cdot\hat{r}$ with $\chi_1(r(\hat{r}),0)=0$. Using 3.8 and 3.15 with $[U(\mathfrak{h}(\hat{r}_1,t)_1)]_{rms}=[U(\mathfrak{h}(\hat{r}_2,t)_1)]_{rms}$ averaged over $\hat{r}$ or t.

10.3a $$\tfrac{1}{2}[U(\mathfrak{h}(\hat{r},t)_1)]^2_{rms}-\frac{mH}{(\mathfrak{h}(\hat{r},t)_1)_{rms}}=C_2<0,$$

where $(\mathfrak{h}(\hat{r},t)_1)_{rms}\equiv r_o+\Delta r_o$ independent of $\hat{r}$.

The average maximum value of $\mathfrak{h}_1$ is evaluated at the instant for which $\overline{U(\mathfrak{h}(\hat{r},t)_1)}=0$ and assuming $\overline{\mathfrak{h}(\hat{r},t)_1}$ is given by $\overline{\mathfrak{h}(\hat{r},t)}_{1,M}=-\frac{mH}{C_2}$, the averaged amplitude of h_o becomes:

$A_{mp}=\overline{\mathfrak{h}(\hat{r},t)}_{1,M}-(\mathfrak{h}(\hat{r},t)_1)_{rms}=-\frac{mH}{C_2}-r_o(1+\frac{\Delta r_o}{r_o})>0$. Set $A_{mp}=a_1\overline{\mathfrak{h}(\hat{r},t)}_{1,M}$ with $0<a_1<1$ and solve for C_2 yielding: $C_2=-\frac{(1-a_1)}{r_o(1+\frac{\Delta r_o}{r_o})}mH$ and $\overline{\mathfrak{h}(\hat{r},t)}_{1,M}=\frac{r_o(1+\frac{\Delta r_o}{r_o})}{(1-a_1)}$ and $A_{mp}=a_1\frac{r_o(1+\frac{\Delta r_o}{r_o})}{(1-a_1)}$

For future use using $|\frac{\Delta r_o}{r_o}|<<1$ and $TE_{int}=BE_{atom}=(T+V)_{atom}=mC_2$:

10.3b $$TE_{int}=BE_{atom}=(T+V)_{atom}=mC_2\doteq-\frac{(1-a_1)}{r_o}(1-\frac{\Delta r_o}{r_o})m^2H \quad \text{with } 0<a_1<1 \text{ and}$$

$$\overline{\mathfrak{h}(\hat{r},t)}_{1,M}\doteq\frac{r_o}{(1-a_1)},\quad A_{mp}=a_1\overline{\mathfrak{h}(\hat{r},t)}_{1,M}\doteq\frac{a_1}{(1-a_1)}\cdot r_o\quad |\frac{\Delta r_o}{r_o}|<<1$$

E. Note that the total internal energy TE_{int} of a given vibrating atom A_o with average radius r_o is, $TE_{int}=mC_1=-\frac{m^2H}{r_o}+\frac{1}{2}mU^2(r_o)=-\frac{(1-a_o)}{r_o}m^2H$, where TE_{int} is independent of density $\rho(r)$ and p for a given r_o where $\rho(r)=\frac{(p+3)m}{4\pi r_o^3}\left(\frac{r}{r_o}\right)^p$ and $-3<p\leq0$. Due to pressure $P_{\Delta r_o}$, (See B.)

i. TE_{int} becomes: $mC_2=-\frac{m^2H}{r_o}(1-\frac{\Delta r_o}{r_o})+\frac{1}{2}mU^2(r_o(1+\frac{\Delta r_o}{r_o}))=-(1-a_1)\frac{m^2H}{r_o}(1-\frac{\Delta r_o}{r_o})$

ii. The interior mass of the atom is compressed maintaining the atom's average radius

$r_o(1+\frac{\Delta r_o}{r_0})=r_f$ and decreasing p from p_i to p_f where $-3<p_f<p_i\leq 0$ and $r_f \equiv r_o+\Delta r_o$. This takes energy $E_{\Delta p}$ where $\Delta p \equiv p_f - p_i$.

iii. The difference in TE_{int} is $\Delta TE_{int}= m(C_2 - C_1)= -(1-a_1)\frac{m^2H}{r_f}[1-\frac{(1-a_0)r_f}{(1-a_1)r_0}]$.

Let A_1 represent the atom with radius r_o and let A_2 represent the atom with radius $r_f \equiv r_o+\Delta r_o$. Using 10.3b, the oscillation amplitude of A_o is $A_{mp}=\frac{a_0}{(1-a_0)}\cdot r_o$ with $0<a_o<1$ and the oscillation amplitude of A_1 is $A_{mp}=\frac{a_1}{(1-a_1)}\cdot r_f$ with $0<a_1<1$.

iv. The total electrical energy E_{El} to change the radius of an atom by $\Delta r_o=r_f-r_o$ and to change p by $\Delta p \equiv p_f - p_i$ is:

10.3c $$E_{El}=E_{\Delta p}+\Delta TE_{int}=E_{\Delta p}-(1-a_1)\frac{m^2H}{r_f}[1-\frac{(1-a_0)r_f}{(1-a_1)r_0}] >0,\ 0<a_o<1,\ 0<a_1<1$$

10.3c is evaluated in section 5 below.

F. The change in internal energy ΔE_{int} of an atom as $A_{mp} \to 0$ is: $\Delta E_{int}=E_{int,f} - E_{int,i}= -\frac{m^2H}{r_f}+(1-a_1)\frac{m^2H}{r_f} = -a_1\frac{m^2H}{r_f}$ and the work done on the external world E_{ex} is $E_{ex}=|\Delta E_{int}|=a_1\frac{m^2H}{r_f}$. It is hypothesized that during an unwanted meltdown of nuclear fuel such as at the Russian Chernobyl 1986 nuclear accident, due to contact with a dissimilar substance, the A_{mp} of the atoms of the nuclear fuel decreases resulting in $|\Delta E_{int}|>0$ and an increase in translational kinetic energy and concomitant temperature and melt of the nuclear fuel and containment vessel.

G. The atomic mass within a sphere of radius $r \leq r_f$ is $M(r) \equiv 4\pi\int_0^r r^2\rho(r)dr=m(\frac{r}{r_f})^{p+3}$

and for $\frac{M(r)}{m_{Fe}}=0.90$: $\frac{r}{r_f}(p)=(\frac{M(r)}{m_{Fe}})^{\frac{1}{p+3}}=(0.90)^{\frac{1}{p+3}}$ and evaluating $\frac{r}{r_f}(p)$ for given p yields:

$\frac{r}{r_f}(0)=0.97$, $\frac{r}{r_f}(-1)=0.95$, $\frac{r}{r_f}(-2)=0.90$, $\frac{r}{r_f}(-2.9)=0.35$, $\frac{r}{r_f}(-2.99)=2.7\cdot 10^{-5}$

With $p=-3^+$ and consequent $\frac{r}{r_f}(-3^+)<<1$, a particle with radius $r_{pa}<<r_f$ plunges through the less dense outer shell of the atom increasing in energy and momentum and strikes the hard nucleus transforming particle kinetic energy into nuclear and particle internal energy and with small enough r, where r is the distance between the center of the atom and the center of the particle, both the atom and the particle explode with energy E_{out} where E_{out} is measured at R as $R \to \infty$.

For some $p=-3^+$ and with $\frac{r}{r_f}(-3^+)<<1$ using 10.3:

10.4a $-\Delta PE(r) = \frac{m_p m_{At} H}{(p+2)r_f}\left[1-(\frac{r}{r_f})^{p+2}\right] = |BE(r_f)_{At}| + |BE_p| + E_{out}$.

And with $E_{el} = -\Delta PE(r)$

10.4b $E_{El} = E_{\Delta p} - (1-a_1)\frac{m_{At}^2 H}{r_f}[1-\frac{(1-a_0)r_f}{(1-a_1)r_0}] = \frac{m_p m_{At} H}{(p+2)r_f}\left[1-(\frac{r}{r_f})^{p+2}\right] = |BE(r_f)_{At}| + |BE_p| + E_{out}$

10.4b is evaluated in section 5.

4. Atom Compression Revisited

Consider a stable atom with initial radius r_o, compressed to create a stable atom with final radius r_f. Using 3.15, the internal energy of the atom in the two states is:

$T_i = \frac{1}{2}mU^2(r) + m\psi(r) = \frac{1}{2}mU^2(r_0) - \frac{m^2 H}{r_0} = mC_1 < 0$, $0 \le r \le r_o$ and C_1=const.

$T_f = \frac{1}{2}mU^2(r) + m\psi(r) = \frac{1}{2}mU^2(r_f) - \frac{m^2 H}{r_f} = mC_2 < 0$, $0 \le r \le r_f$ and C_2=const.

The total internal energy of each atom is:

$$TIE_i = \int_{V_1}\{\frac{1}{2}\rho_1(r)U_{rms}^2(r) + \rho_1(r)\Psi_1(r)\}dV_1 = C_1\int_{V_1}\rho_1(r)dV_1 = mC_1 = \frac{1}{2}mU^2(r_0) - \frac{m^2 H}{r_0} < 0$$

$$TIE_f = \int_{V_2}\{\frac{1}{2}\rho_2(r)U_{rms}^2(r) + \rho_2(r)\Psi_2(r)\}dV_2 = C_2\int_{V_2}\rho_2(r)dV_2 = mC_2 = \frac{1}{2}mU^2(r_f) - \frac{m^2 H}{r_f} < 0$$

To insure stability of the atom, it is required that $\frac{1}{2}mU^2(r_0) << \frac{m^2 H}{r_0}$ and $\frac{1}{2}mU^2(r_f) << \frac{m^2 H}{r_f}$.

The difference in the total internal energy of the two atoms is:

$$\Delta TE_{int} = \frac{1}{2}mU^2(r_f) - \frac{m^2 H}{r_f} - [\frac{1}{2}mU^2(r_0) - \frac{m^2 H}{r_0}] \doteq \frac{m^2 H}{r_0} - \frac{m^2 H}{r_f} = -\frac{m^2 H}{r_0}(\frac{r_0 - r_f}{r_f}) = m(C_2 - C_1) < 0$$

If it turns out the atoms are quasi stable, i.e. $\frac{1}{2}mU^2(r_0) < \frac{m^2 H}{r_0}$ and $\frac{1}{2}mU^2(r_f) < \frac{m^2 H}{r_f}$, then the full expression must be used.

$$\Delta TE_{int} = \frac{1}{2}mU^2(r_f) - \frac{1}{2}mU^2(r_0) - \frac{m^2 H}{r_0}(\frac{r_0 - r_f}{r_f}) = m(C_2 - C_1) < 0$$

From 3.19, $P(r)_{int} = P(r_0)_{ex} + \frac{1}{3}\int_r^{r_0}\rho(w)\frac{d\Psi}{dw}dw$ and by direct computation using 3.15:

$$P(r)_{int} = P(r_0)_{ex} + \frac{1}{24\pi}(\frac{m^2 H}{r_0^4})\frac{(p+3)}{(p+1)}[1-(\frac{r}{r_0})^{2(p+1)}].$$

In order to compress a spherical particle or atom with mass m, from stable radius r_o to radius r_f, requires work $W(t_f)$ where the compression occurs during time $0 \le t \le t_f$. With constant external pressure $P(r_0)_{ex}$ and time dependent internal pressure $P(R,t)_{int}$;

at the surface of the atom at time t_f, $P(r_f,t_f)_{int}=P(r_0)_{ex}$. $W(t_f)$ is:

$W(t_f)=-P(r_0)_{ex}\Delta V=\frac{4}{3}\pi P(r_0)_{ex}(r_0^3-r_f^3)=\frac{4}{3}\pi P(r_0)_{ex}r_0^3[1-(\frac{r_f}{r_0})^3]$ and $r_f^3=r_0^3-\frac{3W(t_f)}{4\pi P(r_0)_{ex}}>0$.

The difference in total internal energy between the stable particle or atom with radius r_f and stable particle or atom with radius r_o is,

$\Delta TE_{int}=\frac{1}{2}mU^2(r_f)-\frac{m^2H}{r_f}-[\frac{1}{2}mU^2(r_0)-\frac{m^2H}{r_0}]\doteq\frac{m^2H}{r_0}-\frac{m^2H}{r_f}=-\frac{m^2H}{r_0}(\frac{r_0-r_f}{r_f})=\frac{m^2H}{r_0}(1-\frac{r_0}{r_f})=m(C_2-C_1)<0$

See chap. 10 sec. 4. Energy in plus energy out becomes:

$W+\Delta TE_{int}=\frac{4}{3}\pi P(r_0)_{ex}r_0^3[1-(\frac{r_f}{r_0})^3]+\frac{m^2H}{r_0}(1-\frac{r_0}{r_f})$ and with $W+\Delta TE_{int}=0$, find $P(r_0)_{ex}=\frac{-\frac{m^2H}{r_0}(1-\frac{r_0}{r_f})}{\frac{4}{3}\pi r_0^3[1-(\frac{r_f}{r_0})^3]}$

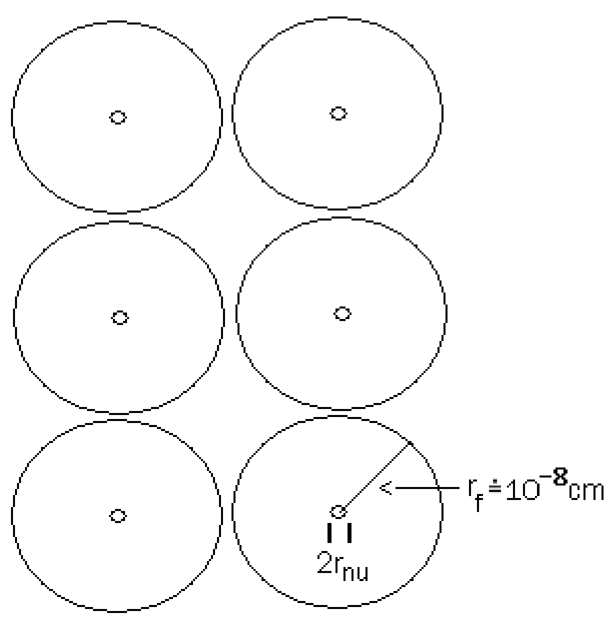
For the special case $\frac{r_f}{r_0}=1-\varepsilon$ with $\varepsilon<<1$, find $P(r_0)_{ex}=\frac{m^2H(1+\varepsilon)}{4\pi r_0^4}$, however require

$\lim_{\varepsilon\to 0}P(r_0)_{ex}=0$ hence, $P(r_0)_{ex}=\frac{m^2H(1+\varepsilon)}{4\pi r_0^4}-\frac{m^2H}{4\pi r_0^4}=\frac{m^2H}{4\pi r_0^4}\varepsilon$.

5. Creation of an Explosion

Continuing the scenario begun above, the energetics of a thermonuclear bomb are developed within the context of a shrunken Fe atom. We consider the bomb material to be 0.5Kg of Fe containing $5.3\cdot 10^{24}$ atoms. With a yield of 30 megatons of TNT, and if each atom contributes to the total yield, E_{out} of each atom is: E_{out} =0.23 erg=140Bev. After the initial p value is decreased from p_i to p_f, it is sputtered into a diatomic thin film. Fig. 10.1 As will be shown with $-3<p\lesssim-2.90$, most of the mass of the atom is contained within a shell of radius $r_{nu}<<r_f\doteq 10^{-8}$cm with $m_{At}-m_{nu}<<m_{At}$.

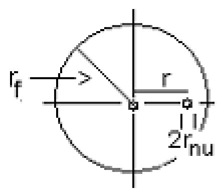
FIGURE 10.1
DIATOMIC THIN FILM



The binding energy of this atom is:
$BE_f=\frac{1}{2}mU^2_{rms}(r_f)-\frac{m^2H}{r_f}=-(1-a_1)\frac{m^2H}{r_f}$, where $A_{mp}=\frac{a_1}{(1-a_1)}\cdot r_f$ with $0< a_1<1$. If $a_1=\frac{1}{3}$, the atoms oscillate with amplitude $\frac{1}{2}r_f$: The average least distance R_{ld} between the center of mass of two adjacent atoms is $R_{ld}=r_f$, the average distance R_{av} is $R_{av}=2r_f$, and the average greatest distance R_{gd} is $R_{gd}=3r_f$.

The thin film is submerged in liquid He causing the mass $m_{At}-m_{nu}$ to shrink so that the average least distance R_{ld} between two adjacent atoms becomes $R_{ld}<r_f$. The two m_{nu} are drawn together on a collision course by the strong nuclear force. Figure 10.2.

FIGURE 10.2



With $m_{nu}\doteq m_{At}$, 10.4b becomes:

10.4c $\quad E_{el}=\frac{m^2_{At}H}{r_f}\left(\frac{1}{p+2}\right)\left[1-\left(\frac{r}{r_f}\right)^{p+2}\right]=2(1+a_1)\frac{m^2_{At}H}{r_f}+E_{out}$

and solving for $\frac{r}{r_f}$ yields:

10.5 $\quad \frac{r}{r_f}=\left[-(2p+3)-(p+2)\{(\frac{r_f}{m^2_{At}H})E_{out}-2a_1\}\right]^{\frac{1}{(p+2)}} \qquad p\neq -2$

$\frac{r}{r_f}=\exp-[2(1+a_1)+(\frac{r_f}{m^2_{At}H})E_{out}] \qquad p=-2$

With E_{out}=280Bev=0.46erg evaluate 10.5 for Fe using $r_f\doteq r_o=1.13\cdot10^{-8}$cm

10.6 $\quad \frac{r}{r_f}=\left[-(2p+3)-\{6.0\cdot10^5-2a_1\}(p+2)\right]^{\frac{1}{(p+2)}} \qquad p\neq -2$

$\frac{r}{r_f}=\exp-[2(1+a_1)+6.0\cdot10^5] \qquad p=-2$

In order for our model for the hydrogen bomb to represent physical reality, the surface density of the atom $\rho(r_f)=\frac{(p+3)m_{Fe}}{4\pi r^3_{f,Fe}}=(1+\frac{p}{3})\bar{\rho}$ must be much smaller that the average density of the atom. i.e. $0<(1+\frac{p}{3})\bar{\rho}<<\bar{\rho}$ and therefore $-3<p\lesssim-2.90$

With $-1\le a_1\le 0$ and $-3<p\lesssim-2.90$, 10.6 becomes:

10.7 $\frac{r}{r_f}\doteq[-(6.0\cdot 10^5)(p+2)]^{\frac{1}{(p+2)}}$

From part G above, $\frac{M(r)}{m_{Fe}}=(\frac{r}{r_f})^{(p+3)}$. Computed values of r, $\frac{M(r)}{m_{Fe}}$, and $\rho(r_f)$ as a function of p are listed in Table 1 and 2 for the special case $a_1=\frac{1}{3}$.

Notice that given E_{out}, a_1, p and the starting atomic species, the theory as derived above does not enable us to know which row of table 10.1 or 10.2 represents physical reality.

The input energy to create the explosion per atom pair is: $E_{in}=2|BE(r_f)|+E_{out}$ and the theoretic yield ratio Φ_y is, $0<\Phi_y\equiv\frac{E_{out}}{E_{in}}<1$, where $\Phi_y=\frac{E_{out}}{2|BE(r_f)|+E_{out}}\doteq 1-\frac{2|BE(r_f)|}{E_{out}}$.

Evaluating Φ_y for Fe with E_{out}=0.46erg yields: $\Phi_y=1-2.2\cdot 10^{-6}$

0.5Kgm of naturally occurring Fe contains $5.3\cdot 10^{24}$ atoms and has dimensions 3.7cm on a side with volume $51cm^3$. To be manageable as a thin film, the bomb material for example might consist of 10^3 strips each of dimension $(10)X(10)X(5.1\cdot 10^{-4})$cm and $\frac{5.1\cdot 10^{-4}}{2r_0}=2.3\cdot 10^4$ atoms thick.

TABLE 10.1

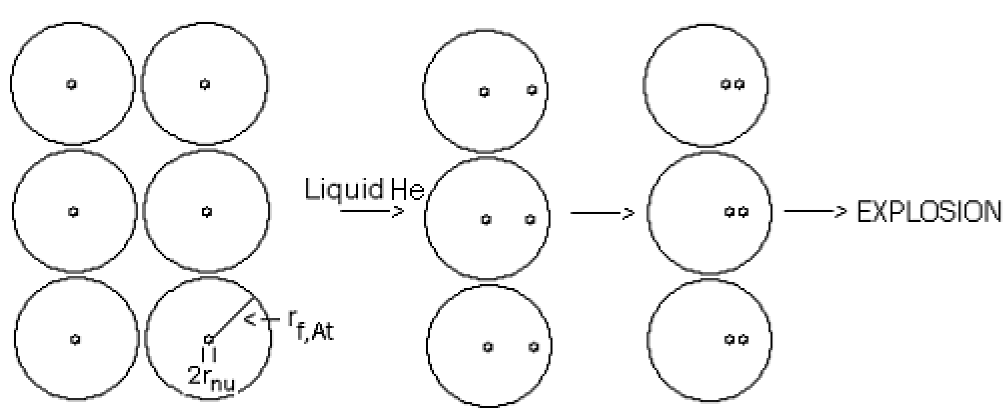
p	r (cm)	$\frac{M(r)}{m_{Fe}}$	$\rho(r_f)$
-2.90	$4.7\cdot 10^{-15}$	0.23	$3.3\cdot 10^{-2}\bar{\rho}$
-2.91	$5.5\cdot 10^{-15}$	0.27	$3.0\cdot 10^{-2}\bar{\rho}$
-2.92	$6.3\cdot 10^{-15}$	0.32	$2.7\cdot 10^{-2}\bar{\rho}$
-2.93	$7.3\cdot 10^{-15}$	0.37	$2.3\cdot 10^{-2}\bar{\rho}$
-2.94	$8.4\cdot 10^{-15}$	0.43	$2.0\cdot 10^{-2}\bar{\rho}$
-2.95	$9.6\cdot 10^{-15}$	0.50	$1.7\cdot 10^{-2}\bar{\rho}$
-2.96	$1.1\cdot 10^{-14}$	0.58	$1.3\cdot 10^{-2}\bar{\rho}$
-2.97	$1.3\cdot 10^{-14}$	0.66	$1.0\cdot 10^{-2}\bar{\rho}$
-2.98	$1.4\cdot 10^{-14}$	0.76	$6.7\cdot 10^{-3}\bar{\rho}$
-2.99	$1.6\cdot 10^{-14}$	0.87	$3.3\cdot 10^{-3}\bar{\rho}$
-3	$1.8\cdot 10^{-14}$	1.0	0

TABLE 10.2

p	r (cm)	$\frac{M(r)}{m_{Fe}}$	$\rho(r_{f,Fe})$
-2.991	$1.64\cdot10^{-14}$	0.89	$3.0\cdot10^{-3}\overline{\rho}$
-2.992	$1.66\cdot10^{-14}$	0.90	$2.7\cdot10^{-3}\overline{\rho}$
-2.993	$1.68\cdot10^{-14}$	0.91	$2.3\cdot10^{-3}\overline{\rho}$
-2.994	$1.70\cdot10^{-14}$	0.92	$2.0\cdot10^{-3}\overline{\rho}$
-2.995	$1.72\cdot10^{-14}$	0.94	$1.7\cdot10^{-3}\overline{\rho}$
-2.996	$1.74\cdot10^{-14}$	0.95	$1.3\cdot10^{-3}\overline{\rho}$
-2.997	$1.77\cdot10^{-14}$	0.96	$1.0\cdot10^{-3}\overline{\rho}$
-2.998	$1.79\cdot10^{-14}$	0.97	$6.7\cdot10^{-4}\overline{\rho}$
-2.999	$1.81\cdot10^{-14}$	0.99	$3.3\cdot10^{-4}\overline{\rho}$
-3	$1.83\cdot10^{-14}$	1.0	0

To repeat:
The binding energy of the atom is:
$BE_f = -(1-a_1)\frac{m^2H}{r_f}$, where $A_{mp}=\frac{a_1}{(1-a_1)}\cdot r_f$ with $0< a_1<1$. If $a_1=\frac{1}{3}$, the atoms oscillate with amplitude $\frac{1}{2}r_f$: The average least distance R_{ld} between the center of mass of two adjacent atoms is $R_{ld}=r_f$, the average distance R_{av} is $R_{av}=2r_f$, and the average greatest distance R_{gd} is $R_{gd}=3r_f$.
To detonate the bomb, the thin film is submerged in liquid He causing the mass $m_{At}- m_{nu}$ to shrink so that the average least distance r_f between two adjacent atoms becomes less than r_f and one atom captures the m_{nu} of the other. Figure 10.3. The captured m_{nu} plunges through the low density (See table 10.1 and 10.2) outer shell of the

FIGURE 10.3



remaining atom increasing in energy and momentum. The $2m_{nu}$ strike one another transforming m_{nu} translational kinetic energy into m_{nu} internal energy and both m_{nu} explode with energy E_{out}=0.46 erg. This is how the A-bomb explosions on Hiroshima and Nagasaki were created; this method is no longer used.

An alternative bomb detonation scenario results if the atom pair do not have enough energy to cause the atoms to explode. The atoms of the thin films coalesce into an incandescent super dense material.

6. Appendix 10A

Given T+V, $\Psi(r)$ and $\rho(r)$ where $T+V=2\pi\int_0^{r_0} r^2\rho(r)U_{rms}^2(r)dr+4\pi\int_0^{r_0} r^2\Psi(r)\rho(r)dr=mC_1=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}$,

$\Psi(r)=-\frac{mH}{r_0}\frac{(p+3)}{(p+2)}[1-\frac{1}{(p+3)}(\frac{r}{r_0})^{p+2}]$ and $\rho(r)=\frac{(p+3)m}{4\pi r_0^3}(\frac{r}{r_0})^p$. Show that:

$V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2.5)}$, $T=\frac{m^2H}{r_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(r_0)$, $-2.5^+\le p\le 0$, $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$

$V=-\infty$, $T=+\infty$, $-3<p\le -2.5^-$, $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$

Formally, V becomes:

$$V=-\frac{m^2H}{r_0}\cdot\frac{(p+3)^2}{(p+2)}\int_0^{r_0}[1-\frac{1}{(p+3)}(\frac{r}{r_0})^{(p+2)}]\frac{r^{(p+2)}}{r_0^{(p+3)}}\cdot dr=-\frac{m^2H}{r_0}\cdot\frac{(p+3)^2}{(p+2)}[\frac{1}{(p+3)}(\frac{r}{r_0})^{(p+3)}-\frac{1}{(p+3)(2p+5)}(\frac{r}{r_0})^{(2p+5)}]\Big|_0^{r_0}$$

For $-2.5^+\le p\le 0$, $V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2.5)}$, and $T=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}-V=\frac{m^2H}{r_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(r_0)$,

For $-3<p\le -2.5^-$, $V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2)}[1-\frac{1}{2(p+2.5)}+\frac{1}{2(p+2.5)}\lim_{r\to 0}(\frac{r_0}{r})^{2|p+2.5|}]=-\infty$ and

$T=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}-V==+\infty$

For $-3<p\le 0$, $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$

QED

It should be noted that one does not measure T or V independent of one another, one always measures the binding energy T+V where $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$ for $-3<p\le 0$. See Chapter 6, Section 5 for an application of T+V.