

## 1. Mathematical Theory of Turbulence and the $(\underline{U} \cdot \nabla) \underline{U}$ Term

The mathematical theory of fluid turbulence is based on the  $(\underline{U} \cdot \nabla) \underline{U}$  term as derived in the Navier-Stokes Equations. It is proved below that this term does not represent physical reality and therefore a new theory is necessary to explain fluid turbulence.

In the derivation of the Navier-Stokes Equations, the assumption is made that the coordinates, velocity and acceleration of a moving point with respect to an inertial frame may be written as  $\underline{x} = \underline{x}(x_0, t)$ ,  $\underline{x}(x_0, 0) = \underline{x}_0$ ,  $\underline{U} = \frac{d\underline{x}}{dt}$  and  $\underline{a} = \frac{d^2\underline{x}}{dt^2}$ .

With  $\underline{U} = \underline{U}[x(x_0, t), y(y_0, t), z(z_0, t), t] = U_x \hat{x} + U_y \hat{y} + U_z \hat{z}$ . Take the derivative of  $\underline{U}$  w.r.t.  $t$ :

$$12.1 \quad \frac{d\underline{U}}{dt} = \frac{\partial \underline{U}}{\partial t} + \frac{\partial \underline{U}}{\partial x} \frac{dx}{dt} + \frac{\partial \underline{U}}{\partial y} \frac{dy}{dt} + \frac{\partial \underline{U}}{\partial z} \frac{dz}{dt} = \frac{\partial \underline{U}}{\partial t} + (\underline{U} \cdot \nabla) \underline{U}$$

i.e. The derivation explicitly uses the coordinates of the moving point  $x, y, z$  which are functions of  $t$ , to be the same as the  $x, y, z$  which represent fixed points in frame  $S$  independent of  $t$ . The notation is inconsistent with itself. One needs a new notation to differentiate the coordinates of a moving point from the fixed coordinates that constitute frame  $S$ . The new notation is as follows.

Let the function  $\underline{h} = (h_x, h_y, h_z)$  represent the position of a moving point in rectangular coordinates where  $h_x, h_y, h_z$  are linear lengths in the  $\hat{x}, \hat{y}, \hat{z}$  directions respectively.

$$12.2 \quad h_x = x + \chi_x(x, y, z, t) \quad \text{with} \quad \chi_x(x, y, z, 0) = 0.$$

$$12.3 \quad h_y = y + \chi_y(x, y, z, t) \quad \text{with} \quad \chi_y(x, y, z, 0) = 0.$$

$$12.4 \quad h_z = z + \chi_z(x, y, z, t) \quad \text{with} \quad \chi_z(x, y, z, 0) = 0.$$

Once chosen,  $x, y, z$ , are stationary in  $S$  and independent of  $t$ :  $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$ .

The velocity of the moving point is  $\underline{U} = \frac{d\underline{h}}{dt} = \frac{\partial h_x}{\partial t} \hat{x} + \frac{\partial h_y}{\partial t} \hat{y} + \frac{\partial h_z}{\partial t} \hat{z}$  and the acceleration

of the moving point is  $\underline{a} = \frac{\partial \underline{U}}{\partial t} = \frac{\partial^2 \underline{h}}{\partial t^2} = \frac{\partial^2 h_x}{\partial t^2} \hat{x} + \frac{\partial^2 h_y}{\partial t^2} \hat{y} + \frac{\partial^2 h_z}{\partial t^2} \hat{z}$  with:

$$12.5 \quad \frac{\partial h_x}{\partial t} = \frac{\partial \chi_x}{\partial t}(x, y, z, t) \quad \text{and} \quad \frac{\partial^2 h_x}{\partial t^2} = \frac{\partial^2 \chi_x}{\partial t^2}(x, y, z, t)$$

$$12.6 \quad \frac{\partial h_y}{\partial t} = \frac{\partial \chi_y}{\partial t}(x, y, z, t) \quad \text{and} \quad \frac{\partial^2 h_y}{\partial t^2} = \frac{\partial^2 \chi_y}{\partial t^2}(x, y, z, t)$$

$$12.7 \quad \frac{\partial h_z}{\partial t} = \frac{\partial \chi_z}{\partial t}(x, y, z, t) \quad \text{and} \quad \frac{\partial^2 h_z}{\partial t^2} = \frac{\partial^2 \chi_z}{\partial t^2}(x, y, z, t)$$

$$\text{and } \underline{a} \neq \frac{\partial \underline{U}}{\partial t} + (\underline{U} \cdot \nabla) \underline{U}.$$

Although 12.1 is mathematically correct, it is not physically correct. The mathematical definition of velocity  $\underline{U}$ ,  $\underline{U} = \frac{d\underline{x}}{dt}$ , is physically incorrect as it miss assigns the stationary coordinates  $x,y,z$  of frame S to the moving point coordinates of the moving point. The mathematical definition of  $\underline{U}$  is inconsistent with the operational definition of  $\underline{U}$  which is  $\underline{U} = \frac{d\underline{h}}{dt}$ .

Newton's Laws as exemplified by  $\rho(\underline{h}) \cdot \frac{d\underline{U}}{dt} = -\nabla P$ , or  $\underline{F} = \frac{d}{dt}(m\underline{U})$  are valid only if position, velocity and acceleration are measured from a frame S which is inertial as determined by on board accelerometers at rest in S. In what follows, S will always be an inertial frame unless otherwise specified.

The modified form of the Navier Stokes Equation in Lagrangian Coordinates becomes:

$$12.8 \quad \rho(\underline{h}) \cdot \frac{d\underline{U}(\underline{h})}{dt} = -\nabla P = \sum_i \underline{F}_{ct}(\underline{h})_i + \sum_j \underline{F}_{fi}(\underline{h})_j \quad \text{where } \underline{h} = \underline{x} + \underline{\chi}(\underline{x}, t) \text{ and } \underline{\chi}(\underline{x}, 0) = 0.$$

$\sum_i \underline{F}_{ct}(\underline{h})_i$  represents the sum of the externally applied contact forces acting at point  $\underline{h}(\underline{x}, t)$  at time t, and  $\sum_j \underline{F}_{fi}(\underline{h})_j$  represents the sum of the field forces, ( $\frac{\text{dyn}}{\text{cm}^3}$ ), acting at point  $\underline{h}(\underline{x}, t)$  at time t where  $\underline{x} \neq \underline{f}(t)$  and  $\frac{d\underline{U}}{dt} = \frac{\partial \underline{U}}{\partial t} + (\underline{U} \cdot \nabla) \underline{U}$ .

FURTHER CORRECTIONS to the Navier Stokes Equation are made in sec. 5.

To sum up. In Lagrangian Coordinates which yield velocity and acceleration following the flow,

$$12.9 \quad \underline{h} = \underline{x} + \underline{\chi}(\underline{x}, t), \underline{U} = \dot{\underline{h}} = \dot{\underline{\chi}}(\underline{x}, t), \dot{\underline{U}} = \ddot{\underline{h}} = \ddot{\underline{\chi}}(\underline{x}, t), \underline{\chi}(\underline{x}, 0) = 0, \underline{x} \neq \underline{f}(t). \underline{x} \text{ arbitrary but fixed}$$

Newton's 3<sup>rd</sup> Law for a discrete, solid mass m, becomes:

$$12.10 \quad \underline{F} = \frac{d}{dt}(m\underline{U}(\underline{h})) \quad \text{where } \underline{h} = \underline{x} + \underline{\chi}(\underline{x}, t) \text{ and } \underline{\chi}(\underline{x}, 0) = 0.$$

Eulerian Coordinates yield velocity and acceleration at a fixed point in inertial frame S. Let  $\underline{x}_0$  be the fixed point in S. We wish to find the velocity and acceleration at  $\underline{x}_0$  at time  $t_0$ , given the flow parameters in Lagrangian coordinates:  $\underline{h} = \underline{x} + \underline{\chi}(\underline{x}, t)$ ,  $\underline{\chi}(\underline{x}, 0) = 0$ ,  $\dot{\underline{h}} = \dot{\underline{\chi}}(\underline{x}, t)$  and  $\ddot{\underline{h}} = \ddot{\underline{\chi}}(\underline{x}, t)$ .

At  $t=0$ ,  $\underline{h}$  satisfies the following,  $\underline{h} = \underline{x}_1 + \underline{\chi}(\underline{x}_1, 0) = \underline{x}_1$  and at time  $t_0$ ,  $\underline{h} = \underline{x}_0 = \underline{x}_1 + \underline{\chi}(\underline{x}_1, t_0)$   
 $\underline{U}(\underline{x}_0, t_0) = \dot{\underline{h}}(\underline{x}_0, t_0) = \dot{\underline{\chi}}(\underline{x}_1, t)|_{t_0}$  and  $\underline{Q}(\underline{x}_0, t_0) = \ddot{\underline{h}}(\underline{x}_0, t_0) = \ddot{\underline{\chi}}(\underline{x}_1, t)|_{t_0}$ . The Eulerian  
 coordinates for velocity and acceleration as functions of the Lagrangian coordinates are:

$$12.11 \quad \underline{U}(\underline{x}_0, t_0) = \dot{\underline{\chi}}(\underline{x}_1, t)|_{t_0}, \underline{Q}(\underline{x}_0, t_0) = \ddot{\underline{\chi}}(\underline{x}_1, t)|_{t_0} \text{ and } \underline{x}_0 - \underline{x}_1 = \int_{t_1}^{t_0} \dot{\underline{\chi}}(\underline{x}_1, t) dt$$

Eqns. 12.11, represent a point-to-point solution for velocity and acceleration at  $(\underline{x}_0, t_0)$ .  
 Specifically 12.11 represent the velocity and acceleration at  $\underline{x}_0$ , for time  $t_0$ .

As  $(\underline{U} \cdot \nabla) \underline{U}$  does not represent physical reality, a new theory for the physical cause of fluid turbulence in water and air and is developed based on a new model for the structure of liquid water and a new theory for the physical cause of the oceanic tides is needed. The new theory of fluid turbulence is developed in Sec. 8 and the new theory of the oceanic tides was developed in Chapter 11, Sec. 7.8.9.

## 2. Water Molecule Dimension and Force Between Water Molecules

It is assumed in the derivation of the Navier-Stokes Equations that solids and liquids are made of continuous matter. It is commonly thought among researchers in fluid flow that no major new physics will emerge if one considers liquids and solids to be made up of atoms instead of continuous matter. In what follows it will be shown that this is not the case, and that major new physics emerges if one considers liquids and solids to be made up of atoms and not made of continuous matter.

Consider a closed container of liquid water in pressure equilibrium with an atmosphere of air at  $T=293^\circ\text{K}=20^\circ\text{C}$ , at rest on the earth's surface.

With  $P=nKT$  and with  $n_a=2.48 \cdot 10^{19} \frac{1}{\text{cm}^3}$ , the pressure of air on the water surface  $P_a$  is

$P_a=n_aKT=10^6 \frac{\text{dy}}{\text{cm}^2}$  and if there were no bond force between water molecules and

with  $n_w=3.33 \cdot 10^{22} \frac{1}{\text{cm}^3}$ , and using 4.6, the pressure of water on the air is  $P_w=4 \cdot 10^{11} \frac{\text{dy}}{\text{cm}^2}$

and the water would explosively evaporate. This does not occur and therefore there is a bond force between water molecules. However removing the air above water in a

closed container down to the water vapor pressure  $P_v=2.34 \cdot 10^4 \frac{\text{dy}}{\text{cm}^2}$  causes the

water to explosively evaporate. External pressure (e.g. atmospheric pressure), the attractive field force  $f_{f,w-w}$  between water molecules, and the gravitational force are what cause water to remain in the liquid state.

Using 4.6, the average contact pressure  $P_{c,w-w}$  between 2 adjacent water molecules bonded together is  $P_{c,w-w} = \left( \frac{KT}{\bar{R}(2r_0 + \bar{R})} \right)^2$ .  $r_0$  is the effective radius of the water molecule and  $2r_0 + \bar{R}$  is

the average center to center distance of 2 adjacent water molecules. The average

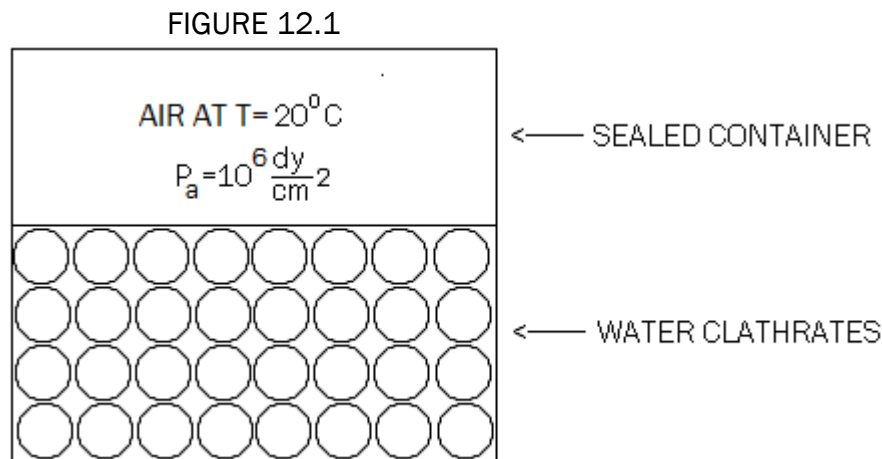
contact force  $\bar{f}_{c,w-w}$  between 2 adjacent water molecules is given by  $\bar{f}_{c,w-w} = P_{w-w}(2r_0 + \bar{R})^2 = \left( \frac{KT}{\bar{R}} \right)$ .

Atmospheric pressure, the attractive field force and gravity effect  $\bar{f}_{c,w-w}$  by decreasing  $\bar{R}$ .

As derived below,  $\bar{R} \approx 10^{-9}$  cm and with  $T = 293^{\circ}\text{K}$ ,  $\bar{f}_{c,w-w} \approx 10^{-5}$  dy. The force due to atmospheric pressure is,  $f_{at-w-w} = 10^6 (2r_0 + \bar{R})^2 \approx 10^{6-16} = 10^{-10}$  dy and the force due to gravity  $f_{g,w-w}$  is,  $f_{g,w-w} = \rho gh (2r_0 + \bar{R})^2 \approx 10^3 (2r_0 + \bar{R})^2 h \approx 10^{-13}$  h and even at the extreme depth of  $h = 10^6$  cm,  $f_{g,w-w} \approx 10^{-7}$  dy.

The attractive field force  $f(R)_{f,w-w}$  is hypothesized to be  $f(R)_{f,w-w} = -f_0 \exp(-\frac{R}{R_0})$ , where  $f_0$  and  $R_0$  are determined below.

The attractive force between water molecules results in a molecular structure called a clathrate. Figure 12.1. The clathrate is hypothesized to consist of spheres



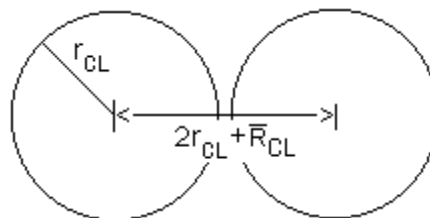
composed of tightly packed water molecules.

Considering the liquid water, let  $r_{CL}$  be the radius of the assumed spherical water clathrate;  $M_{CL}$  is its' mass with volume  $v_{CL} = \frac{4}{3}\pi r_{CL}^3$  and effective volume  $V_{CL} = (2r_{CL} + \bar{R}_{CL})^3$  where  $(2r_{CL} + \bar{R}_{CL})$  is the average distance between 2 clathrates, Figure 12.2.

Let  $N_W$  represent the number of water molecules in the clathrate. The mass  $m_W$  of 1 water molecule is  $3.01 \cdot 10^{-23}$  gm and  $\frac{M_{CL}}{V_{CL}} = \frac{m_W N_W}{(2r_{CL} + \bar{R}_{CL})^3} = 1 \frac{\text{gm}}{\text{cm}^3}$ .

Let  $L_i$  be the initial length of a vertical water column where  $L_i = N(2r_{CL} + \bar{R}_{CL})$  and  $N$  is the number of water clathrates along the length. Let  $L_f$  be the final length under compression

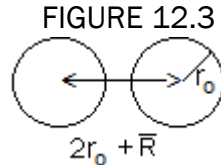
FIGURE 12.2



where  $L_f = N(2r_{CL})$  Experimentally it is shown that for compressive pressures up to the bulk modulus,  $\frac{L_f}{L_i} \doteq 1$  where  $B_{\Theta_{EX}} = 2.1 \cdot 10^{10} \frac{dy}{cm^2}$ . Consequently

$$\frac{L_f}{L_i} = \frac{1}{(1 + \frac{\bar{R}_{CL}}{2r_{CL}})} \doteq 1 \quad \text{and} \quad \frac{\bar{R}_{CL}}{2r_{CL}} \ll 1 \quad \text{and therefore} \quad \frac{M_{CL}}{V_{CL}} = 1 \doteq \frac{m_w N_w}{(2r_{CL})^3} \frac{gm}{cm^3}.$$

As above,  $(2r_o + \bar{R})$  is the average distance between 2 adjacent water molecules within the clathrate, Figure 12.3.



Compute the radius of the water molecule using  $\rho_w = 1 \frac{gm}{cm^3} = \frac{m_w}{(2r_o + \bar{R})^3} \cdot \frac{\frac{4}{3}\pi r_{CL}^3}{(2r_{CL})^3} = \frac{m_w}{1.9(2r_o + \bar{R})^3}$

$$\text{and } (2r_o + \bar{R}) = \left(\frac{m_w}{1.9}\right)^{\frac{1}{3}} = 2.5 \cdot 10^{-8} \text{ cm}$$

$$12.12 \quad (2r_o + \bar{R}) = 2.5 \cdot 10^{-8} \text{ cm}, \quad N_w \doteq 2.66 \cdot 10^{23} \cdot r_{CL}^3$$

In the following,  $\bar{R}(293)$  is computed using the speed of sound in water where  $2r_o + \bar{R}(293)$  is the center to center distance between two adjacent water molecules in the clathrate.

The speed of sound in water is given by (Eq. 4.14):

$$C_s = \left\{ \frac{(C_p + 3K) P_o}{C_p P_o} \right\}^{1/2} = \left\{ \frac{(C_p + 3K) KT}{C_p \bar{R} (2r_o + \bar{R})^2} \right\}^{1/2}. \quad \text{Solve for } \bar{R}(293) \text{ using the above}$$

derived value  $(2r_o + \bar{R}(293)) = 2.5 \cdot 10^{-8} \text{ cm}$ , and the experimentally determined values,

$$C_p = 9.06K, \quad m = 3.01 \cdot 10^{-23} \text{ gm}, \quad \text{and } C_s = 1.48 \cdot 10^5 \frac{cm}{sec}. \quad \text{This yields: } \bar{R}(293) = 3.8 \cdot 10^{-9} \text{ cm}.$$

Using  $\bar{R}(293)$ , find  $r_o = 1.1 \cdot 10^{-8} \text{ cm}$ . The accepted radius of the water molecule is  $1.4 \cdot 10^{-8} \text{ cm}$ ,

however  $1.4 \cdot 10^{-8} \text{ cm}$  is computed using the erroneous assumption that  $\rho_w = 1 \frac{gm}{cm^3} = \frac{m_w}{(2r_o + \bar{R}_w)^3}$

Instead of (from above), the correct assumption that  $\rho_w = 1 \frac{gm}{cm^3} = \frac{m_w}{(2r_o + \bar{R})^3} \cdot \frac{\frac{4}{3}\pi r_{CL}^3}{(2r_{CL})^3} = \frac{m_w}{1.9(2r_o + \bar{R})^3}$ .

Returning to the field force, consider a molecule in the interior of a clathrate. Molecule 2 is bonded to the six molecules to which it is in contact; two in the x direction, two in the y

direction and two in the z direction. Consider the two molecules, molecule 1 and 3 to which molecule 2 is bonded in the x direction. Ignoring the effects, for the moment, of the molecules in the y and z direction on the motion of molecule 2 in the x direction, the field force on molecule 2 in the x direction is:

$$\underline{f}(R_{12})_{f,21} + \underline{f}(R_{23})_{f,23} = -\underline{f}(R_{12})_{f,21} \hat{x}_{12} - \underline{f}(R_{23})_{f,23} \hat{x}_{32} = -\frac{m^2 H_{ch}}{(2r_o + R_{12})^2} \hat{x}_{12} + \frac{m^2 H_{ch}}{(2r_o + R_{23})^2} \hat{x}_{12}$$

and expanding in a Taylor Series yields:  $\underline{f}(R_D)_{f,T,2} \hat{x} \doteq \frac{m^2 H_{ch}}{4r_o^3} (R_{12} - R_{23}) \hat{x}_{12}$  where

$\underline{f}(R_D)_{f,T,2} \hat{x}$  is the total field force acting on molecule 2 in the  $\hat{x}$  direction ignoring the effects of the molecules in the y and z direction on the motion of molecule 2 in the x direction:  $R_D \equiv (R_{12} - R_{23})$ .  $H_{ch}$  is the chemical bond constant where  $H_{ch} \ll H = 1.0 \cdot 10^{30} \frac{\text{erg cm}}{\text{gm}^2}$ ;

All measurements are made from an inertial frame.

$$12.12A \quad \underline{f}(R_D)_{f,T,2} \hat{x} \doteq f_1 (R_{12} - R_{23}) \hat{x}_{12}, \quad f_1 \equiv \frac{m^2 H_{ch}}{4r_o^3}$$

Consider now a water molecule on the surface of a clathrate and on the water air interface subjected to collisions with atmospheric molecules. The water surface is in the x-y plane with the z direction positive upwards. Let molecule 1 represent a molecule on the surface of a clathrate. Molecule 1 is bonded to 4 water molecules to which it is in contact in the x-y plane and also bonded to molecule 2 directly beneath molecule 1 in the negative z direction. As an approximation to the field force acting on molecule 1, consider the 5 water molecules to which molecule 1 is bonded. Let the chemical bond constant  $H_{ch}$  be modified to  $H_1$  so that 5 times the field force of molecule 2 acting on molecule 1 in the  $-\hat{z}_{21}$  direction is approximately the same as the 5 molecules on molecule 1 with chemical bond constant  $H_{ch}$ . Let  $\underline{f}(R)_{f,52}$  represent the instantaneous vector field force of the 5 molecules bonded

to molecule 1 on the surface of the clathrate where  $\underline{f}(R_{12})_{f,52} = -5 \frac{m^2 H_1}{(2r_o + R_{12})^2} \hat{z}_{21}$

and expanding in a Taylor Series yields:  $\underline{f}(R_{12})_{f,52} \doteq -5 \frac{m^2 H_1}{(2r_o)^2} (1 - \frac{R_{12}}{r_o}) \hat{z}_{21}$ .

$\hat{z}_{21}$  is a unit vector pointing from the center of molecule 2 to the center of molecule 1 where  $2r_o + R_{12}$  is the center to center distance between molecule 2 and molecule 1. All measurements are made from an inertial frame.

$$12.12B \quad \underline{f}(R_{12})_{f,52} = -5f_o (1 - \frac{R_{12}}{r_o}) \hat{z}_{21}, \quad f_o \equiv \frac{m^2 H_1}{(2r_o)^2}$$

Using conservation of energy principles with  $R \equiv R_{12}$

$$12.12C \quad \Delta K.E. = \int_0^R \{f(r)_f\} dr = -5f_0 \left( r - \frac{1}{2} \frac{r^2}{r_0} \right) \Big|_0^R = -5f_0 R \left( 1 - \frac{1}{2} \frac{R}{r_0} \right), \quad 0 \leq R \leq R_M \leq r_0, \quad R_M = \text{const.}$$

$$\frac{1}{2} m U(R) = \frac{1}{2} m U(0)^2 - 5f_0 R, \quad U(R_M) = 0$$

With  $U(R_M) = 0$ ,  $\frac{1}{2} m U(0)^2$  becomes,  $\frac{1}{2} m U(0)^2 = 5f_0 R_M \left( 1 - \frac{1}{2} \frac{R_M}{r_0} \right)$  and 12.12C becomes:

$$12.12D \quad \Delta K.E. = -5f_0 R \left( 1 - \frac{1}{2} \frac{R}{r_0} \right), \quad 0 \leq R \leq R_M \leq r_0$$

$$\frac{1}{2} m U(R)^2 + 5f_0 R \left( 1 - \frac{1}{2} \frac{R}{r_0} \right) = 5f_0 R_M \left( 1 - \frac{1}{2} \frac{R_M}{r_0} \right)$$

The average K.E. on the interval  $0 \leq R \leq R_M$  is:

$$\overline{K.E.} = \frac{5}{R_M} \int_0^{R_M} f_0 R \left( 1 - \frac{1}{2} \frac{R}{r_0} \right) dr + 5f_0 R_M \left( 1 - \frac{1}{2} \frac{R_M}{r_0} \right) = 5 \frac{f_0 R_M}{2} \left( 1 - \frac{2}{3} \frac{R_M}{r_0} \right).$$

Note that

$$\overline{K.E.} < \frac{1}{2} m U(0)^2.$$

With  $\overline{K.E.}(T) = 5 \frac{f_0 R_M}{2} \left( 1 - \frac{2}{3} \frac{R_M}{r_0} \right) = \frac{1}{2} \cdot \frac{3}{2} k \cdot T$  and  $\overline{K.E.}(373^\circ) = 5 \frac{f_0 r_0}{6} = \frac{3}{4} k \cdot 373^\circ = 3.86 \cdot 10^{-14}$  erg.

Solve for  $f_0$  to find,  $f_0 = 4.2 \cdot 10^{-6}$  dy.

$$12.12E \quad f(R)_f = -4.2 \cdot 10^{-6} \left( 1 - \frac{R}{r_0} \right), \quad f_0 = 4.2 \cdot 10^{-6} \text{ dy.}$$

$$\int_{R_{12}}^{R_{52}} f_{52} = -2.1 \cdot 10^{-5} \left( 1 - \frac{R_{12}}{r_0} \right) \Big|_{21}^{21}$$

The energy necessary to heat water from  $293^\circ\text{K}$  to  $373^\circ\text{K}$  consists of three parts; The work  $W_f$  done against the field force, the energy,  $E_{KE}$ , necessary to increase the temperature by  $80^\circ\text{K}$  and the energy changing the internal energy of the water molecule  $mC_1$  i.e.  $W_{m\Delta C_1}$ . See 3.16. The total work  $W_T$  is  $W_T = W_f + E_{KE} + W_{m\Delta C_1}$ .

Using 12.12E, the work done against the field force  $W_f$  is:

$$12.12F \quad W_f = -5 \int_{\bar{R}(293)}^{r_0} \{f(r)_f\} dr = 5f_0 r \left( 1 - \frac{1}{2} \frac{r}{r_0} \right) \Big|_{\bar{R}(293)}^{r_0} = 5.0 \cdot 10^{-14} \text{ erg}$$

and the energy,  $E_{KE}$ , necessary to increase the temperature by  $80^\circ\text{K}$  is:

$$E_{KE} = \frac{3}{2} k(80) = 1.66 \cdot 10^{-14} \text{ erg. } W_T \text{ becomes:}$$

$$12.13 \quad W_T = W_f + E_{KE} + W_{m\Delta C_1} = 5.0 \cdot 10^{-14} + 1.7 \cdot 10^{-14} + W_{m\Delta C_1} = 6.7 \cdot 10^{-14} + W_{m\Delta C_1} \text{ erg}$$

The experimentally determined specific heat of a water molecule between  $293^\circ\text{K}$  and  $373^\circ\text{K}$  is  $9.06 \text{ cal} = 1.25 \cdot 10^{-15} \frac{\text{erg}}{\text{K}^\circ}$ . The total experimentally determined energy  $W_{\text{ex}}$  to take a water molecule from  $293^\circ\text{K}$  to  $373^\circ\text{K}$  is:  $W_{\text{ex}} = 1.25 \cdot 10^{-15} (80) = 1.0 \cdot 10^{-13} \text{ erg}$ . The difference  $W_{\text{ex}} - W_T = W_{m\Delta C_1} = 3.3 \cdot 10^{-14} \text{ erg}$  goes into increasing the internal energy of the water molecule. The change in radius  $\Delta r_o$  of the water molecule due to an increase in internal energy of  $3.3 \cdot 10^{-14} \text{ erg}$  is  $\Delta r_o = 4.3 \cdot 10^{-15} \text{ cm}$ . See chap.3, sec. 8.

Compiling Results for a water molecule on the air clathrate interface:

$$12.14 \quad \begin{aligned} \text{a. } \Delta \text{K.E.} &= -2.1 \cdot 10^{-5} R_{12} \left(1 - \frac{R_{12}}{2.2 \cdot 10^{-8}}\right) \Big|_{R_{\min}}^{R_{\max}} - 5f_o R \left(1 - \frac{R_{12}}{2r_o}\right) \\ \text{b. } \frac{1}{2} m U^2(R_{12}) + 2.1 \cdot 10^{-5} R_{12} \left(1 - \frac{R_{12}}{2.2 \cdot 10^{-8}}\right) &= \frac{1}{2} m U^2(0) \\ \text{c. } \frac{1}{2} m U^2(0) &= 2.1 \cdot 10^{-5} R_{\max} \left(1 - \frac{R_{\max}}{2.2 \cdot 10^{-8}}\right), \quad U^2(R_{\max}) = 0 \\ \text{d. } U(0) \text{ es} \Big|_{373^\circ} &= \left(5 \frac{f_o r_o}{m}\right)^{\frac{1}{2}} = 0.88 \cdot 10^5 \frac{\text{cm}}{\text{sec}}, \quad R_{\max} \Big|_{373^\circ} = r_o \\ \text{e. } H_1 &= \frac{(2r_o)^2 f_o}{m^2} = 2.26 \cdot 10^{24} = 2.26 \cdot 10^{-6} \text{ H} \frac{\text{erg cm}}{\text{gm}^2} \\ \text{f. } W_{\text{ex}} - W_T &= W_{m\Delta C_1} = 3.3 \cdot 10^{-14} \text{ erg} \end{aligned}$$

Boiling is considered in section 4.

### 3. Dimensions of Water Clathrate

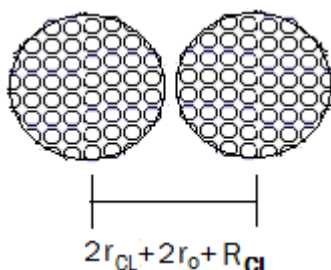
In what follows the experimenter is at sea level and the surrounding atmosphere is at STP. In order to account for the pressure equilibrium across the air-water interface and the fact that water is difficult to compress but can be parted with one's hand, and the fact that water is a good solvent; our model for a clathrate consists of water molecules held together in spherical clathrates, fig. 12.1 by field force  $\underline{f}(R)_{f,12}$ , within the clathrate and with still to be determined, clathrate to clathrate force  $\underline{f}(R_{CL})_{f,cl-cl}$ . The center to center distance between two adjacent clathrates is  $2r_{CL} + \bar{R}_{CL}$  and as above, the center to center distance between two adjacent water molecules



in a clathrate is  $2r_o + \bar{R}$ . To avoid future confusion,  $\bar{R}_{w-w}$  will only be used for water molecules that are in the clathrate.

Two adjacent clathrates are bound together by water molecules on the surface of clathrate 1 bonded to water molecules on the surface of adjacent clathrate 2. A 2 dimensional representation of 2 spherical water clathrates made up of water molecules bound to one another is given in figure 12.4

FIGURE 12.4



The force binding water molecule A on the surface of clathrate 1 to 1 molecule on the surface of adjacent clathrate 2 is:  $f(R_{CL})_{f,cl-cl} = f_o(1 - \frac{R_{CL}}{1.1 \cdot 10^{-8}}) = -4.2 \cdot 10^{-6}(1 - \frac{R_{CL}}{1.1 \cdot 10^{-8}})$

The total field force holding clathrate 1 and 2 together is:  $N_T f(R_{CL})_{f,cl-cl} = -4.2 \cdot 10^{-6} N_T (1 - \frac{R_{CL}}{1.1 \cdot 10^{-8}})$  where  $N_T$  is the total number of molecules on the surface of clathrate 1 bonded to an equal number of molecules on the surface of clathrate 2, each molecule bound to 1 molecule on the opposite surface.

Using 4.6, the average pressure  $P_{cl-cl}$  that 2 clathrates exert on one another is:

$$P_{cl-cl} = \frac{KT}{(2r_{CL})^2 \bar{R}(T)_{CL}} = 10^6 \frac{dy}{cm}^2, \text{ and assuming } \bar{R}(293)_{CL} = \bar{R}(293)_{w-w} \text{ where}$$

$$\bar{R}(293)_{w-w} = 3.8 \cdot 10^{-9} \text{ cm, find } r(293)_{CL} = 1.6 \cdot 10^{-6} \text{ cm.}$$

$$P_{cl-cl} \text{ is also equal to; } P_{cl-cl} = \frac{N_T \bar{f}(R_{CL})_{f,cl-cl}}{(2r_{CL})^2} = \frac{4.2 \cdot 10^{-6} N_T}{(2r_{CL})^2} (1 - \frac{\bar{R}_{CL}}{2.2 \cdot 10^{-8}}) = 10^6 \text{ solve for}$$

$$N_T \text{ and find: } N_T = 2.9 \approx 3.0$$

A table of values of  $r_{CL}$  is given in table 12,  $N_w$  = number of water molecules per clathrate.

TABLE 12 FOR  $\bar{R}(293)_{CL}$ ,  $r_{CL}$ ,  $m_{CL}$ ,  $N_W$ 

$P_{cl-cl} \frac{dy}{cm^2}$	$\bar{R}(293)_{CL} cm$	$r_{CL} cm$	$m_{CL} gm$	$N_W$
$10^6$	$3.8 \cdot 10^{-9}$	$1.6 \cdot 10^{-6}$	$3.3 \cdot 10^{-17}$	$1.1 \cdot 10^6$

By direct computation, the effective density of a clathrate is:

$$1 \frac{gm}{cm^3} = \frac{m_{CL}}{V_{eff}} = \frac{\frac{m_W \frac{4}{3} \pi r_{CL}^3}{(2r_o + \bar{R}_{w-w})^3}}{(2r_{CL})^3} = \frac{4.2 m_W}{8(2r_o + \bar{R}_{w-w})^3} = 1.$$

The momentum of the clathrates in still water has implications for Brownian motion.

A. In the following, the Bulk Modulus of Water is computed.

The isothermal bulk modulus  $B_\Theta$  is defined by  $B_\Theta = -V \left( \frac{dP}{dV} \right)_\Theta$  where  $P_E$  is the total external pressure applied to a solid, liquid or gas sample. The pressure between 2 adjacent water molecules in a clathrate is:  $P_{WW} = \frac{KT}{\bar{R}_W(2r_o + \bar{R}_W)^2} = \frac{KT}{(V^{\frac{1}{3}} - 2r_o)V^{\frac{2}{3}}} = 2.0 \cdot 10^{10} \frac{dy}{cm^2}$  and

$$B_\Theta = KT \left\{ \frac{2}{3} \frac{1}{(V^{\frac{1}{3}} - 2r_o)V^{\frac{2}{3}}} + \frac{1}{3} \frac{1}{(V^{\frac{1}{3}} - 2r_o)^2 V^{\frac{1}{3}}} \right\} = KT \left\{ \frac{\frac{2}{3} r_o + \bar{R}_W}{(2r_o + \bar{R}_W)^2 \bar{R}_W} \right\}$$

Using 12.12,  $(2r_o + \bar{R}(293)_{w-w}) = 2.5 \cdot 10^{-8} cm$ ,  $r_o = 1.1 \cdot 10^{-8} cm$ , and  $\bar{R}(293)_{w-w} = 3.8 \cdot 10^{-9} cm$ .

Compute:  $B_\Theta = 4.9 \cdot 10^{10} \frac{dy}{cm^2}$

$B_\Theta$  is 2.3 times the experimental value  $B_{\Theta_{Ex}} = 2.1 \cdot 10^{10} \frac{dy}{cm^2}$ . However, the experimental value  $B_{\Theta_{Ex}}$  has been directly determined by compressing a water sample isothermally in an enclosed pressure cell and measuring the change of length  $(-\Delta L)_{ex}$  as a function of applied pressure. The basic measurement  $(-\Delta L)_{ex}$ , is made by measuring the electrical resistance in a Nirex wire between a point of attachment on the pressure piston and a fixed point on the pressure casing: Once at external pressure  $P_{ex} = 0 \frac{dy}{cm^2}$  and again at a final external pressure  $P_{ex,f}$ . Given the difficulty in making consistent electric resistance measurements at high pressure, it is understandable that human error may account for the experimentally determined value,  $B_{\Theta_{Ex}} = 2.1 \cdot 10^{10} \frac{dy}{cm^2}$ .

The basic measurement  $(-\Delta L)_{ex}$ , should be made using x-ray negatives, one taken at external pressure  $P_{ex} = 0 \frac{dy}{cm^2}$  and one taken at final external pressure  $P_{ex,f}$ .

B. Given a spherical stationary drop of water at the mouth of a faucet, what is the radius  $r_{dr}$  of the spherical drop of water that can be supported by the annular area  $A_{an}=\pi(r_o^2-r_i^2)$  where  $r_o$  is the outer radius of the faucet mouth and  $r_i$  is the inner radius of the mouth of the faucet?

The number of clathrates that will fit into  $A_{an}$  is  $\frac{A_{an}}{(2r_{cl})^2}=9.8 \cdot 10^{10} A_{an}$  and the total average force  $\bar{f}_{an}$  holding up the spherical stationary drop of water is:

$$\bar{f}_{an}=(9.8 \cdot 10^{10} A_{an}) \bar{f}_{cl-cl}=(9.8 \cdot 10^{10} A_{an})(2.1 \cdot 10^{-5})=2.1 \cdot 10^6 A_{an}=M_{dr}g, \text{ with } M_{dr}=2.1 \cdot 10^3 A_{an}g$$

with drop volume,  $V_{dr}=2.1 \cdot 10^3 A_{an} \text{ cm}^3$ . With  $r_o=0.5\text{cm}$  and  $r_i=0.2\text{cm}$ , the radius of the drop becomes:  $r_{dr}=7.9(A_{an})^{\frac{1}{3}}=7.9(\pi(r_o^2-r_i^2))^{\frac{1}{3}}=7.9(.66)^{\frac{1}{3}}=6.9\text{cm}$ .  $r_{dr}=6.9\text{cm}$  is much larger than the observed drop radius  $r_{dr} \sim 0.5\text{cm}$ . The clathrate to metal

faucet force  $\bar{f}_{cl-m}$  is therefore  $\bar{f}_{cl-m} \ll \bar{f}_{cl-cl}$  and:  $\bar{f}_{an}=(9.8 \cdot 10^{10} (.66)) \bar{f}_{cl-m}=M_{dr}g=490 \text{ dy}$  with  $\bar{f}_{cl-m}=0.75 \cdot 10^{-8} \text{ dy}$ .

#### 4. Vapor Pressure, Water to Steam, the Latent Heat of Vaporization, Evaporation

##### a. Vapor Pressure

As air pressure over water is lowered to the vapor pressure, the water explosively vaporizes. At  $293^{\circ}\text{K}$ , the vapor pressure of water is  $2.3 \cdot 10^4 \frac{\text{dy}}{\text{cm}^2}$ . Using 4.6, the average pressure  $P_{cl-cl}$  that 2 clathrates exert on one another is:

$$P_{cl-cl}=\frac{KT}{(2r_{cl})^2 \bar{R}(T)_{cl}}=\frac{10^{-14}}{(r_{cl})^2 \bar{R}(T)_{cl}} \text{ and at } P_{cl-cl}=2.3 \cdot 10^4 \frac{\text{dy}}{\text{cm}^2}:$$

$$f(R_{cl})f_{cl-cl}=-f_o(1-\frac{R_{cl-cl}}{1.1 \cdot 10^{-8}})=f(R)_{f,w-w}=-f_o(1-\frac{R_{w-w}}{1.1 \cdot 10^{-8}})=0 \text{ and } R_{cl-cl}=R_{w-w}=1.1 \cdot 10^{-8} \text{ cm}$$

and the water explosively vaporizes drawing heat energy from the unvaporized water. If the process is continued, the unvaporized water turns to ice.

With  $\bar{R}(T)_{cl}=1.1 \cdot 10^{-8} \text{ cm}$  and  $P_{cl-cl}=\frac{10^{-14}}{(r_{cl})^2 \bar{R}(T)_{cl}}=2.3 \cdot 10^4 \frac{\text{dy}}{\text{cm}^2}$ , find  $r_{cl}=6.3 \cdot 10^{-6} \text{ cm}$

TABLE 12A FOR  $\bar{R}(293)_{cl}$ ,  $r_{cl}$ ,  $m_{cl}$ ,  $N_w$

$P_{cl-cl} \frac{\text{dy}}{\text{cm}^2}$	$\bar{R}(293)_{cl} \text{ cm}$	$r_{cl} \text{ cm}$	$m_{cl} \text{ gm}$	$N_w$
$10^6$	$3.8 \cdot 10^{-9}$	$1.6 \cdot 10^{-6}$	$3.3 \cdot 10^{-17}$	$1.1 \cdot 10^6$
$2.3 \cdot 10^4$	$1.1 \cdot 10^{-8}$	$6.3 \cdot 10^{-6}$	$3.3 \cdot 10^{-17}$	$1.1 \cdot 10^6$

The question remains, why isn't decreasing atmospheric pressure to the vapor pressure sufficient to vaporize water without cooling the un-vaporized water.

The answer is that at the instant a water molecule reaches  $R_{CL-CL}=1.1 \cdot 10^{-8}$  cm with  $f(R_{CL})f_{CL-CL}=0$ , the water molecules speed is zero and the energy it takes to increase its temperature to that of the remaining liquid water comes from the liquid water where the energy required is initially  $C_V \Delta T = 1.25 \cdot 10^{-15} (293) = 3.7 \cdot 10^{-13}$  erg.

#### b. Water to Steam, the Latent Heat of Vaporization

The latent heat of vaporization per water molecule,  $e_V$ , for water at  $100^\circ\text{C}$  at 1atms., pressure is:  $e_V = 0.730 \cdot 10^{-12} \frac{\text{erg}}{\text{W}} = 5.29 \cdot 10^3 \text{K} = 0.456 \frac{\text{eV}}{\text{W}}$ . Note that the specific heat of water between  $0^\circ$  and  $99.9^\circ$  degrees C is  $9.06 \text{K} = 1.25 \cdot 10^{-15} \frac{\text{erg}}{\text{deg}}$ .

The total energy  $E_T$  necessary to bring water from  $293^\circ\text{K}$  to water vapor at  $373^\circ\text{K}$  at 1atmos. is,  $E_T = e_V + C_V \Delta T = 0.730 \cdot 10^{-12} + 1.25 \cdot 10^{-15} (80) = 0.830 \cdot 10^{-12} \frac{\text{erg}}{\text{W}}$ . The

energy  $e_V$  is  $e_V = 0.730 \cdot 10^{-12} \frac{\text{erg}}{\text{W}} = m_w \Delta C_1 + .772 \cdot 10^{-13}$  where  $m_w \Delta C_1$  represents the increase in internal energy of one water molecule on going from water at  $100^\circ\text{C}$  at 1 atm to water vapor at  $100^\circ\text{C}$  at 1atms. see 3.27, and where  $m_w \Delta C_1 = 0.65 \cdot 10^{-12} \frac{\text{erg}}{\text{W}}$

The energy  $0.0772 \cdot 10^{-12} \text{erg} = \frac{3}{2} kT|_{373}$  is the energy necessary to increase the kinetic energy of the just released water molecule from 0 to  $\frac{3}{2} kT|_{373}$ .

#### c. Evaporation

In the following it is understood that water temperature and air temperature are at  $T=293^\circ\text{K}$  with no wind and the water in a closed windowless room. The absolute humidity is considered as a variable, where experimentally in earth's atmosphere,  $0 < \text{A.H.} < 3.0 \cdot 10^{-5} \frac{\text{gm}}{\text{cm}^3} = 1.0 \cdot 10^{18} \frac{\text{W.M.}}{\text{cm}^3}$ . The absolute humidity controls the measured evaporation rate by controlling the rate at which water molecules in the air strike the water surface and are absorbed on contact with the water surface. Thus even in ideal conditions where water temperature and air temperature are at  $T=293^\circ\text{K}$  with no wind and in a closed room, the evaporation rate will be a variable dependent on the absolute humidity. Under these conditions the evaporation rate will vary from 0 to 10mm of water per 24 hours which equals  $1.2 \cdot 10^{-5} \frac{\text{cm}}{\text{sec}}$ . With  $N_W \equiv$  number of escaped water molecules, the evaporation rate  $R_E$  is:

$0 < R_E < 1.2 \cdot 10^{-5} (2r_0 + \bar{R}) = 1.2 \cdot 10^{-5} (2.5 \cdot 10^{-8}) = 480 \frac{N_W}{\text{wsec}}$ , read as number of escaped water molecules per second per water molecule on the air water surface bound to the clathrate.

The operational area of a water clathrate is  $A_{CL} = ((2)1.6 \cdot 10^{-6})^2 = 1.0 \cdot 10^{-11} \text{ cm}^2$  and  $R_E$  becomes:  $0 \leq R_E \leq 480 \frac{A_{CL}}{A_W} = 7.7 \cdot 10^6 \frac{N_W}{\text{SEC CL}}$ , read as number of escaped water molecules per second per clathrate for clathrates on the air water surface.

Using 12.12F and 12.13 the total energy it takes to remove a water molecule at  $T=293^\circ \text{ K}$  at atmospheric pressure at the air water interface from a clathrate resulting in a water molecule free from the clathrate floating in air at  $T=293^\circ \text{ K}$  at atmospheric pressure is:  $W_{T,W} = W_f + E_{KE} + W_{m\Delta C_1} = W_f + e(293)_V$  were experimentally the latent heat

of vaporization at  $293^\circ \text{ K}$  is  $7.6 \cdot 10^{-13} \frac{\text{erg}}{W}$  read as erg per escaped water molecule.

$W_{T,W}$  becomes,  $W_{T,W} = 5.0 \cdot 10^{-14} + 7.6 \cdot 10^{-13} = 8.1 \cdot 10^{-13} \frac{\text{erg}}{W}$ . The total energy per second per clathrate is,  $Q = W_{T,W} R_E = (8.1 \cdot 10^{-13}) R_E \frac{\text{erg}}{\text{SEC CL}}$  where the power comes

from a temperature gradient  $\Delta T(z)$  in the water where  $\Delta T(z) = \frac{Q \Delta L}{K \cdot A}$ , z Positive

Downwards. As applied to our model for water,  $\frac{Q}{A}$  is the energy per sec as measured crossing area A parallel to the plane of the air-water interface.  $\Delta L$  is a chosen length in the z direction normal to the plane of the air-water interface and into the fluid, and K is the coefficient of thermal conductivity of water where  $K = 0.606 \cdot 10^5 \frac{\text{erg}}{\text{sec cm K}^\circ}$ .

With  $Q = (8.1 \cdot 10^{-13}) R_E \frac{\text{erg}}{\text{SEC CL}}$  and  $A = 1.0 \cdot 10^{-11} \text{ cm}^2$ ,  $\Delta L = 1 \text{ cm}$ ,  $\Delta T(z)$  becomes,

$\Delta T(z) = (1.3 \cdot 10^{-6}) R_E K^\circ \frac{1}{\text{cm}}$ . Evaluating  $R_E$  for an evaporation rate of 1mm per 24

hours yields  $R_E = 7.7 \cdot 10^5 \frac{N_W}{\text{SEC CL}}$  resulting in  $\Delta T(z) = 1.0 K^\circ \frac{1}{\text{cm}}$ .

$\Delta T(z)$  represents the temperature gradient necessary so that the water surface will emit  $7.7 \cdot 10^5$  independent water molecules per second per clathrate which is the equivalent of 1mm of water per 24 hours.

In general for an evaporation rate, ER, of  $0 < ER < 10 \text{ mm}$  of water per 24 hours requires a  $\Delta T(z)$  of:  $0 < \Delta T(z) < 10 K^\circ \frac{1}{\text{cm}}$ .

At the molecular level, the power  $W_{T,CL}$  is carried by three water molecules binding

two clathrates in the z direction. With  $V_{rms} = 6.3 \cdot 10^4 \frac{\text{cm}}{\text{sec}}$  and  $\bar{R}(293)_{CL} = 3.8 \cdot 10^{-9} \text{ cm}$  the collision frequency of the three molecules on clathrate A with their opposite

number on clathrate B is:  $\frac{1}{\tau} = 8.3 \cdot 10^{12} \frac{\text{collision}}{\text{sec}}$  and each of the three water

molecules carries  $0 < \frac{1}{3} Q \cdot \tau = E(\text{carry})_{CL-CL} < 3.2 \cdot 10^{-26} \frac{\text{erg}}{\text{carry}}$ . As there are more molecules within the clathrate than the three molecules binding two

clathrates together and with  $\overline{R(293)}_{CL} = \overline{R(293)}_{W-W}$  it follows that  $E(\text{carry})_{W-W} < E(\text{carry})_{CL-CL}$  where  $E(\text{carry})_{W-W}$  is the energy per carry within the clathrate.

## 5. Rewriting the Navier-Stokes Equation

As normally written, the Navier-Stokes Equations are:  $\rho(\underline{x}, t) \cdot \frac{d\underline{U}}{dt} = -\nabla P + (\underline{U} \cdot \nabla) \underline{U} + \sum \underline{F}_i$ . This is wrong on three counts.

1.  $\underline{x}$  is assumed to be a function of time  $t$ :  $\underline{x} = \underline{x}(\underline{x}_0, t)$ ,  $\underline{x}(\underline{x}_0, 0) = \underline{x}_0$ ,
2. The sum of the external forces term,  $\sum \underline{F}_i \frac{dy}{cm^3}$  is incorrectly placed.
3. The  $-\nabla P$  is incorrect for fluids where the pressure at a point like volume, is dependent on flow direction and is replaced by  $-\left\{ \frac{\partial P_x}{\partial h_x} \hat{x} + \frac{\partial P_y}{\partial h_y} \hat{y} + \frac{\partial P_z}{\partial h_z} \hat{z} \right\}$  in the Lagrangian form, where  $\underline{P}(\underline{h}) = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}$ ,  $\underline{h} = \underline{h}(\underline{x}, t) = \underline{x} + \underline{\chi}(\underline{x}, t)$

As shown in section 1 of the present chapter, the derivation of the  $(\underline{U} \cdot \nabla) \underline{U}$  term is physically false and does not represent physical reality. The term is however used in the derivation of the oceanic tides and the derivation of mathematical fluid turbulence. The correct derivation of the oceanic tides was derived in Chap. 11, Sec. 7, 8, 9 and the new derivation of the fluid turbulence is derived in Sec. 8.

As regards the  $\sum \underline{F}_i$  term. From Newton's 3<sup>rd</sup> Law,  $\rho(\underline{h}) \cdot \frac{d^2 \underline{h}}{dt^2} = \sum \underline{F}(\underline{h})_i$ . The pressure term is a result of external forces and is not an innate field force such as gravity or a contact force such as a propeller in water. Consequently the pressure term is on the wrong side of the = sign. Without external forces there is no pressure term and (incidentally no liquid water) and with a pressure term, there are external forces.

As regards the  $-\nabla P$  term. If the pressure at a point like volume is independent of direction, then the derivation of the  $-\nabla P$  term is correct. However if

$\underline{P}(\underline{h}) = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}$  i.e. the pressure at a point like volume is dependent on direction, then  $\underline{P}(\underline{h})$  replaces  $P$  in the derivation of the force per unit volume acting on a point like mass.

The Navier Stokes Equation is a generalization of Newton's Third Law  $\underline{F} = m \underline{a}$  written for a continuous medium and as such, an unbalanced force in the  $x_i$  direction creates an acceleration in the  $x_i$  direction and no acceleration in the  $x_j$  direction.

By direct computation the Lagrangian form of the Navier Stokes Equation following the flow becomes:

$$12.14A \quad \rho(\underline{h}) \cdot \frac{d\underline{U}(\underline{h})}{dt} = \sum \underline{F}(\underline{h})_i = -\left\{ \frac{\partial P_x}{\partial h_x} \hat{x} + \frac{\partial P_y}{\partial h_y} \hat{y} + \frac{\partial P_z}{\partial h_z} \hat{z} \right\}, \quad \underline{P}(\underline{h}) = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}$$

$$\underline{h} = \underline{h}(\underline{x}, t) = \underline{x} + \underline{\chi}(\underline{x}, t) \quad \text{with } \underline{\chi}(\underline{x}, 0) = 0, \quad \frac{d\underline{x}}{dt} = 0$$

And the Eulerian form of the Navier Stokes Equation computing the flow parameters at a fixed point in the flow is:

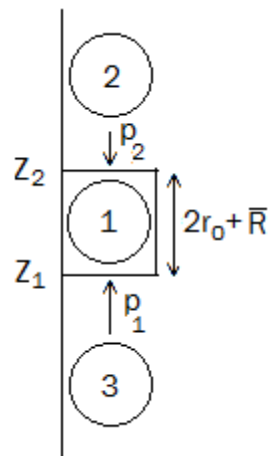
$$12.13B \quad \rho(\underline{x}, t) \cdot \frac{d\underline{U}(\underline{x}, t)}{dt} = \sum \underline{F}(\underline{x}, t)_i = -\left\{ \frac{\partial P_x}{\partial x} \hat{x} + \frac{\partial P_y}{\partial y} \hat{y} + \frac{\partial P_z}{\partial z} \hat{z} \right\},$$

$$\underline{P}(\underline{x}, t) = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}, \quad \underline{x} \neq \underline{x}(t) \quad \text{i.e. } \frac{d\underline{x}}{dt} = 0$$

The equation of state for water is derived in chapter 12, sec 8 and substituted for  $\underline{P}(\underline{h})$  in 12.14A. and for  $\underline{P}(\underline{x}, t)$  in 12.13B. Notice the absence of the  $(\underline{U} \cdot \nabla) \underline{U}$  term as it is dependent on the physically false assumption that  $\underline{x}$  is a function of t (Chap. 12, sec 1).

Consider a fluid at rest under the influence of gravity near the earth's surface such as water in a glass on a stationary table top.  $-g \hat{z}$  is the local acceleration of gravity and in this case as the pressure in all directions at a given point in the fluid is uniform, it is permissible to use scalar P in,  $\underline{F} = -\rho(\underline{h}) g \hat{z} \frac{dy}{cm^3} = -\nabla P$ .

FIGURE 12.5A



How is it that an external force acting in the  $-\hat{z}$  direction can create a force acting in the  $+\hat{z}$  direction to support stationary water in a glass of water?

It is the rebound of neutral charge, solid mass atoms and molecules from the solid earth, conserving energy and momentum, that enable a field force per unit volume  $\underline{F} = -\rho g \hat{z}$  in the  $-\hat{z}$  direction, to cause a contact force in the  $+\hat{z}$  direction on water molecules within a clathrate. Using Fig 12.5A and 12.12,  $r_0$  is the effective radius of

the water molecule,  $r_0=1.1 \cdot 10^{-8}$  cm and  $\bar{R}(293)=3.8 \cdot 10^{-9}$  cm is the average distance traveled by atom 1 between collisions at  $Z_1$  and  $Z_2$ .

As an aside: In the transition metals  $\bar{R} \approx 10^{-10}$  cm. It is commonly claimed by many chemists and physicists that for solids  $\bar{R} \approx 10^{-8}$  cm. Using the  $\bar{R} \approx 10^{-8}$  cm value to compute the mass density of solid materials such as the transitions metals yields a density too small by a factor of  $\frac{1}{3}$  while using the figure  $\bar{R} \approx 10^{-10}$  cm yields the experimentally determined mass density value as computed in chap. 4, sec 2.

Considering Fig 12.5A, to represent atoms in water; from chap.4, sec 3 the average pressure that two water molecules in a clathrate exert on one another at  $Z_2$  is:

$$12.15 \quad P(293) = \left(\frac{1}{\bar{R}}\right) \frac{KT}{(2r_0 + \bar{R})^2} = 1.7 \cdot 10^{10} \frac{\text{dy}}{\text{cm}^2}$$

For the case under consideration under the influence of gravity, the time  $t_d$  taken by atom 1 to fall a distance  $\bar{R}$  is:  $\bar{R} = U_{\text{rms},z,i} t_d + \frac{1}{2} g t_d^2$  with solution

$$t_d = \frac{U_{\text{rms},z,i}}{g} \left\{ \left[ 1 + 2 \frac{\bar{R}g}{U_{\text{rms},z,i}^2} \right]^{\frac{1}{2}} - 1 \right\} \text{ where } U_{\text{rms},z,i} \text{ is the rms speed of atom 1 in the } -\hat{z}$$

direction when the atom is at  $Z_2$  directly after rebound at time  $0^+$ , where

$$U_{\text{rms},i}^2 = U_{\text{rms},x,i}^2 + U_{\text{rms},y,i}^2 + U_{\text{rms},z,i}^2 \text{ and } \frac{1}{3} U_{\text{rms},i}^2 = U_{\text{rms},x,i}^2 = U_{\text{rms},y,i}^2 = U_{\text{rms},z,i}^2$$

Let  $U_{\text{rms},z,f}$  represent the rms speed of atom 1 in the  $-\hat{z}$  direction when the atom is at  $Z_1$  directly before the collision with atom 3 at time  $t_d^-$  where:

$$12.15A \quad U_{\text{rms},z,f} = U_{\text{rms},z,i} + g t_d = U_{\text{rms},z,i} + U_{\text{rms},z,i} \left\{ \left[ 1 + 2 \frac{\bar{R}g}{U_{\text{rms},z,i}^2} \right]^{\frac{1}{2}} - 1 \right\} = U_{\text{rms},z,i} \left[ 1 + 2 \frac{\bar{R}g}{U_{\text{rms},z,i}^2} \right]^{\frac{1}{2}}$$

Note that  $t_d = t_u$  where  $t_u$  is the time taken for atom 1 after collision with atom 3 to go up the distance  $\bar{R}$  and collide with atom 2. In the derivation of 12.15 the down and back time  $t_d + t_u = 2t_d$  is used, however in the derivation of 12.15A the down time  $t_d$  is used. The average pressure is now averaged over  $t_d$  not  $2t_d$  with resultant pressure

$$P(Z_1) = \left(\frac{1}{2\bar{R}}\right) \frac{KT_f}{(2r_0 + \bar{R})^2} \text{ and } P(Z_2) = \left(\frac{1}{2\bar{R}}\right) \frac{KT_i}{(2r_0 + \bar{R})^2}. \text{ Derive } \frac{T_f}{T_i} = \frac{U_{\text{rms},f}^2}{U_{\text{rms},i}^2} = \frac{U_{\text{rms},z,f}^2}{U_{\text{rms},z,i}^2} = \left[ 1 + 2 \frac{\bar{R}g}{U_{\text{rms},z,i}^2} \right] = \frac{P(Z_1)}{P(Z_2)}$$

and with  $U_{\text{rms},z,i}^2 = \frac{1}{3} U_{\text{rms},i}^2 = \frac{KT_i}{m}$ .  $T_f$  becomes:  $T_f = \left[ 1 + 2 \frac{\bar{R}g}{U_{\text{rms},z,i}^2} \right] T_i = \left( 1 + 2 \frac{mg\bar{R}}{KT_i} \right) T_i$ .  $-\nabla P$  becomes:

$$-\nabla P \approx - \left\{ \frac{P(Z_2) - P(Z_1)}{Z_2 - Z_1} \right\} \hat{z} = - \left\{ \frac{P(Z_2) - P(Z_1)}{(2r_0 + \bar{R})} \right\} \hat{z} = - \left( \frac{1}{2\bar{R}} \right) \frac{K}{(2r_0 + \bar{R})^3} (T_i - T_f) \hat{z} \text{ And:}$$



$$12.16 \quad -\nabla P = -\left(\frac{1}{2\bar{R}}\right) \frac{K}{(2r_0 + \bar{R})^3} (T_i - T_f) \hat{z} = \frac{mg}{(2r_0 + \bar{R})^3} \hat{z} = +\rho g \hat{z}, \quad T_i = T_f < 0$$

Note the positive sign in front of  $\rho g \hat{z}$ . It is the rebound of atom 1 from atom 3, conserving energy and momentum, that enable a field force  $\underline{f} = -\rho g \hat{z}$  in the  $-\hat{z}$  direction, to cause a contact force in the  $+\hat{z}$  direction.

Consider an airplane wing moving through the air where the pressure on the wing

$P_w(\underline{h})$  is,  $P_w(\underline{h}) = n(\underline{h})KT(\underline{h})$ ,  $\frac{dx}{dt} = 0$  and  $n(\underline{h}) = \#$  of molecules in the air per  $\text{cm}^3$ .

Dropping the subscript where it is understood,  $-\nabla P(\underline{h})$  becomes:

$$-\nabla P(\underline{h}) = -\left\{ \frac{\partial P}{\partial x} \hat{x} + \frac{\partial P}{\partial y} \hat{y} + \frac{\partial P}{\partial z} \hat{z} \right\} = -\left\{ \left( \frac{\partial n}{\partial x} KT + n_o K \frac{\partial T}{\partial x} \right) \hat{x} + \left( \frac{\partial n}{\partial y} KT + n_o K \frac{\partial T}{\partial y} \right) \hat{y} + \left( \frac{\partial n}{\partial z} KT + n_o K \frac{\partial T}{\partial z} \right) \hat{z} \right\}$$

and

$$12.17 \quad \rho(\underline{h}) \cdot \frac{dU(\underline{h})}{dt} = -\nabla P(\underline{h}) = -\left\{ \left( \frac{1}{n_o} \frac{\partial n}{\partial x} + \frac{1}{T} \frac{\partial T}{\partial x} \right) \hat{x} + \left( \frac{1}{n_o} \frac{\partial n}{\partial y} + \frac{1}{T} \frac{\partial T}{\partial y} \right) \hat{y} + \left( \frac{1}{n_o} \frac{\partial n}{\partial z} + \frac{1}{T} \frac{\partial T}{\partial z} \right) \hat{z} \right\} P(\underline{h}), \quad \frac{dx}{dt} = 0$$

The  $\frac{\partial n}{\partial x}, \frac{\partial n}{\partial y}, \frac{\partial n}{\partial z}$  terms are hypothesized to be caused by the separation of the flow from the upper side of the wing resulting in a partial vacuum on the trailing edge of the upper side of the wing resulting in a decrease in pressure on the trailing edge of the upper side of the wing and lift. This is further examined in Sec. 8, part B: Separation.

In order to further develop a model for water, it is necessary to develop a model for a water droplet, sec 6, and absolute humidity, sec 7, as set down below.

## 6. Formation of Micelle Water Droplets, Mists, Fogs and Clouds in Earth's Atmosphere

Mists, Fogs, Clouds, the Marine Layer, a steaming cup of coffee are composed of water droplets. e.g. A lake steaming on an autumnal day before sunrise, create visible water droplets  $\sim 0.1$  to  $\sim 1\text{mm}$  in diameter. In order to understand how water droplets are formed and why they appear to float, or rise or fall in the atmosphere, it is necessary to have a model for the atmosphere. The model created here is based on the following equations.

$$12.18 \quad \text{a. } P = nKT$$

$$\text{b. } \rho(z) \cdot \frac{dW}{dt} \hat{z} = -\nabla P = +\rho g \hat{z}$$

$$\text{c. } \frac{1}{2} \bar{m} V_{rms}^2 + \bar{m} gz = \frac{3}{2} KT(z) + \bar{m} gz = \frac{3}{2} KT(0), \quad 0 \leq z \leq 10^6 \text{ cm} = 10\text{km}$$

12.18c hypothesizes that the average kinetic energy plus average potential energy is a constant for every parcel of air at  $z$  for all  $z$ ,  $0 \leq z \leq 10\text{km}$ . Errors are to be expected especially for  $12\text{km} < z$  where  $T(z)$  increases with  $z$  due to warming from the

sun. Using  $T(0)=293^{\circ}\text{K}$ , and  $\bar{m}=4.82\cdot 10^{-23}\text{gm}$ , 12.18c becomes:  $T(z)=293-22.8\cdot 10^{-5}z(\text{cm})$ .

12.18c is of interest as it is a direct analogue of chap 3, eq. 3.15.

The standard model of the atmosphere is  $T(z)=293-7.3\cdot 10^{-5}z$ ,  $0\leq z\leq 10^6\text{cm}$

and is used here. The difference between the standard model and 12.18c is;  $15.5\cdot 10^5z(\text{cm})$

Using 12.18a and b, find:  $\frac{dp}{dz}=-n\bar{m}g=K(T\frac{dn}{dz}+n\frac{dT}{dz})$  and  $-\frac{\bar{m}g}{KT}=(\frac{d\ln n}{dz}+\frac{d\ln T}{dz})$ . With

$dz=-1.37\cdot 10^4dT$ , integrate  $-\frac{\bar{m}g}{KT}$  to find  $\ln\left\{\frac{n(z)}{n(0)}\left(\frac{T(0)}{T(z)}\right)^{3.7}\right\}=0$  and  $n(z)=n(0)[1-2.5\cdot 10^{-7}z]^{3.7}$ .

Using 12.18a,  $P=n(0)KT(0)[1-2.5\cdot 10^{-7}z]^{4.7}=10^6[1-2.5\cdot 10^{-7}z]^{4.7}$ . At 10km  $P(10)=0.26\cdot 10^6\frac{\text{dy}}{\text{cm}^2}$

Where  $10^6\frac{\text{dy}}{\text{cm}^2}=1$  atmosphere. Collating results:

12.19 With  $T(0)=293^{\circ}\text{K}$ ,  $0\leq z\leq 10^6\text{cm}=10\text{km}$

a.  $P=10^6[1-2.5\cdot 10^{-2}z]^{4.7}\frac{\text{dy}}{\text{cm}^2}$  z in km

b.  $n(z)=2.5\cdot 10^{19}[1-2.5\cdot 10^{-2}z]^{3.7}\frac{1}{\text{cm}^3}$

c.  $\rho(z)=1.2\cdot 10^{-3}[1-2.5\cdot 10^{-2}z]^{3.7}\frac{\text{gm}}{\text{cm}^3}$

d.  $T(z)=[293-7.3]^{\circ}\text{K}$ ,

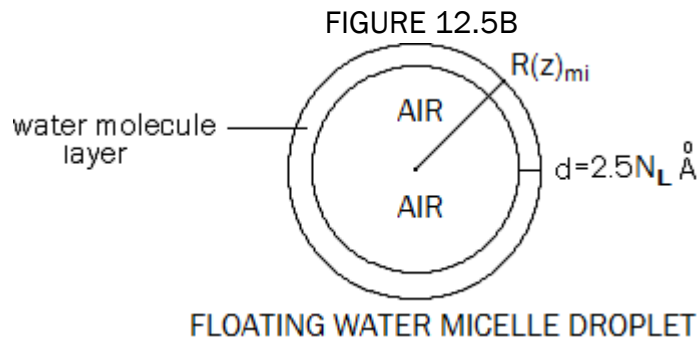
A table of values of 12.19 is listed in Table 12.1

TABLE 12.1

Z km	$P\frac{\text{dy}}{\text{cm}^2}$	$n\frac{1}{\text{cm}^3}$	$\rho\frac{\text{gm}}{\text{cm}^3}$	T $^{\circ}\text{K}$
0	$10^6$	$2.5\cdot 10^{19}$	$1.2\cdot 10^{-3}$	293
1	$0.89\cdot 10^6$	$2.3\cdot 10^{19}$	$1.1\cdot 10^{-3}$	286
2	$0.79\cdot 10^6$	$2.1\cdot 10^{19}$	$1.0\cdot 10^{-3}$	278
5	$0.53\cdot 10^6$	$1.5\cdot 10^{19}$	$0.73\cdot 10^{-3}$	257
10	$0.26\cdot 10^6$	$0.85\cdot 10^{19}$	$0.41\cdot 10^{-3}$	220

Let  $M_{mi}$  represent the mass of a spherical water vapor micelle droplet and  $R_{mi}$  its radius.

Using 12.18b;  $-\nabla P=\rho_{\text{air}}g\cong\frac{\Delta P}{\Delta z}\cong\frac{3M_{mi}g}{4\pi R_{mi}^3}$  and  $\rho(z)_{\text{air}}=\frac{3M_{mi}}{4\pi R_{mi}^3}\cong 1.2\cdot 10^{-3}[1-2.5\cdot 10^{-7}z]^{3.7}$ .



$M_{mi}$  is considered to be a micelle filled with air at S.T.P., Fig. 12.5B. With  $m_w$  the

mass of a water molecule,  $m_w = 3.01 \cdot 10^{-23}$  gm, and  $N_w$  the number of water molecules on the droplet surface the mass of the surface water molecules  $M_w$  becomes,  $M_w = m_w N_w$ . The mass of the air molecules  $M_A$  in the interior of the micelle is  $M_A = \rho(z)_{air} \left( \frac{4}{3} \pi R_{mi}^3 \right)$  where  $\rho(z)_{air}$  is the density of air at the altitude at which the micelle is formed. The mass of the droplet becomes:

$$M_{mi} = M_w + M_A = m_w N_w + \rho(z)_{air} \left( \frac{4}{3} \pi R_{mi}^3 \right).$$

How micelles are formed is discussed in below.

$$N_w = \frac{A_{mi}}{A_w} N_L = \frac{4 \pi R_{mi}^2}{(2r_w + \bar{r}_w)^2} N_L \text{ where } N_L \text{ is the number of layers of water molecules.}$$

$M_{mi}$  becomes:  $M_{mi} = m_w \frac{4 \pi R_{mi}^2}{(2r_w + \bar{r}_w)^2} N_L + \rho(z)_{air} \left( \frac{4}{3} \pi R_{mi}^3 \right)$ . The density of the micelle  $\rho(z)_{mi}$  becomes:

$$\rho(z)_{mi} = \frac{3M_{mi}}{4 \pi R_{mi}^3} = \frac{3m_w N_L}{(2r_w + \bar{r}_w)^2 R_{mi}} + \rho(z)_{air}. \text{ In order to float at altitude } z \text{ with no updraft, it}$$

is required that  $\rho(z)_{mi} = \frac{3M_{mi}}{4 \pi R_{mi}^3} = \rho(z)_{air}$  and consequently  $\frac{3m_w N_L}{(2r_w + \bar{r}_w)^2 R_{mi}} \ll \rho(z)_{air}$ . As a

trial, assume  $\frac{3m_w N_L}{(2r_w + \bar{r}_w)^2 R_{mi}} = 10^{-2} \rho(z)_{air}$  and solve for  $R(z)_{mi}$  where  $R(z)_{mi}$  represents the radius of the micelle that will float and remain at height  $z$ . This yields:

$$12.20 \quad R(z)_{mi} = 1.2 \cdot 10^{-2} [1 - 2.5 \cdot 10^{-2} z]^{-3.7} N_L, \quad z \text{ in km.}$$

Values for  $R_{mi}$ , and  $M_{mi} = \rho(z)_{air} \left( \frac{4}{3} \pi R_{mi}^3 \right)$  have been computed and are tabulated in Table 12.2.

The resulting values for  $R_{mi}$  are observable and lend credence to the assumption

$$\text{that } \frac{3m_w N_L}{(2r_w + \bar{r}_w)^2 R_{mi}} < 10^{-2} \rho(z)_{air}.$$

TABLE 12.2

Z km	N <sub>L</sub>	R <sub>mi</sub> cm	M <sub>mi</sub> gm	mN <sub>w</sub> gm	Z km	N <sub>L</sub>	R <sub>mi</sub> cm	M <sub>mi</sub> gm	mN <sub>w</sub> gm
0	1	1.2·10 <sup>-2</sup>	8.7·10 <sup>-9</sup>	8.7·10 <sup>-11</sup>	0	10	1.2·10 <sup>-1</sup>	8.7·10 <sup>-6</sup>	8.7·10 <sup>-8</sup>
1	1	1.3·10 <sup>-2</sup>	1.0·10 <sup>-8</sup>	1.0·10 <sup>-10</sup>	1	10	1.3·10 <sup>-1</sup>	1.0·10 <sup>-5</sup>	1.0·10 <sup>-7</sup>
2	1	1.4 ·10 <sup>-2</sup>	1.3·10 <sup>-8</sup>	1.3·10 <sup>-10</sup>	2	10	1.4 ·10 <sup>-1</sup>	1.3·10 <sup>-5</sup>	1.3·10 <sup>-7</sup>
5	1	2.0·10 <sup>-2</sup>	2.3·10 <sup>-8</sup>	2.3·10 <sup>-10</sup>	5	10	2.0·10 <sup>-1</sup>	2.3·10 <sup>-5</sup>	2.3·10 <sup>-7</sup>
10	1	3.5·10 <sup>-2</sup>	7.3·10 <sup>-8</sup>	7.3·10 <sup>-10</sup>	10	10	3.5·10 <sup>-1</sup>	7.3·10 <sup>-5</sup>	7.3·10 <sup>-7</sup>

If  $\rho(z)_{\text{air}} < \frac{3M_{\text{mi}}}{4\pi R_{\text{mi}}^3}$ , then the micelle will descend to a new level until  $\rho(z)_{\text{air}} = \frac{3M_{\text{mi}}}{4\pi R_{\text{mi}}^3}$ . If

$\rho(z)_{\text{air}} > \frac{3M_{\text{mi}}}{4\pi R_{\text{mi}}^3}$ , then the micelle will ascend until  $\rho(z)_{\text{air}} = \frac{3M_{\text{mi}}}{4\pi R_{\text{mi}}^3}$ .

### Formation of Micelles

The marine layer, fogs, mists and clouds can exist at heights ranging from sea level to an altitude of ~50 miles for noctiluent clouds. Individual water molecules have a mass of 18amu, O<sub>2</sub> has a mass of 32amu, and N<sub>2</sub> has a mass of 28amu. Individual water molecules will float upwards through still air and can be found at heights of ~50 miles. (H<sub>2</sub>O)<sub>n</sub> has a mass of 18n amu and will descend to earth in still air for n ≥ 2. For n large enough, (H<sub>2</sub>O)<sub>n</sub> will fall to earth as a rain drop.

Observationally marine layer, fogs, mists and clouds exist as individual droplets and do not exist as individual water molecules at constant height Z. In still air, individual water molecules continue to rise until they combine with one another and descend to earth. Clouds therefore do not exist as water molecules with n=2,3,, bound to one another as they will not form a cloud at constant Z, but descend from the sky to earth for small enough n, n ≥ 2 and fall from the sky as a rain drop for large enough n. Clouds therefore exist as micelle droplets, Fig 12.6, bound to one another with a density equal to the atmospheric density at the altitude at which the cloud is found.

It is hypothesized that individual water molecules evaporating from the oceans, lakes and rivers rise up into the atmosphere, form bonds with one another and then form a micelle at altitude Z filled with air at the P,T,ρ of the air at Z. Assuming still air, there are two possibilities:

a. For cold enough air; Immediately on leaving the water source, the water molecules form a marine layer or fogs or mists composed of individual micelles and/or micelles bonded to one another and filled with air with P,T,ρ equal to that of air at ground level.

b. individual water molecules rise up to altitude Z forming clouds at that altitude composed of individual micelles and/or micelles formed at altitude Z bonded to one another and filled with air with P,T, $\rho$  equal to that of air at altitude Z.

c. For large enough Z, T will be  $T < 273^{\circ}\text{K}$ . If the pressure at Z where the micelle is formed is less than the ice vapor pressure at Z, then it is conjectured that once the micelle is formed, individual water molecules will fill the micelle until the internal pressure is at the vapor pressure of ice at height Z. The micelle will then ascend or descend until the density of the atmosphere equals the density of the micelle. Perhaps this is how noctilucent clouds are formed.

## 7. Absolute Humidity

The absolute humidity is defined as the mass of water vapor in grams, per cubic meter of air.  $AH = n_W m_W (\frac{\text{gm}}{\text{m}^3})$ , where  $n_W$  is the number of water molecules per cubic meter and  $m_W = 3.01 \cdot 10^{-23}$  gm is the mass of 1 water molecule. At ground level with no micelles present, water vapor exists as unbonded water molecules and for such air  $\rho_{\text{air}} = 0.0012 \frac{\text{gm}}{\text{cm}^3} = 1200 \frac{\text{gm}}{\text{m}^3}$  and the measured absolute humidity has values  $0 < A.H. < 30 \frac{\text{gm}}{\text{m}^3}$

By direct observation clouds and fogs are composed of floating droplets and although the floating droplets do not substantially contribute to atmospheric pressure, they do contribute substantially to the absolute humidity.

If water micelles with  $N_L = 1$  at  $z = 2\text{km}$  (Table 12.2 and fig. 12.6) compose 5% by volume of a cloud, then by direct computation there are  $n_{\text{mi}} = 2.3 \cdot 10^9 \frac{\text{mi}}{\text{m}^3}$  with total water mass per micelle  $1.3 \cdot 10^{-10}$  gm and  $AH_{\text{mi}} = 2.3 \cdot 10^9 (1.3 \cdot 10^{-10}) = 0.30 \frac{\text{gm}}{\text{m}^3}$  and if with the same values as above the water molecules compose 10% by volume of a cloud then  $AH_{\text{mi}} = 0.60 \frac{\text{gm}}{\text{m}^3}$  and if 100% by volume, then  $AH_{\text{mi}} = 6.0 \frac{\text{gm}}{\text{m}^3}$  where

$AH_{\text{mi}}$  = the absolute humidity due to water molecules in micelles. A table of values of  $AH_{\text{mi}}$  is tabulated in Table 12.2A. Note that for a given Z and % vol.,  $AH_{\text{mi}}$  is independent of  $N_L$ . Why?

The computed values are calculated for a standard atmosphere given by 12.19 and do not include free floating  $\text{H}_2\text{O}$ . With  $AH_M$  = the total absolute humidity due to the mass of all water molecules per  $\text{m}^3$ , then there are  $AH_M - AH_{\text{mi}}$  gm of free floating  $\text{H}_2\text{O}$  present in the air per  $\text{m}^3$ .

What physically causes micelles to break apart and reform as liquid water drops with  $\rho = 1 \frac{\text{gm}}{\text{cm}^3}$  at the dew point temperature? Consider the following scenario. With  $N_L = 1$ ,

table 12.2, the radius of a micelle  $R_{\text{mi}}$  at 5 km is:  $R_{\text{mi}} = 2.0 \cdot 10^{-2}$  cm and contains

TABLE 12.2A

Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$	Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$	Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$
0	1	5	0.31	0	1	10	0.62	0	1	100	6.2
1	1	5	0.30	1	1	10	0.60	1	1	100	6.0
2	1	5	0.26	2	1	10	0.52	2	1	100	5.2
3	1	5	0.24	3	1	10	0.48	3	1	100	4.8
4	1	5	0.20	4	1	10	0.40	4	1	100	4.0
5	1	5	0.18	5	1	10	0.36	5	1	100	3.6
6	1	5	0.17	6	1	10	0.34	6	1	100	3.4
10	1	5	0.11	10	1	10	0.22	10	1	100	2.2

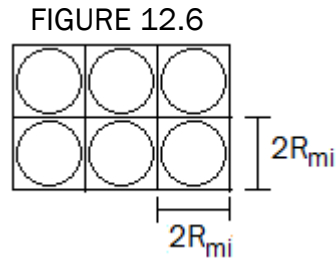
And Surprisingly

Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$	Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$	Z km	N <sub>L</sub>	% vol	AH <sub>mi</sub> $\frac{\text{gm}}{\text{m}^3}$
0	10	5	0.31	0	10	10	0.62	0	10	100	6.2
1	10	5	0.30	1	10	10	0.60	1	10	100	6.0
2	10	5	0.26	2	10	10	0.52	2	10	100	5.2
3	10	5	0.24	3	10	10	0.48	3	10	100	4.8
4	10	5	0.20	4	10	10	0.40	4	10	100	4.0
5	10	5	0.18	5	10	10	0.36	5	10	100	3.6
6	10	5	0.17	6	10	10	0.34	6	10	100	3.4
10	10	5	0.11	10	10	10	0.22	10	10	100	2.2

With micelle volume comprising 100% by volume of the atmosphere of a cloud, the micelles are space filling, fig. 12.6 and the air is saturated with water vapor, and at 5 km using table 12.2A the AH is  $3.6 \frac{\text{gm}}{\text{m}^3}$  with  $8.0 \cdot 10^{12}$  water molecules per micelle with mass  $2.3 \cdot 10^{-10}$  gm. The average center to center distance between two micelles is  $(2R_{mi} + \bar{r}_{mi})$  and with  $\bar{r}_{mi} \ll 2R_{mi}$  the maximum number of micelles per unit volume is  $n_{M,mi} = \frac{1}{(2R_{mi})^3} = 0.125 \frac{1}{(R_{mi})^3}$ .

Evaluating  $n_{M,mi}$  at 5km,  $N_L = 1$  and  $R_{mi} = 2.0 \cdot 10^{-2}$  cm yields,  $n_{M,mi} = 1.6 \cdot 10^4 \frac{\text{mi}}{\text{cm}^3} = 1.6 \cdot 10^{10} \frac{\text{mi}}{\text{m}^3}$ , This is the saturation number of  $\frac{\text{mi}}{\text{m}^3}$ . The micelles are now in constant contact with volume  $V_{mi} = \frac{4}{3} \pi R_{mi}^3$  inside the spectral volume  $V_{sp} = (2R_{mi})^3$ . With no water vapor in the atmosphere, the pressure at 5 km is  $P_1 = 0.53 \cdot 10^6 \frac{\text{dy}}{\text{cm}^2}$  however with  $3.6 \frac{\text{gm}}{\text{m}^3}$  of water in the air, the Oxygen and Nitrogen molecules are squeezed into the volume  $8R_{mi}^3 - \frac{4}{3} \pi R_{mi}^3 = 3.8R_{mi}^3$  and  $P_1 V_1 = P_1 (8R_{mi}^3) = P_2 V_2 = P_2 (3.8R_{mi}^3)$  and  $P_2 = 2.1 P_1 = 1.1 \cdot 10^6 \frac{\text{dy}}{\text{cm}^2}$  It is hypothesized that the increased pressure  $P_2$  on the micelles breaks apart the micelles, and if the temperature is at the dew point, the water molecules will bond

together to form rain drops with density  $1 \frac{\text{gm}}{\text{cm}^3}$  and experimentally determined radii  $0.05 \leq r_w \leq 0.4 \text{cm}$ .



Consider a rectangular volume of height 1km stretching from 4.5km to 5.5km with cross section A at micelle saturation with  $AH_{mi}=3.6 \frac{\text{gm}}{\text{m}^3}$  and with micelle volume comprising 100% by volume of the atmosphere of a cloud. When at the dew point, the micelles condense into water drops and the drops fall from the sky, the area A on the ground will be covered by water to a depth of  $0.36 \text{cm}=0.14 \text{inches}$ . This strongly suggests that during the duration of a heavy rain,  $0.36 N_L \text{cm}=0.14 N_L \text{inches}$  of rain will fall with  $N_L > 1$  and/or, moist air is blown into the atmospheric volume experiencing rain, cools, condenses and forms rain drops and/or there is sufficient free floating  $\text{H}_2\text{O}$  water vapor to account for the total rain fall.

For example using the data at 5km from table 12.1, 12.2 and 12.2A, and  $AH_{mi}=3.6 \frac{\text{gm}}{\text{m}^3}$ , and given that  $AH_M=30 \frac{\text{gm}}{\text{m}^3}$  derive  $AH_M - AH_{mi}=26.4 \frac{\text{gm}}{\text{m}^3}$  of free floating  $\text{H}_2\text{O}$  present in the air in the volume  $8R_{mi}^3 - \frac{4}{3}\pi R_{mi}^3 = 3.8R_{mi}^3 = 3.0 \cdot 10^{-5} \text{cm}^3$ . Find  $7.2 \cdot 10^{-10} \text{gm}$  and  $2.4 \cdot 10^{13}$  free floating  $\text{H}_2\text{O}$  present in the air in the volume  $3.0 \cdot 10^{-5} \text{cm}^3$  creating a partial pressure  $2.8 \cdot 10^4 \frac{\text{dy}}{\text{cm}^2}$ .

Relative humidity, RH is defined as  $RH \equiv \frac{P_{ww}}{P_v} \cdot 10^2$  where  $P_{ww}$  is the partial pressure of water molecules striking one another and  $P_v$  is the vapor pressure  $P_v$  of air striking liquid water at temperature T and RH is expressed as a percent. There is an immediate problem with the definition of RH. The assumption is made that the water in the air is composed of individual water molecules unbonded to one another. Clouds -the origin of rain- are by inspection most assuredly not composed of individual water molecules unbonded to one another. Clouds are composed of clathrates. Given two parcels of air with the same absolute humidity, one of which is composed of clathrates with partial pressure  $P_{cl-cl}$  and the other composed of individual water molecules unbonded to one another with partial pressure  $P_{w-w}$ . By direct calculation  $P_{cl-cl} \ll P_{w-w}$  with  $RH_{cl-cl} \ll RH_{w-w}$  and the formula  $RH \equiv \frac{P_{ww}}{P_v} \cdot 10^2$  is in general false for clouds. It follows that if experimentally the formula is correct, then the water exists as individual water molecules.

With the same absolute humidity, if the water vapor exists as micelles, it is likely that  $\Delta T$  as determined by a sling hygrometer will be a monotonically decreasing function of  $N_L$ , i.e.  $\Delta T = \Delta T(N_L)$  and given  $\Delta N_L > 0$ , then  $\frac{\Delta^2 T}{\Delta N_L} < 0$ . This means that if water vapor exists as micelles, then measuring  $\Delta T$  will not result in accurate values of RH. There is a caveat however, it is possible that a sling hygrometer in earth's atmosphere will break up the micelles into individual water molecules. If so, both the individual water molecules case and the micelle case will have the same  $\Delta T$  reading as a result of the breakup of the micelles into individual water molecules.

## 8. Turbulent Flow in Water

Consider a volume of water in a trough with a free surface and with the water initially at rest in inertial frame S.

Using 4.6 and Chap. 12, Sec. 3, the average absolute pressure that 2 adjacent clathrates in still water exert on one another is:

$$12.21 \quad P_{CL} = \frac{KT}{\bar{R}(h_d)(2R_{CL})^2} = \rho g h_d + 10^6 \frac{dy}{cm^2}, \quad h_d > 2R_{CL} + \bar{R}_{CL} \doteq 2R_{CL}$$

$$P_{CL} \doteq 10^6 \frac{dy}{cm^2}, \quad \text{for } h_d \lesssim 1m$$

Where  $h_d$  is the depth of the water at the point where  $P_{CL}$  is measured,  $R_{CL}$  is the radius of a clathrate and  $2R_{CL} + \bar{R} \doteq 2R_{CL}$  is the average distance between two adjacent clathrates. Note that although the gravitational force  $\underline{f} = -m_{CL}g\hat{z}$  is in the  $-\hat{z}$  direction, the pressure due to gravity  $P_g = \rho g h_d$ , is spherically symmetric about  $h_d$ .

The atmospheric pressure term  $P_{Atm} = 10^6 \frac{dy}{cm^2}$  is necessary as experimentally  $\bar{R} = f(P_{Atm})$ . 12.21 is measured using an aneroid barometer.

If there is fluid flow, then fluid pressure is not spherically symmetric in both Lagrangian and Eulerian measurements. Using sec.5, number 3, the Lagrangian form of the pressure following the flow become:

$$12.21A \quad \underline{P}(\underline{h})_{CL} = P_x \hat{x} + P_y \hat{y} + P_z \hat{z} = KT \left\{ \frac{\hat{x}}{\bar{R}(h_x)_x (2R_{CL})^2} + \frac{\hat{y}}{\bar{R}(h_y)_y (2R_{CL})^2} + \frac{\hat{z}}{\bar{R}(h_z)_z (2R_{CL})^2} \right\}$$

$$\bar{R}(\underline{h}) = \bar{R}(h_x)_x \hat{x} + \bar{R}(h_y)_y \hat{y} + \bar{R}(h_z)_z \hat{z}$$

$$\underline{h} = \underline{x} + \underline{\chi}(x,y,z,t), \quad \underline{\chi}(x,y,z,0) = 0, \quad \underline{x} \neq \underline{x}(t)$$

$$\underline{h} = h_x \hat{x} + h_y \hat{y} + h_z \hat{z} = (x + \chi_x(x,y,z,t)) \hat{x} + (y + \chi_y(x,y,z,t)) \hat{y} + (z + \chi_z(x,y,z,t)) \hat{z}$$



And using 12.21A,  $\rho(\underline{h}) \cdot \frac{d\underline{U}(\underline{h})}{dt} = \sum \underline{F}(\underline{h})_i = -\left\{ \frac{\partial P_x}{\partial h_x} \hat{x} + \frac{\partial P_y}{\partial h_y} \hat{y} + \frac{\partial P_z}{\partial h_z} \hat{z} \right\}$ , the Lagrangian form of the Navier-Stokes Equation following the flow becomes:

$$12.21B \quad \rho(\underline{h}) \cdot \frac{d\underline{U}(\underline{h})}{dt} = \sum \underline{F}(\underline{h})_i = \left\{ \frac{P_x}{\bar{R}(h_x)_x} \cdot \frac{\partial \bar{R}(h_x)_x}{\partial h_x} \hat{x} + \frac{P_y}{\bar{R}(h_y)_y} \cdot \frac{\partial \bar{R}(h_y)_y}{\partial h_y} \hat{y} + \frac{P_z}{\bar{R}(h_z)_z} \cdot \frac{\partial \bar{R}(h_z)_z}{\partial h_z} \hat{z} \right\}$$

$$P_x = \frac{KT}{\bar{R}(h_x)_x (2R_{CL})^2}, \quad P_y = \frac{KT}{\bar{R}(h_y)_y (2R_{CL})^2}, \quad P_z = \frac{KT}{\bar{R}(h_z)_z (2R_{CL})^2}$$

The Eulerian form of the pressure, evaluating pressure at a fixed point  $\underline{x}$  at time  $t$  becomes:

$$12.21C \quad \underline{P}(\underline{x}, t)_{CL} = P_x \hat{x} + P_y \hat{y} + P_z \hat{z} = KT \left\{ \frac{\hat{x}}{\bar{R}(\underline{x}, t)_x (2R_{CL})^2} + \frac{\hat{y}}{\bar{R}(\underline{x}, t)_y (2R_{CL})^2} + \frac{\hat{z}}{\bar{R}(\underline{x}, t)_z (2R_{CL})^2} \right\}$$

And using 12.21C, the Eulerian form of the Navier-Stokes Equation at a fixed point  $\underline{x}$  at time  $t$  becomes:

$$12.21D \quad \rho(\underline{x}, t) \cdot \frac{d\underline{U}(\underline{x}, t)}{dt} = \sum \underline{F}(\underline{x}, t)_i = \left\{ \frac{P_x}{\bar{R}(\underline{x}, t)_x} \cdot \frac{\partial \bar{R}(\underline{x}, t)_x}{\partial x} \hat{x} + \frac{P_y}{\bar{R}(\underline{x}, t)_y} \cdot \frac{\partial \bar{R}(\underline{x}, t)_y}{\partial y} \hat{y} + \frac{P_z}{\bar{R}(\underline{x}, t)_z} \cdot \frac{\partial \bar{R}(\underline{x}, t)_z}{\partial z} \hat{z} \right\}$$

$$P_x = \frac{KT}{\bar{R}(\underline{x}, t)_x (2R_{CL})^2}, \quad P_y = \frac{KT}{\bar{R}(\underline{x}, t)_y (2R_{CL})^2}, \quad P_z = \frac{KT}{\bar{R}(\underline{x}, t)_z (2R_{CL})^2}$$

It is emphasized that  $\underline{P}$  for both the Lagrangian and the Eulerian form is measured by instruments freely moving with the flow.

In the Lagrangian form, one instrument moving with the flow, measures  $\underline{P}$  for a parcel of water moving with the flow.

In the Eulerian form, many instruments are required to measure  $\underline{P}$  at fixed point  $\underline{x}$  at time  $t$ . A reading of the instrument is taken as the instrument passes fixed point  $\underline{x}$ . For sufficiently "Strong" flows, the reading of an instrument fixed to the containment vessel (the bottom of the ocean or the sides of a lab flume etc.) at  $\underline{x}$  will be approximately equal to the reading of the moving instruments as the moving instruments pass fixed point  $\underline{x}$  if the instrument fixed to the containment vessel does not appreciably alter the flow.

The containment vessel must be at rest in an inertial frame, otherwise the acceleration of the frame must be taken into account when computing  $\underline{P}$ .

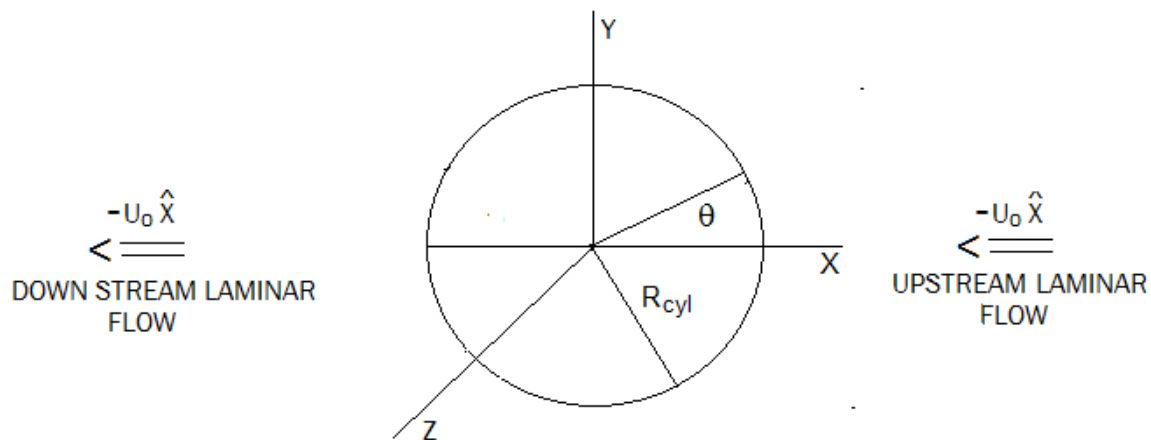
For a general explanation of the cause of fluid turbulence, see Chap. 2, Sec. 8, Fig. 2.7.

Consider a solid cylinder of radius  $R_{cyl}$  and length  $L_0$  in the  $z$  direction, Fig. 12.7, held stationary in flowing water. The cylinder is sticking out of the water in the positive  $z$  direction so that water does not flow around the positive  $z$  end of the cylinder and the negative  $z$  end of the cylinder is far enough away from  $z=0$  so as not to disturb creep flow at  $z=0$ . The figure shows a cross section of the cylinder with the supports not shown. The sides and bottom of the tank are far enough away from the cylinder so that wall effects are unimportant. Note that using the ideal gas law  $P_{CL} = nKT + 10^6 = \rho gh + 10^6$  in place of 12.21 is incorrect as for a constant  $gh$ , doubling  $T$  does not double  $\rho$ .

From table 12, still water is composed of clathrates each of which contains  $N_W = 2.1 \cdot 10^6$  water molecules. At  $T = 293^0 K$ , with average pressure  $P_{CL-CL} = 1.0 \cdot 10^6 \frac{dy}{cm^2}$  the average contact force  $f_{CL-CL}$  that two clathrates exert on one another is  $f_{CL-CL} = 1.0 \cdot 10^{-5}$  dyn and with average pressure  $P_{W-W} = 1.7 \cdot 10^{10} \frac{dy}{cm^2}$ , the average force that two water molecules within the clathrate exert on one another is  $f_{W-W} = 1.0 \cdot 10^{-5}$  dyn

The flow of water around the cylinder as the upstream velocity  $U \hat{x}$  is increased is observationally as follows: Creep flow, followed by Laminar flow, followed by Flow Separation, followed by the creation of Vortices, followed by Fluid Turbulence, followed by full blown Fluid Turbulence. Reference 12.1

FIGURE 12.7



CYLINDER HELD STATIONARY IN WATER FLOWING IN A TROUGH, TOP VIEW,  $Z=0$  AT WATER SURFACE

#### A. Creep Flow

In the upstream and downstream side of the cylinder the far field flow velocity  $-U_0 \hat{x}$  is small enough so that the flow is everywhere laminar and there is no separation of the flow from the cylinder for  $0 < \theta < 180^0$ ,  $180 < \theta < 360^0$ . The upstream side of creep flow is very symmetric to that of the downstream side of creep flow.

With  $x^2+y^2=R_{cyl}^2$ , those clathrates on the cylinder with  $y \geq 0$  will collide at  $x=-R_{cyl}$ ,  $y=0$  with those clathrates with  $y \leq 0$  at  $x=-R_{cyl}$ ,  $y=0$  and push each other off the cylinder with  $x < -R_{cyl}$ . Figure 12.7. Eq. 12.21B enables us to follow an individual clathrate as it strikes and is pushed around the cylinder by the upstream flow.

The position, velocity and acceleration of the center of mass of a clathrate in contact with the cylinder is represented by  $\underline{h}$ ,  $\dot{\underline{h}}$ , and  $\ddot{\underline{h}}$ . fig. 12.7. From symmetry, the dynamics of creep flow for  $0 < \theta < 180^\circ$ , are the same as for  $180^\circ < \theta < 360^\circ$ . Clathrate collisions occurring at  $0$  and  $180^\circ$  are discussed below.

$$\underline{h} = R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right) \{\cos\theta(t)\hat{x} + \sin\theta(t)\hat{y}\}, \quad 0 < \theta < 180^\circ, \quad \underline{h}(t)|_{t=0} = R_{cyl} + R_{cl} + \bar{R}(0)_R \hat{x}, \quad \theta(t)|_{t=0} = 0^\circ$$

and Fig. 12.2

$$\dot{\underline{h}} = R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right) \{[-\sin\theta(t)\hat{x} + \cos\theta(t)\hat{y}]\dot{\theta}(t)\} + \frac{\dot{\bar{R}}_R}{2} \{\cos\theta(t)\hat{x} + \sin\theta(t)\hat{y}\}$$

$$\ddot{\underline{h}} = -R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right) \{[\cos\theta(t)\dot{\theta}(t)^2 + \sin\theta(t)\ddot{\theta}(t)]\hat{x} + [-\sin\theta(t)\dot{\theta}(t)^2 + \cos\theta(t)\ddot{\theta}(t)]\hat{y}\} + \frac{\ddot{\bar{R}}_R}{2} \{\cos\theta(t)\hat{x} + \sin\theta(t)\hat{y}\} + \frac{\dot{\bar{R}}_R}{R_{cyl}} \{-\sin\theta(t)\hat{x} + \cos\theta(t)\hat{y}\}\dot{\theta}(t)$$

$$\underline{a}_{cp} = -R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right) \{\cos\theta(t)\hat{x} + \sin\theta(t)\hat{y}\}\dot{\theta}(t)^2 \text{ is the centripetal acceleration}$$

Using  $\hat{x} = \cos\theta(t)\hat{R}_{cyl} - \sin\theta(t)\hat{\theta}$  and  $\hat{y} = \sin\theta(t)\hat{R}_{cyl} + \cos\theta(t)\hat{\theta}$  with,  $\hat{R}(t)_{cyl} = \cos\theta(t)\hat{x} + \sin\theta(t)\hat{y}$  and  $\hat{\theta} = -\sin\theta(t)\hat{x} + \cos\theta(t)\hat{y}$ . Find;

$$\dot{\hat{R}}(t)_{cyl} = \dot{\theta}(t)\hat{\theta}, \quad \dot{\hat{\theta}} = -\dot{\theta}(t)\hat{R}(t)_{cyl}$$

$$\underline{h} = \left\{R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right)\right\} \hat{R}(t)_{cyl}$$

$$\dot{\underline{h}} = \left\{R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right)\dot{\theta}(t)\right\} \hat{\theta}(t)_{cyl} + \frac{\dot{\bar{R}}_R}{2} \hat{R}(t)_{cyl}, \quad \text{for } 0 < \theta < 180^\circ,$$

$$\ddot{\underline{h}} = -\left\{R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right)\dot{\theta}(t)^2 - \frac{\ddot{\bar{R}}_R}{2}\right\} \hat{R}(t)_{cyl} + \left\{R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right)\ddot{\theta}(t) + \frac{\dot{\bar{R}}_R}{2}\dot{\theta}(t)\right\} \hat{\theta}(t)_{cyl}$$

$$\underline{a}_{cp} = -R_{cyl} \left(1 + \frac{R_{cl} + \bar{R}_R/2}{R_{cyl}}\right)\dot{\theta}(t)^2 \hat{R}(t)_{cyl} \text{ is the centripetal acceleration.}$$

As above,  $-U_0\hat{x}$  is the upstream far field velocity of the fluid flow actively creating the pressure force to push the clathrate around the cylinder.

The centripetal force  $\underline{f}_{cp}$  causing a clathrate to move in a circular arc

of radius  $R_{cyl}$  is:  $\underline{f}_{cp} = -f_{cp} \hat{R}_{cyl} = -\frac{M_{CL} U_{\theta}^2}{R_{cyl}} \hat{R}_{cyl}$ . If the fluid caused pressure force  $\underline{f}$  on an individual clathrate is  $\underline{f} = \underline{f}_{cp}$ , then the clathrate will move in a circle of radius  $R_{cyl}$  with pressure  $P_{cp}$  and constant speed  $U_{\theta}$ . Note that given  $R_{cyl} = \text{const.}$  and assuming  $\underline{f} = \underline{f}_{cp}$ ,  $U_{\theta}$  does not have to be a constant in order that the clathrate move in a circle of radius  $R_{cyl}$  as long as  $\underline{f} = \underline{f}_{cp}$ . The more general case of  $\underline{f} \neq \underline{f}_{cp}$  is discussed below.

Assuming  $\underline{f} = -f_{cp} \hat{R}_{cyl}$ : i.e.  $\ddot{\theta}(t) = \dot{\bar{R}}_R = \ddot{\bar{R}}_R = 0$

$$P_{cp} \hat{R}(t)_{cyl} = \frac{\underline{f}}{(2R_{cl} + \bar{R}_R)^2} = \frac{\underline{f}}{(2R_{cl})^2} \left(1 - \frac{\bar{R}_R}{R_{cl}}\right) = \frac{\underline{f}_{cp}}{(2R_{cl})^2} \left(1 - \frac{\bar{R}_R}{R_{cl}}\right) = \frac{M_{CL} U_{\theta}^2}{R_{cyl} (2R_{cl})^2} \left(1 - \frac{\bar{R}_R}{R_{cl}}\right) \hat{R}(t)_{cyl}$$

$$\text{and } P_{cp} = \frac{M_{CL}}{(2R_{cl})^2} \left(1 - \frac{\bar{R}_R}{R_{cl}}\right) R_{cyl} \dot{\theta}(t)^2.$$

Rewriting 12.21A using  $\hat{R}(t)_{cyl}$  and  $\hat{\theta}(t)_{cyl}$  yields:

$$\underline{P}_{cyl} = -P_R \hat{R}(t)_{cyl} + P_{\theta} \hat{\theta}(t)_{cyl} = (-P_R \cos\theta(t) - P_{\theta} \sin\theta(t)) \hat{x} + (-P_R \sin\theta(t) + P_{\theta} \cos\theta(t)) \hat{y}.$$

$$12.22A \quad P_R = P_{cp} = \frac{M_{CL}}{(2R_{cl})^2} R_{cyl} \dot{\theta}(t)^2 = 3.3 \cdot 10^{-6} \frac{U_{\theta}^2}{R_{cyl}} \frac{dy}{cm^2}$$

$$-f_R \hat{R}(t)_{cyl} = -3.3 \cdot 10^{-17} \frac{U_{\theta}^2}{R_{cyl}} \hat{R}(t)_{cyl} dy$$

where  $P_R$  is the pressure in the  $-\hat{R}(t)_{cyl}$  direction due to  $U_{\theta}$ , that is exerted on a clathrate in order that  $\underline{f} = \underline{f}_{cp} = -f_R \hat{R}(t)_{cyl}$ .

$$12.22B \quad P_{\theta,T} = P(R_{cyl} \cdot [\theta(t) + \frac{2R_{cl}}{R_{cyl}}])_{\theta} - P(R_{cyl} \cdot [\theta(t)])_{\theta} = \frac{M_{CL}}{(2R_{cl})^2} R_{cyl} \ddot{\theta}(t) = 0$$

$$\underline{f}_{\theta,T} \hat{\theta}(t)_{cyl} = f(R_{cyl} \cdot [\theta(t) + \frac{2R_{cl}}{R_{cyl}}])_{\theta} - f(R_{cyl} \cdot \theta(t))_{\theta} = 3.3 \cdot 10^{-17} \dot{U}_{\theta} \hat{\theta}(t)_{cyl} = 0$$

12.22B represents the idealized case  $U_{\theta} = \text{const.}$  with no frictional or field forces acting on the clathrate. In experimental reality in water on the earth's surface, there are always frictional or field forces acting on the clathrate so that if experimentally  $U_{\theta} = \text{const.}$ , then  $\underline{f}_{\theta,T} = \underline{f}_{\theta} + \underline{f}_{fr} = 0$ .

If  $U_{\theta} = \text{const.}$  then  $U_{\theta} t = R_{cyl} \theta(t)$ . A table of values is given in table 12.3

The background pressure, equal in all directions except as noted below, remains at  $1.0 \cdot 10^6 \frac{dy}{cm^2}$ .

It is hypothesized that 1. Chemical bonding between clathrates and the cylinder and 2, Wall effects between the wall, the clathrates and the cylinder do not keep the clathrate in contact with the cylinder during creep flow.

TABLE 12.3

$U_\theta \frac{\text{cm}}{\text{sec}}$	$R_{\text{cyl}} \text{ cm}$	$t \text{ sec}$	$\theta(t)$
0.1	1	31	$\pi$
1	1	3.1	$\pi$
0.1	10	310	$\pi$
1	10	31	$\pi$

On the downstream side of the cylinder,  $\frac{\pi}{2} \leq \theta < \pi$  and  $\pi < \theta \leq \frac{3\pi}{2}$ , for creep flow there is no back flow and the force  $-f_R \hat{R}(t)_{\text{cyl}}$  keeping the clathrate in contact with the cylinder is created by atmospheric pressure,  $P_{\text{atm}} = 10^6 \frac{\text{dy}}{\text{cm}^2}$ .

For  $-\frac{\pi}{2} \leq \theta < 0$  and  $0 < \theta \leq \frac{\pi}{2}$ , on the upstream side of the flow, the derived pressure due to fluid flow is  $P_R = P_{\text{cp}} = \frac{M_{\text{CL}}}{(2R_{\text{CL}})^2} 2R_{\text{cyl}} \dot{\theta}(t)^2 = 3.3 \cdot 10^{-6} \frac{U_\theta^2}{R_{\text{cyl}}} \frac{\text{dy}}{\text{cm}^2}$  and with  $U_\theta = 10^{-1} \frac{\text{cm}}{\text{sec}}$  and  $R_{\text{cyl}} = 1 \text{ cm}$ ,  $P_R$  becomes  $P_R = 3.3 \cdot 10^{-8} \frac{\text{dy}}{\text{cm}^2}$  while atmospheric pressure is  $10^6 \frac{\text{dy}}{\text{cm}^2}$ .

The atmospheric pressure force  $f_{\text{apf}}$  is balanced with in the fluid, but on the surface of the cylinder as a clathrate reflects away from the cylinder a distance  $d < R_{\text{CL}}$  in the  $\hat{R}(t)_{\text{cyl}}$  direction, the  $f_{\text{apf}}$  is unbalanced and the unbalanced atmospheric pressure force, pushes the clathrate back onto the cylinder. It is therefore hypothesized that it is not the pressure created by the upstream flow, 12.22A, but atmospheric pressure that keeps the clathrate in contact with the cylinder. This can be tested by decreasing atmospheric pressure while maintaining  $\underline{U}_\theta = -U_\theta \hat{x}$ .

The flow bifurcates at  $\theta = 0$  due to collision with the cylinder and leaves the cylinder at  $\theta = \pi$  due to collision of the flow at  $\theta = \pi^-$  with the flow at  $\theta = \pi^+$ . This is examined in the following section, Flow Separation.

It remains to examine the coupling between the upstream flow  $\underline{U}_\theta = -U_\theta \hat{x}$ , and the flow around the cylinder,  $\underline{U}_\theta = U_\theta \hat{\theta}(t)_{\text{cyl}}$ .

The unidirectional average atmospheric background pressure  $P_{\text{CL-CL}}$  that 2 clathrates exert on one another on the surface of the cylinder in the  $\theta$  direction is:

$$P_{CL-CL} = \frac{KT}{(2R_{CL})^2 \overline{R(T)}_{\hat{\theta}}} = 1.0 \cdot 10^6 \frac{dy}{cm^2} \text{ with } f_{atm} = \frac{KT}{\overline{R(T)}_{\hat{\theta}}} = 1.0 \cdot 10^{-5} dy.$$

The over force on a clathrate in contact with and traveling around the cylinder in the

$\theta$  direction is:  $\Delta f_{atm} = F_{\theta} = \frac{KT}{\overline{R(T)}_{\hat{\theta}}} \cdot \frac{\Delta \overline{R(T)}_{\hat{\theta}}}{\overline{R(T)}_{\hat{\theta}}}$ . As above,  $f_R$  is hypothesized to be

$f_R = 10^6 (2R_{CL})^2 = 1.0 \cdot 10^{-5} dy$  with  $f_R \neq 3.3 \cdot 10^{-17} \frac{U_{\theta}^2}{R_{cyl}} dy$ , it is hypothesized that

$$F_{\theta} = \frac{KT}{\overline{R(T)}_{\hat{\theta}}} \cdot \frac{\Delta \overline{R(T)}_{\hat{\theta}}}{\overline{R(T)}_{\hat{\theta}}} = 3.3 \cdot 10^{-17} \frac{U_{\theta}^2}{R_{cyl}} dy \text{ and with } U_0 = 10^{-1} \frac{cm}{sec} \text{ and } R_{cyl} = 1cm,$$

$$F_{\theta} = 3.3 \cdot 10^{-19} dy \ll f_{atm} = f_R = f_{\theta} - F_{\theta} \doteq f_{\theta}.$$

Solving for  $\Delta \overline{R(T)}_{\hat{\theta}}$  with  $\overline{R(T)}_{\hat{\theta}} = 3.8 \cdot 10^{-9} cm$  yields:  $\Delta \overline{R(T)}_{\hat{\theta}} = 3.3 \cdot 10^{-21} \frac{U_{\theta}^2}{R_{cyl}} cm$ .  $\Delta \overline{R(T)}_{\hat{\theta}}$

becomes:  $\Delta \overline{R(T)}_{\hat{\theta}} = 3.3 \cdot 10^{-23} cm$ .

A clathrate on the x axis with  $x > R_{cyl}$ ,  $y = 0$ , is pushed in the  $-x$  direction with force  $f_{-x}$

and assuming  $f_{-x} = F_{\theta}$ ,  $f_{-x}$  becomes  $f_{-x} = 3.3 \cdot 10^{-17} \frac{U_{\theta}^2}{R_{cyl}} = 3.3 \cdot 10^{-19} dy$ . On striking the

cylinder at  $x = R_{cyl}$  the clathrates are pushed in the  $\pm \hat{\theta}(t)_{cyl}$  direction.

An alternative model is to consider a rise in temperature  $\Delta T$  as the cause of the increase in pressure  $\Delta P_{CL-CL}$  and force  $f_{-x}$  necessary to move the water with speed  $U_0$ .

$$\text{Consider: } \Delta P_{CL-CL} = \frac{KT}{(2R_{CL})^2 \overline{R(T)}_{CL}} \frac{\Delta T}{T} = 1.0 \cdot 10^6 \frac{dy}{cm^2}. \Delta T \text{ becomes: } \Delta T = 0.97 \cdot 10^{-11} K^{\circ}.$$

## B. Flow Separation

With increasing  $U_0$ , the flow ceases to be creep flow and a given clathrate no longer maintains constant contact with the cylinder as it is pushed around the cylinder and separates from the cylinder at some angle  $\theta$ . It is hypothesized, subject to experimental verification, that as  $U_0$  is slowly increased, the downstream angle of

separation  $2\phi$ , fig. 12.8, continuously changes from  $2\phi = 0$  to  $2\phi = \pi$ .

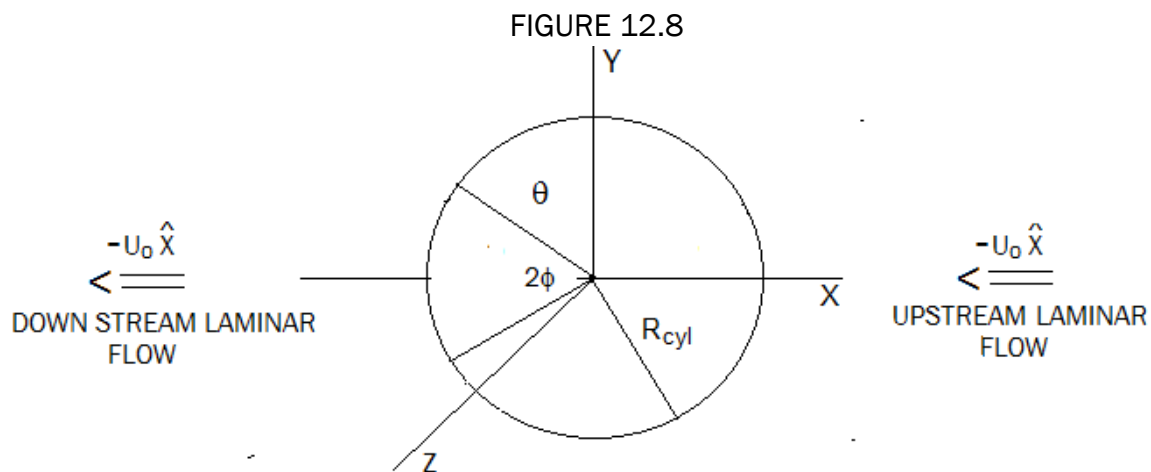
For small enough upstream flow speed  $U_0$ , the downstream angle of separation  $2\phi$  is

$2\phi = 0$  and the flow continues along the  $-x$  axis. At collision, the clathrates at  $\theta = \pi^-$

and  $\theta = -\pi^+$  strike one another with average force  $f_{\theta} + F_{\theta} \doteq f_R = 1.0 \cdot 10^{-5} dy$ .

Not all collisions are center of mass on center of mass collisions and colliding clathrate #1 pushes itself between clathrate #2 and the cylinder and after repeated collisions between #1, the cylinder and #2; #1 pushes #2 off the cylinder and downstream along the  $-x$  axis.

As upstream flow speed  $U_0$  is increased,  $\bar{R}(x,t)_x$  decreases and  $\bar{R}(x,t)_{yz}$  increases so that  $P_x = \frac{KT}{\bar{R}(x,t)_x(2R_{cl})^2} > P_{yz} = \frac{KT}{\bar{R}(x,t)_{yz}(2R_{cl})^2}$  and downstream of the cylinder two circular flows are established, clockwise for  $y > 0$  and counterclockwise for  $y < 0$ . As  $U_0$  is increased, the radius of the circular flows is increased and the clathrates next to the cylinder collide with the circular flows resulting in an increase in  $2\phi$ . To translate this verbal model into mathematics is left as a problem for the reader.



### C. Vortices and Turbulence in Water at $T=293^\circ\text{K}$

Circular flow or vortices are formed in the fluid wake of a solid object by collision of the fluid with the solid object. Traditionally vortices are mathematically expressed by  $\omega = \nabla \times \mathbf{U}$ . The problem with this approach is that the physical cause of the centripetal force necessary to cause a fluid element to move with circular motion is not stated. The approach taken here is to realize that water is made of clathrates and to investigate the physical circumstances that result in a centripetal force. The physical circumstances that are investigated here are:

1. The centripetal force is a result of molecular bonding holding water molecules together in solid body rotation.
2. The centripetal force is a result of the Coriolis Force for large scale vortices: i.e.  $r \gtrsim 100\text{km}$

As regards 1.

In the fluid wake of a solid object, vortices are hypothesized to be the result of induced clathrate rotation caused by the collision of a clathrate with the solid object resulting in solid body clathrate angular momentum  $I\omega$ . There are two possibilities:

a. The standard clathrate with diameter  $3.2 \cdot 10^{-6}$  cm and  $1.1 \cdot 10^6$  water molecules, breaks up into  $N_p$  smaller pieces each containing  $N_w$  water molecules with average angular momentum  $\overline{\omega}$  averaged over  $1.1 \cdot 10^6$  water molecules where  $N_w = \frac{1.1 \cdot 10^6}{N_p}$ .

In this case the pressure that the smaller pieces exert on the surrounding standard clathrates is increased resulting in an expansion of the fluid. The pressure  $P_{CL' \cdot CL'}$  that two adjacent clathrates exert on one another using an aneroid barometer is:

$$P_{CL' \cdot CL'} = \frac{KT}{\overline{R(T)}_{CL'} (2R_{CL'} + \overline{R(T)}_{CL'})^2}$$

where  $R_{CL'}$  is the radius of the clathrates containing  $N_w$

water molecules and  $2R_{CL'} + \overline{R(T)}_{CL'}$  is the average clathrate center to clathrate center distance between two adjacent clathrates each containing  $N_w$  water molecules.

FIGURE 12.9

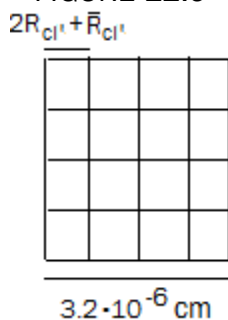


Figure 12.9 represents the effective cross section of a standard spherical clathrate taken along a diameter with clathrate effective volume  $V_{\text{eff}} = (3.2 \cdot 10^{-6})^3 = 3.3 \cdot 10^{-17}$  cm

and actual volume  $V = \frac{4}{3}\pi(1.6 \cdot 10^{-6})^3 = 1.7 \cdot 10^{-17}$  cm<sup>3</sup>.  $2R_{CL'} + \overline{R(T)}_{CL'}$  represents the effective cross section of a clathrate containing  $N_w$  water molecules taken along a diameter with clathrate effective volume  $V'_{\text{eff}} = (2R_{CL'} + \overline{R(T)}_{CL'})^3$  and actual volume

$V' = \frac{4}{3}\pi(R_{CL'})^3$ . Using figure 12.9,  $N_p = 64$  and  $N_w = 1.7 \cdot 10^4$  water molecules.

With  $(V'_{\text{eff}} N_p)^{\frac{1}{3}} = (V_{\text{eff}})^{\frac{1}{3}}$  resulting in  $2R_{CL'} + \overline{R(T)}_{CL'} = (3.2 \cdot 10^{-6})(N_p)^{-\frac{1}{3}} = 3.1 \cdot 10^{-8}(N_w)^{\frac{1}{3}}$ .

There is a small round off error as for  $N_w = 1$ ,  $2R_{CL'} + \overline{R(T)}_{CL'} = 2.5 \cdot 10^{-8}$  cm, see sec. 2.

So for  $1 \leq N_w \leq 10$ ,  $2R_{CL'} + \overline{R(T)}_{CL'} = 2.5 \cdot 10^{-8}(N_w)^{\frac{1}{3}}$  is used and for  $10 < N_w \leq 1.1 \cdot 10^6$ ,

$2R_{CL'} + \overline{R(T)}_{CL'} = 3.1 \cdot 10^{-8}(N_w)^{\frac{1}{3}}$  is used. Computed values of  $2R_{CL'} + \overline{R(T)}_{CL'}$  are tabulated in Table 12.4



TABLE 12.4,  
 $2R_{CL} + \overline{R(T)}_{CL}$  in cm,  $P_{CL \cdot CL}$  in  $\frac{dy}{cm^2}$ ,  $I$  in  $gmcm^2$

$N_p$	$1.1 \cdot 10^6$	$5.5 \cdot 10^5$	$1.1 \cdot 10^5$	$1.1 \cdot 10^4$
$N_w$	1	2	10	$10^2$
$2R_{CL} + \overline{R(T)}_{CL}$	$2.5 \cdot 10^{-8}$	$3.2 \cdot 10^{-8}$	$5.5 \cdot 10^{-8}$	$1.4 \cdot 10^{-7}$
$P_{CL \cdot CL}$	$1.6 \cdot 10^{10}$	$9.8 \cdot 10^9$	$3.3 \cdot 10^9$	$5.1 \cdot 10^8$
$I$	$6.9 \cdot 10^{-40}$	$2.6 \cdot 10^{-39}$	$4.3 \cdot 10^{-38}$	$5.4 \cdot 10^{-36}$
$N_p$	$1.1 \cdot 10^3$	$1.1 \cdot 10^2$	$1.1 \cdot 10^1$	1
$N_w$	$10^3$	$10^4$	$10^5$	$1.1 \cdot 10^6$
$2R_{CL} + \overline{R(T)}_{CL}$	$3.1 \cdot 10^{-7}$	$6.5 \cdot 10^{-7}$	$1.4 \cdot 10^{-6}$	$3.2 \cdot 10^{-6}$
$P_{CL \cdot CL}$	$1.0 \cdot 10^8$	$2.4 \cdot 10^7$	$5.1 \cdot 10^6$	$1.0 \cdot 10^6$
$I$	$2.9 \cdot 10^{-34}$	$1.2 \cdot 10^{-32}$	$5.4 \cdot 10^{-31}$	$3.4 \cdot 10^{-29}$

Previously, sec. 2 and 3,  $\overline{R(T)}_{CL}$  was computed to be  $3.8 \cdot 10^{-9}$  cm for  $N_w=1$  and was assumed to be true for  $N_w=1.1 \cdot 10^6$  and is now assumed to be true for  $1 \leq N_w \leq 1.1 \cdot 10^6$

With  $T=293^\circ K$ :  $P_{CL \cdot CL} = \frac{KT}{\overline{R(T)}_{CL} (2R_{CL} + \overline{R(T)}_{CL})^2} = \frac{1.0 \cdot 10^{-5}}{(2R_{CL} + \overline{R(T)}_{CL})^2}$ . Computed values of  $P_{CL \cdot CL}$  are tabulated in Table 12.4.

For a spherical clathrate angular momentum  $I\omega = \frac{2}{5} M_{cl} R_{cl}^2 \omega$ , where  $M_{cl} = V_{eff} = (2R_{CL} + \overline{R(T)}_{CL})^3$ .

$I\omega$  becomes,  $I\omega = 10^{-1} (2R_{CL} + \overline{R(T)}_{CL})^3 (2R_{CL} + \overline{R(T)}_{CL} - 3.8 \cdot 10^{-9})^2 \omega$ . Computed values of  $I$  are tabulated in Table 12.4.

In the wake of a solid object and for large enough fluid flow speed  $U_0$  measured w.r.t. the solid object, turbulence, disorderly highly non laminar flow results and is modeled to be due to the collision of clathrates with a solid object and the breakup of individual clathrates into  $N_p$  equal pieces each containing  $N_w$  water molecules where  $1 \leq N_w \leq 1.1 \cdot 10^6$  and  $N_p N_w = 1.1 \cdot 10^6$ . Assuming each of the  $N_p$  clathrate pieces is bonded to 6 adjacent clathrate pieces by 6 chemical bonds, then an upper limit to the energy required to break up a standard clathrate into  $N_p$  clathrate pieces is

$\frac{1}{2}(6) (0.80 \cdot 10^{-12}) N_p = 2.4 \cdot 10^{-12} N_p$  erg. The total kinetic energy of the upstream standard clathrate is  $\frac{1}{2} M_{cl} U_0^2 = 2.4 \cdot 10^{-12} N_p$  and  $U_0 = 3.7 \cdot 10^2 N_p^{\frac{1}{2}} \frac{cm}{sec} = 8.3 N_p^{\frac{1}{2}}$  mph.

On striking a stationary solid, if all of the total kinetic energy of the upstream standard clathrate goes into rotational kinetic energy  $KE_{rot}$ , then  $KE_{rot} = \frac{1}{2} I \omega^2 = 2.4 \cdot 10^{-12} N_p$  and  $\omega = 2.2 \cdot 10^{-6} \left(\frac{N_p}{l}\right)^{\frac{1}{2}}$ . Using table 12.4, values of  $U_0$  and  $\omega$  have been computed and are listed in table 12.5.

There is a caveat however: Assuming  $KE_{Tr} = KE_{rot}$  and that the density of  $\frac{m_{cl}}{N_p}$  is a constant:

$KE_{rot} = \frac{1}{2} I \omega^2 = \frac{2 m_{cl} R_{cl}^2}{5 N_p} \omega^2 = \frac{2 m_{cl} R_{cl}^2}{5 N_p} \frac{U_1^2}{R_{cl}^2} = \frac{2 m_{cl} U_1^2}{5 N_p} = \frac{1}{2} \frac{m_{cl}}{N_p} U_0^2$  and  $U_1 = 1.1 U_0$  and for  $N_p > 1$ ,  $N_p > 1 \Rightarrow \frac{m_{cl}}{N_p} < m_{cl}$ . With  $\frac{m_{cl}}{N_p} < m_{cl}$ , it is conceivable that  $U_1 > U_0$ .

TABLE 12.5

$N_p$  = number of clathrate pieces,  $U_0$  in mph,  $l$  in  $gmcm^2$ ,  $\omega$  in  $\frac{R}{sec}$

$N_p$	$1.1 \cdot 10^6$	$5.5 \cdot 10^5$	$1.1 \cdot 10^5$	$1.1 \cdot 10^4$
$U_0$	$8.7 \cdot 10^3$	$6.2 \cdot 10^3$	$2.7 \cdot 10^3$	$8.7 \cdot 10^2$
$l$	$6.9 \cdot 10^{-40}$	$2.6 \cdot 10^{-39}$	$4.3 \cdot 10^{-38}$	$5.4 \cdot 10^{-36}$
$\omega$	$0.88 \cdot 10^{17}$	$3.2 \cdot 10^{16}$	$3.5 \cdot 10^{15}$	$9.9 \cdot 10^{13}$
$N_p$	$1.1 \cdot 10^3$	$1.1 \cdot 10^2$	$1.1 \cdot 10^1$	1
$U_0$	$2.7 \cdot 10^2$	$8.7 \cdot 10^1$	$2.7 \cdot 10^1$	8.3
$l$	$2.9 \cdot 10^{-34}$	$1.2 \cdot 10^{-32}$	$5.4 \cdot 10^{-31}$	$3.4 \cdot 10^{-29}$
$\omega$	$4.3 \cdot 10^{12}$	$2.1 \cdot 10^{11}$	$9.9 \cdot 10^9$	$3.8 \cdot 10^8$

Once the standard clathrate has broken up into  $N_p$  pieces the stream flow pushes each of the  $N_p$  pieces so that their translational kinetic energy  $KE_{Tr} = \frac{1}{2} \frac{m_{cl}}{N_p} U_0^2$

Table 12.6 below, is an expansion of Table 12.5 for reasonable  $U_0$  flow speeds.

TABLE 12.6

$N_p$ =number of clathrate pieces,  $N_w$ =number of water molecules per clathrate piece

$2R_{CL} + \overline{R(T)}_{CL}$  in cm,  $P_{CL-CL}$  in  $\frac{dy}{cm^2}$ ,  $l$  in  $gmcm^2$ ,  $U_o$  in mph,  $\omega$  in  $\frac{R}{sec}$

$N_p$	1	2	3	4	5	10
$N_w$	$1.1 \cdot 10^6$	$5.5 \cdot 10^5$	$3.7 \cdot 10^5$	$2.7 \cdot 10^5$	$2.2 \cdot 10^5$	$1.1 \cdot 10^5$
$2R_{CL} + \overline{R(T)}_{CL}$	$3.2 \cdot 10^{-6}$	$2.5 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$2.0 \cdot 10^{-6}$	$1.9 \cdot 10^{-6}$	$1.5 \cdot 10^{-6}$
$P_{CL-CL}$	$1.0 \cdot 10^6$	$1.6 \cdot 10^6$	$2.1 \cdot 10^6$	$2.5 \cdot 10^6$	$2.8 \cdot 10^6$	$4.4 \cdot 10^6$
$l$	$3.3 \cdot 10^{-29}$	$9.8 \cdot 10^{-30}$	$5.1 \cdot 10^{-30}$	$3.2 \cdot 10^{-30}$	$2.5 \cdot 10^{-30}$	$7.6 \cdot 10^{-31}$
$U_o$	8.3	12	14	17	19	26
$\omega$	$3.8 \cdot 10^8$	$1.0 \cdot 10^9$	$1.4 \cdot 10^9$	$1.7 \cdot 10^9$	$2.0 \cdot 10^9$	$3.6 \cdot 10^9$
$N_p$	20	30	40	50	60	100
$N_w$	$5.5 \cdot 10^4$	$3.7 \cdot 10^4$	$2.7 \cdot 10^4$	$2.2 \cdot 10^4$	$1.8 \cdot 10^4$	$1.1 \cdot 10^4$
$2R_{CL} + \overline{R(T)}_{CL}$	$1.2 \cdot 10^{-6}$	$1.0 \cdot 10^{-6}$	$9.3 \cdot 10^{-7}$	$8.7 \cdot 10^{-7}$	$8.1 \cdot 10^{-7}$	$6.9 \cdot 10^{-7}$
$P_{CL-CL}$	$7.0 \cdot 10^6$	$1.0 \cdot 10^7$	$1.2 \cdot 10^7$	$1.3 \cdot 10^7$	$1.5 \cdot 10^7$	$2.1 \cdot 10^7$
$l$	$2.5 \cdot 10^{-31}$	$1.0 \cdot 10^{-31}$	$7.0 \cdot 10^{-32}$	$5.0 \cdot 10^{-32}$	$3.5 \cdot 10^{-32}$	$1.6 \cdot 10^{-32}$
$U_o$	37	45	52	59	64	83
$\omega$	$2.0 \cdot 10^{10}$	$3.8 \cdot 10^{10}$	$5.3 \cdot 10^{10}$	$7.0 \cdot 10^{10}$	$9.1 \cdot 10^{10}$	$1.7 \cdot 10^{11}$

In the above, it has been assumed that the standard upstream clathrate strikes a stationary solid object and after one strike, breaks up into  $N_p$  clathrates each containing  $N_w$  water molecules. This is an idealization and the physical reality is that the breakup of the standard clathrate may involve multiple strikes resulting in many different size clathrates each with its own  $(\overline{R(T)}_{CL})_i$  and  $(2R_{CL} + \overline{R(T)}_{CL})_i$ . To calculate the average downstream pressure requires computing  $(\overline{R(T)}_{CL})_i(2R_{CL} + \overline{R(T)}_{CL})_i$  over  $i$ , to yield:

$$P = \frac{KT}{(\overline{R(T)}_{CL})_i(2R_{CL} + \overline{R(T)}_{CL})_i} \text{ where } \overline{(\overline{R(T)}_{CL})_i(2R_{CL} + \overline{R(T)}_{CL})_i} = \frac{1}{n_p} \sum_{i=1}^{n_p} n_i (\overline{R(T)}_{CL})_i (2R_{CL} + \overline{R(T)}_{CL})_i$$

and where each of the subscript  $i$  clathrate pieces contains the same number of water molecules and  $n_i$  is the number of clathrate pieces per unit volume that contain the same number of water molecules.  $n_p$  is the total number of clathrate pieces per unit volume

$P$  will be time dependent following the downstream flow as clathrate pieces break up or recombine,

As the first step toward computing average P for clathrate pieces that contain unequal numbers of water molecules consider:

To remove 1 water molecule from a clathrate at  $T=293^{\circ}\text{K}$  to a free water molecule at

$T=293^{\circ}\text{K}$  takes  $0.80 \cdot 10^{-12}$  erg (see sec.4, part b) and to completely pull apart 1

standard clathrate containing  $1.1 \cdot 10^6$  water molecules takes  $0.88 \cdot 10^{-6}$  erg.

Assuming all the translational KE of 1 clathrate is used to remove  $N_w$  water molecules from a clathrate on striking a solid stationary object in midstream, the

incoming speed of the clathrate is  $\frac{1}{2}m_{cl}U_0^2 = 0.80 \cdot 10^{-12}N_w$ . This yields:

$U_0 = 220(N_w)^{\frac{1}{2}} \frac{\text{cm}}{\text{sec}} = 5.0(N_w)^{\frac{1}{2}} \text{mph}$  with  $U_0(1) = 5.0 \text{mph}$  and  $U_0(1.1 \cdot 10^6) = 5.3 \cdot 10^3 \text{mph} = \text{mach } 7.0$   
Speeds of mach 7.0 are reached and exceeded by meteorites plunging into the ocean.

As a problem compute P for  $N_w$  assuming all of the  $N_w$  and the clathrate piece

containing  $1.1 \cdot 10^6 - N_w$  water molecules, are confined within the volume of the original upstream clathrate.

As regards b above.

Instead of the standard clathrate with radius  $R_{cl} = 1.6 \cdot 10^{-6} \text{cm}$ , consider  $N_{cl}$  clathrates bonded together to form a supra clathrate. For large enough  $N_{cl}$ , the supra clathrate approximates a sphere with radius  $R_{s-cl}$  containing  $N_{cl}$  clathrates and assuming the density remains constant,  $M_{s-cl} = V_{s-cl,eff} = (2R_{s-cl})^3$ .

With  $V_{s-cl,eff} = N_{cl}(2R_{cl})^3$  the length of one side of the cube is  $2R_{cl}N_{cl}^{\frac{1}{3}}$  with  $2R_{cl}N_{cl}^{\frac{1}{3}} = 2R_{s-cl}$ .

As above with  $T=293^{\circ}\text{K}$  and  $\bar{R}_{s-cl} = 3.8 \cdot 10^{-9} \text{cm}$ ;  $P_{s-cl-s-cl} = \frac{KT}{\bar{R}_{s-cl}(2R_{s-cl})^2} = 1.0 \cdot 10^6 N_{cl}^{-\frac{2}{3}}$

and  $I\omega$  becomes,  $I\omega = 10^{-1}(2R_{s-cl})^5 \omega = 3.4 \cdot 10^{-29} N_{cl}^{\frac{5}{3}}$

Computed values of  $2R_{s-cl}$ ,  $P_{s-cl-s-cl}$  and  $I$  have been computed and are tabulated in Table 12.7.

TABLE 12.7,

$2R_{s-cl}$  in cm,  $P_{s-cl-s-cl}$  in  $\frac{\text{dy}}{\text{cm}^2}$ ,  $I$  in  $\text{gmcm}^2$

$N_{cl}$	1	10	$10^2$	$10^3$
$2R_{s-cl}$	$3.2 \cdot 10^{-6}$	$6.9 \cdot 10^{-6}$	$1.5 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$
$P_{s-cl-s-cl}$	$1.0 \cdot 10^6$	$2.1 \cdot 10^5$	$4.6 \cdot 10^4$	$1.0 \cdot 10^4$
$I$	$3.4 \cdot 10^{-29}$	$1.6 \cdot 10^{-27}$	$7.3 \cdot 10^{-26}$	$3.4 \cdot 10^{-24}$

For speeds  $V < 5.0$  mph, it is conceivable that standard clathrate collisions with a solid object would yield resonant radial oscillation rates on those molecules on the surface of the clathrate that would bind one clathrate to another yielding a reduced pressure between clathrates: See chap. 5, sec 4 and Table 12.5. This might play a role in the adherence of a flow on the downstream side of a solid during creep flow.

**NOTE THAT SEC. D "TURBULENCE" HAS BEEN INCORPORATED INTO SEC. C ABOVE  
THE REST OF CHAPTER 12 IS UNDER REVIEW BUT STILL WORTH A READ**

### 9. Creation of Vortices and Vortex-Turbulent Flow in Air

Historically, the creation of vortices and vortex-turbulent flow has been investigated using the Navier-Stokes Equations. The equations are wrong on three counts.

1. The equations assume liquids, solids and gases are continuous i.e. there are no empty spaces in the medium under consideration. This is wrong. Liquids, solids and gases are made of atoms and molecules, that are not continuous and for which there are empty spaces between the atoms.

Consider a bar of Platinum in empty space with no external pressure on it. Assuming it is continuous, it is impossible to derive the force necessary to pull the bar apart and it is impossible to derive the force necessary to compress the bar. However once one acknowledges the existence of Platinum Atoms and using the measured mass, atomic radius, coefficient of expansion and Newton's 3<sup>rd</sup> Law, one can derive the average pressure that atoms exert on one another, the bulk modulus and the speed of sound in Platinum as is done in chapter 4. In addition, using the bond energy between two Platinum Atoms, one can derive the tensile strength of Platinum.

2. Historically the statistical properties of vortices and turbulence have been derived using the  $(\underline{U} \cdot \nabla) \underline{U}$  term of the Navier-Stokes Equations. As is shown in section 1 of this chapter, the  $(\underline{U} \cdot \nabla) \underline{U}$  term does not represent physical reality and should not be used to derive the properties of vortices and vortex-turbulent flow.

3. Circular motion of a fluid or gas is introduced as  $\omega = \nabla \times \underline{U}$  without giving an explanation of the origin of the centrifugal force  $\underline{f} = -\frac{mU^2}{r} \hat{i}_r$  that holds  $m$  in a circular path or radius  $r$ . That is rectified in this section.

The hypothesis is made that the onset of vortices in:

- A. Von Karman street flow in air past a stationary cylinder is due to a chemical bond change in air upon striking the cylinder as elucidated below.
- B. Full blown turbulence caused by air ejected from a circular tube into still air is due to the induced rotation  $\underline{\omega}$  of  $N_2$ ,  $O_2$ ,  $H_2O$  molecules upon striking the sides of the tube and the still air as elucidated below.

As regards A.

If upon striking the stationary cylinder, H<sub>2</sub>O and/or N<sub>2</sub> and/or O<sub>2</sub> are set into radial oscillation such that a resonant chemical bond forms between the molecules to produce (H<sub>2</sub>O)<sub>n<sub>1</sub></sub>≡(X<sub>1</sub>)<sub>n<sub>1</sub></sub> and/or (N<sub>2</sub>)<sub>n<sub>2</sub></sub>≡(X<sub>2</sub>)<sub>n<sub>2</sub></sub> and/or (O<sub>2</sub>)<sub>n<sub>3</sub></sub>≡(X<sub>3</sub>)<sub>n<sub>3</sub></sub> and/or (N<sub>2</sub>O<sub>2</sub>)<sub>n<sub>4</sub></sub>≡(X<sub>4</sub>)<sub>n<sub>4</sub></sub>, (see chap. 5, sec. 4), then a decrease in n<sub>0</sub> occurs where n<sub>0</sub> is the number of molecules per cm<sup>3</sup> before the creation of the hypothesized (X<sub>i</sub>)<sub>n<sub>i</sub></sub> i=1,2,3,4. The creation of (X<sub>i</sub>)<sub>n<sub>i</sub></sub> creates a low pressure center in those volumes VLP that contain the afore mentioned molecules. The air surrounding the VLP's flows into the lowered pressure VLP's preserving angular momentum and creating vortices. With n<sub>i</sub>=2,3... the resulting molecule (X<sub>i</sub>)<sub>n<sub>i</sub></sub> is more massive than any of the other (X<sub>i</sub>)<sub>1</sub> and will descend in the atmosphere. However with n<sub>i</sub>=2, it is hypothesized that (X<sub>i</sub>)<sub>2</sub> will descend slowly enough so that motion in the - $\hat{z}$  direction of the hypothesized vortices will not be observable for the experimentally observable lifetime of the vortex. Whether or not (H<sub>2</sub>O)<sub>n<sub>1</sub></sub> has anything to do with the onset of vortices can be tested by using dry air in a wind tunnel i.e. air with no water vapor in it.

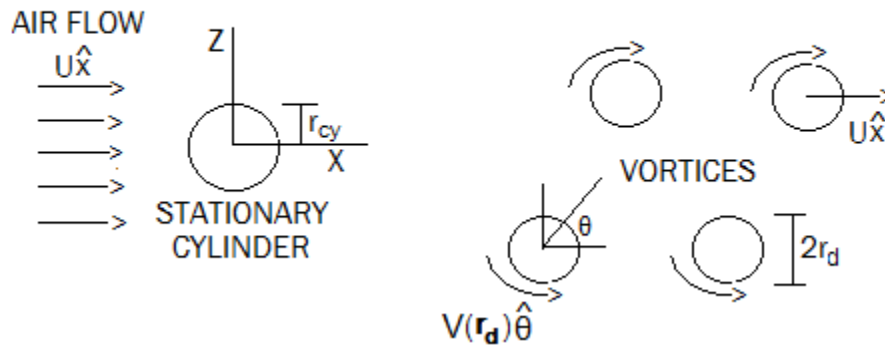
It is observed, see fig. 12.6A, that a back flow in the shadow of the cylinder with a component in the - $\hat{x}$  direction for 0<x<r<sub>cy</sub>, -L≤y≤L<r<sub>cy</sub> and with r<sub>cy</sub>≤r≤r<sub>bl</sub><<1.1r<sub>cy</sub> and a forward flow in the  $\hat{x}$  direction for 0<x<r<sub>cy</sub>, -L≤y≤L<r<sub>cy</sub> and with r<sub>bl</sub><r≤1.1r<sub>cy</sub>, creates a shear flow. It is believed that nascent vortices are created by this shear flow without considering the possibility that nascent vortices create the shear flow. In particular downstream of the cylinder for x>>r<sub>cy</sub> where the radius of the vortex is much larger than the radius of the nascent vortex, what is the origin of the centrifugal force  $f = -\frac{mU^2}{r} \hat{i}_r$  that holds the vortex together?

It is hypothesized that the (X<sub>i</sub>)<sub>n<sub>i</sub></sub> molecules created by striking the leading edge of the cylinder are swept into the afore mentioned shadow zone creating a low pressure volume in the shadow zone that is swept downstream. Air flowing into the low pressure zone and preserving angular momentum forms vortices.

Let n<sub>LP</sub> represent the number of molecules per cm<sup>3</sup> in VLP. In still air at sea level in earth's atmosphere with P<sub>0</sub>=10<sup>6</sup> $\frac{dy}{cm^3}$  and T<sub>0</sub>=293<sup>0</sup>K, n<sub>0</sub> is n<sub>0</sub>=2.5·10<sup>19</sup> $\frac{1}{cm^3}$ . At sea level and in VLP the pressure is: P<sub>LP</sub>= $\frac{n_{LP}}{n_0}$ P<sub>atm</sub>= $\frac{n_{LP}}{n_0}$ 10<sup>6</sup><P<sub>0</sub> with obvious generalization

$P(z)_{LP} = \frac{n(z)}{n(z)_0} P_{LP}(z)_{atm} < P(z)_{LP}$  where z is the distance above sea level. The creation of an idealized vortex follows.

FIGURE 12.6A  
VON KARMAN VORTEX STREET



Consider a circular disk of radius  $r_d$  and thickness  $L$  where  $L \ll r_d$ . Assuming solid

body rotation, what must  $n(r)_{LP}$  be such that  $f(r) = \frac{m_a V(r)^2}{r} = m_a r \omega^2 = \frac{m_a V(r_d)^2}{r_d} \left(\frac{r}{r_d}\right)$ .

$m_a$  is the average mass of an air molecule where  $m_a = 4.8 \cdot 10^{-23}$  gm and effective area

$$A_m \doteq 3.5 \cdot 10^{-16} \text{ cm}^2. \text{ Let } m_x \text{ represent the average mass of the } (X_i)_{n_i}. \quad m_x = \frac{\sum_{i=1}^4 N_i m_i}{\sum_{i=1}^4 N_i}$$

Where  $N_i$  is the number of  $(X_i)_{n_i}$  in VLP and  $m_i$  is the mass of  $(X_i)_{n_i}$ . As will be shown,

$$m_x \sum_{i=1}^4 N_i \ll m_a N_a \text{ where } N_a \text{ is the number of air molecules in VLP.}$$

The pressure acting on  $m_a$  is:  $p = \frac{1}{3} n(r)_{LP} m_a \bar{u}^2$  where  $\bar{u}^2$  is the average speed squared of  $m_a$  where  $\bar{u}^2 = 0.856 u_{rms}^2$ .  $p$  becomes,  $p = 0.856 n(r)_{LP} K T$  and:

$$\frac{dp}{dr} = 0.856 K T \cdot \frac{dn(r)_{LP}}{dr} = f \cdot (2r_m A_m)^{-1} = \frac{m_a V(r_d)^2}{r_d} \left(\frac{r}{r_d}\right) (2r_m A_m)^{-1} \text{ and with } r_m \doteq 0.75 \cdot 10^{-8}$$

$$\text{cm; } \frac{m_a V(r_d)^2}{r_d} \left(\frac{r}{r_d}\right) (2r_m A_m)^{-1} \doteq 9 \frac{V(r_d)^2}{r_d} \left(\frac{r}{r_d}\right) = 0.856 K T \cdot \frac{dn(r)_{LP}}{dr}. \text{ Solving for } \frac{dn(r)_{LP}}{dr}$$

$$\text{using } K T = 4 \cdot 10^{-14} \text{ yields: } \frac{dn(r)_{LP}}{dr} = 2.6 \cdot 10^{14} \frac{V(r_d)^2}{r_d} \left(\frac{r}{r_d}\right) \text{ with}$$

$$n(r)_{LP} = 1.3 \cdot 10^{14} \left(\frac{V(r_d)}{r_d}\right)^2 r^2 + K_0$$

$$\text{and with } n(r_d)_{LP} = n_0 = 2.5 \cdot 10^{19} \frac{1}{\text{cm}^3}, \text{ find } n(r)_{LP} = 2.5 \cdot 10^{19} -$$

$$1.3 \cdot 10^{14} V(r_d)^2 \left[1 - \left(\frac{r}{r_d}\right)^2\right] \left(\frac{1}{\text{cm}^3}\right),$$

For future use:

$$12.20D \quad n(r)_{LP} = 2.5 \cdot 10^{19} - 1.3 \cdot 10^{14} V(r_d)^2 \left[ 1 - \left( \frac{r}{r_d} \right)^2 \right] \left( \frac{1}{cm^3} \right),$$

$$\frac{dn(r)_{LP}}{dr} = 2.6 \cdot 10^{14} \frac{V(r_d)^2}{r_d} \left( \frac{r}{r_d} \right) \frac{1}{cm^4}$$

$$P(r) = P(r_d) - 4.4 V(r_d)^2 \left[ 1 - \left( \frac{r}{r_d} \right)^2 \right] \frac{dy}{cm^2}$$

$$\frac{dp}{dr} = 8.8 \frac{V(r_d)^2}{r_d} \left( \frac{r}{r_d} \right) \frac{dy}{cm^3}$$

$$f(r) = \frac{m_a V(r_d)^2}{r_d} \left( \frac{r}{r_d} \right) dy$$

$$\frac{df}{dr} = m_a \left( \frac{V(r_d)^2}{r_d} \right) \frac{2 dy}{cm}$$

With Reynolds number  $\sim 300$  and using Fig. 12.6A, evaluate 12.20D using  $U = 2 \frac{cm}{sec}$  and  $V = 2 \frac{cm}{sec}$  and  $r_d = 5cm$  and  $P(r_d) = 1 \text{ atm} = 10^6 \frac{dy}{cm^2}$ . Using these values it is seen that for all practical purposes  $n(r)_{LP}$  is constant,  $n(r)_{LP} \doteq n_0 = 2.5 \cdot 10^{19} \frac{1}{cm^3}$  and  $P(r) \doteq P(r_d) = 10^6 \frac{dy}{cm^2}$  and  $\frac{df}{dr} = 0.77 \cdot 10^{-23} \frac{dy}{cm}$ .

It is conjectured that the creation of the  $(X_i)_{n_i}$   $i=1,2,3,4$  by collision with the upper leading edge of an airplane wing creates a low pressure on the upper surface of the wing by the process enumerated above, resulting in lift.

12.20D has been evaluated and the results are given in Table 12.4

As the vortices are pushed downstream, the air on the periphery of the vortex diffuses into the vortex destroying the origin of the centrifugal force by destroying the pressure gradient. The centripetal force is now not balanced by the centrifugal force and the vortex explodes resulting in turbulence. As regards B.

TABLE 12.4

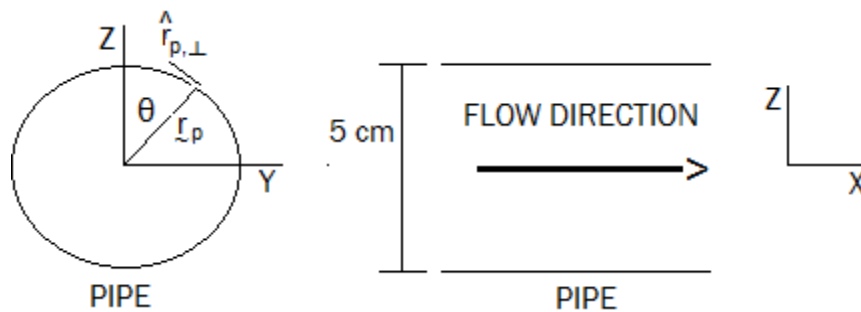
$\frac{r}{r_d}$	$n(r)_{LP} \frac{1}{cm^3}$	$\frac{dn(r)_{LP}}{dr} \frac{1}{cm^4}$	$P(r) \frac{dy}{cm^2}$	$\frac{dp}{dr} \frac{dy}{cm^3}$	$f(r)dy$	$\frac{df}{dr} \frac{dy}{cm}$
1	$2.5 \cdot 10^{19}$	$2.1 \cdot 10^{14}$	$10^6$	7.0	$3.8 \cdot 10^{-23}$	$0.77 \cdot 10^{-23}$
0.9	$2.5 \cdot 10^{19}$	$1.9 \cdot 10^{14}$	$10^6$	6.3	$3.4 \cdot 10^{-23}$	$0.77 \cdot 10^{-23}$
0.5	$2.5 \cdot 10^{19}$	$1.0 \cdot 10^{14}$	$10^6$	3.5	$1.9 \cdot 10^{-23}$	$0.77 \cdot 10^{-23}$
0.2	$2.5 \cdot 10^{19}$	$0.4 \cdot 10^{14}$	$10^6$	1.4	$0.77 \cdot 10^{-23}$	$0.77 \cdot 10^{-23}$



0.1	$2.5 \cdot 10^{19}$	$0.21 \cdot 10^{14}$	$10^6$	0.70	$0.38 \cdot 10^{-23}$	$0.77 \cdot 10^{-23}$
0	$2.5 \cdot 10^{19}$	0	$10^6$	0	0	$0.77 \cdot 10^{-23}$

Consider a round pipe 5cm in diameter emitting air at velocity  $V_m \hat{x} = 1.2 \cdot 10^3 \frac{\text{cm}}{\text{sec}}$  with Reynolds Number  $Re = 4 \cdot 10^4$  into still air. See Fig. 12.6B

FIGURE 12.6B



The air flow emitted from the pipe is laminar, but at a distance of  $\sim 5\text{cm}$  downstream from the pipe, the flow abruptly develops into full blown turbulence. Any vortices that develop are not visible and have a diameter smaller than  $\sim 0.1\text{mm}$ . It is hypothesized that as an air molecule repeatedly strikes the wall of the pipe, it acquires a rotational speed of  $U_m \leq 1.2 \cdot 10^3 \frac{\text{cm}}{\text{sec}}$  about the air molecule's center of mass where as above,

$1.2 \cdot 10^3 \frac{\text{cm}}{\text{sec}}$  is the air molecule's translational speed. With the molecule now in solid body rotation, i.e. spun up,  $\underline{\omega}_m$  is  $\underline{\omega}_m = \frac{U_m}{r_m} \hat{r}_{p,\perp} = \frac{U_m}{r_m} (-\cos\theta \hat{y} + \sin\theta \hat{z})$ . Fig. 12.6C: The

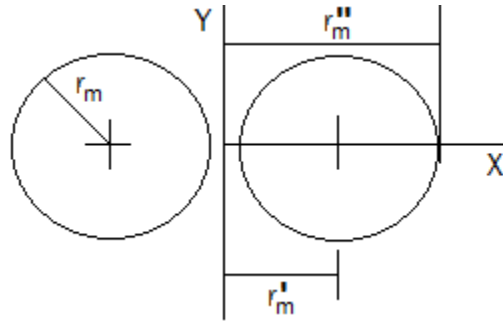
air molecules adjacent to the pipe exit the pipe with their direction of rotation

$\hat{r}_{p,\perp} = (-\cos\theta \hat{y} + \sin\theta \hat{z})$  dependent on their position on the pipe periphery at exit.

The collision of the spinning molecules with the molecules that are not spinning is hypothesized to be the cause of turbulence.

It is possible that vortices develop by the method enumerated in part A, but until experimental evidence exists that demonstrates that vortices exist for this experiment, the exercise is academic.

FIGURE 12.6C  
AIR MOLECULE

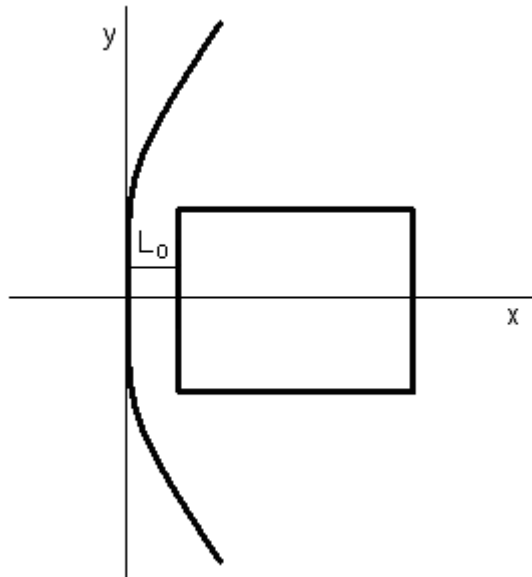


It is interesting to note that an object such as an asteroid or reentry vehicle would have to be traveling some  $\sim 18,000$  mph to break apart Nitrogen or Oxygen molecules and thus increase  $n(\underline{x}, t)$ .

#### 10. Shock Wave in Earth's Atmosphere

Leading any axis-symmetric solid object traveling faster than or equal to the local speed of sound (Mach 1) is a high pressure, high-density surface of revolution called a shock wave. In Figure 12.8, a solid cube 2 cm on a side is moving from right to left at Mach 2.0. All measurements are made by an observer O in an inertial frame at rest with respect to the ground.

Figure 12.8- Shock Wave



The shock wave acts as a shield in front of the solid object and the majority of incident air molecules reflect off of its surface and a minority penetrate the shock. Figure 12.9 represents the solid moving into still air at arbitrary Mach number  $M$  but before the formation of the shock. The air molecules at S.T.P. in A are idealized to be in stationary serried ranks separated by the average distance  $\lambda_0$  between the molecules. Note that assuming that the molecules in serried ranks are separated by

the mean free path, yields an air density  $\sim 10^3$  times smaller than the measured air density.

The average velocity of the molecules in A is taken to be  $0 \frac{\text{cm}}{\text{sec}}$  except for the molecules directly in contact with the solid. For  $M \gg 1$  this is a good approximation, but for  $M \lesssim 1$ , a bad approximation. The struck molecules in A travel the distance  $\lambda_0$  to the left in time  $t_0$  with average velocity  $-2MC_0 \hat{x}$ ; Strike the molecules in the 2<sup>nd</sup> rank and exchange velocities. In time  $t_0$ , the solid object travels distance  $\frac{\lambda_0}{2}$  with velocity  $-MC_0 \hat{x}$ . Considering A through H, it is apparent that  $n_1 \approx 2n_0$  where  $n_0$  is the molecule number density in A and  $n_1$  is the molecule number density in H. Also  $\rho_1 \approx 2\rho_0$  where  $\rho_0$  is the mass density in A and  $\rho_1$  is the mass density in H. The percussive wave generated by the moving object travels with speed  $2MC_0$  as measured by observer O. However the average speed of the molecules is  $MC_0$ , i.e.  $[(2MC_0) \cdot 0.8 + 0.8] \cdot 16 \cdot 1 = MC_0$

The adiabatic sound speed in A is  $C_0 = \left(\gamma \frac{P_0}{\rho_0}\right)^{\frac{1}{2}} = \left(\gamma \frac{n_0 K T_0}{\rho_0}\right)^{\frac{1}{2}}$  and the adiabatic sound

speed in H as measured w.r.t. the moving object is,  $C_1 = \left(\gamma \frac{P_1}{\rho_1}\right)^{\frac{1}{2}} = \left(\gamma \frac{n_1 K T_1}{\rho_1}\right)^{\frac{1}{2}} = \left(\gamma$

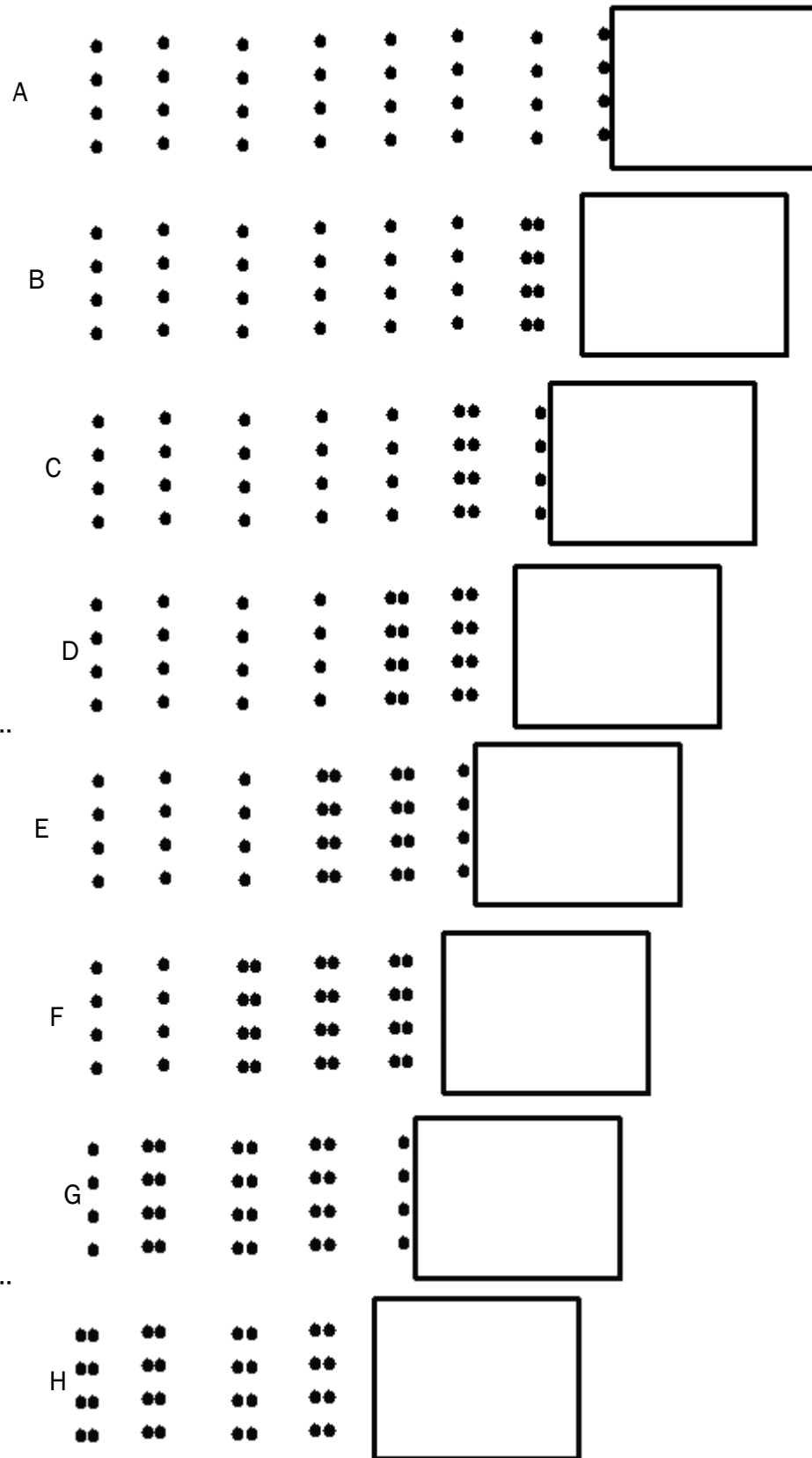
$$\frac{n_0 K T_0 T_1}{\rho_0 T_0} \right)^{\frac{1}{2}} = C_0 \left(\frac{T_1}{T_0}\right)^{\frac{1}{2}}.$$

For a shock to form with  $C_S = M \cdot C_0 = C_0 \left(\frac{T_1}{T_0}\right)^{\frac{1}{2}}$  yields:  $T_1 = M^2 T_0$ . This is not observed and consequently a shock wave does not form in air that is in accord with the ideal gas law and  $\frac{n_0}{\rho_0} = \frac{n_1}{\rho_1} = \frac{1}{m_0}$ .

The density at fig. 12.9 H in front of the moving object  $\rho_f$  is  $\rho_f = 2\rho_0$  where  $\rho_0$  is the background air density before the passage of the moving object, fig. 12.9 A. The leading edge separating  $\rho = \rho_0$  from  $\rho = 2\rho_0 = \rho_f$  is traveling with speed  $2MC_0$ : The moving object is traveling with speed  $MC_0$ .

The movement of the solid object causes the breakup of water droplets directly in front of the moving object resulting in an increase in the number of molecules per unit volume from  $n_0$  to  $n_1 = n_0 + n_{RW}$  where  $n_{RW}$  is the number of newly released water molecules per unit volume (Not Shown). As the water droplets break up, the number density increases from  $n_0$  to  $n_1$  but the mass density  $\rho_0$ , at the instant of the

Figure 12.9 Water Droplets Not



creation of  $n_{rw}$  directly in front of the moving object, remains the same, fig. 12.9 A (Water droplets not shown).

As the water droplets are pushed away from the moving object, fig. 12.9 H, the number density increases from  $n_1$  to  $n_f=2n_1$  and the mass density increases from  $\rho_0$  to  $\rho_f=2\rho_0$ .

The phase speed of sound  $C_2$  at fig. 12.9 H directly in front of the moving object is the speed of sound with respect to the air plus the speed of the air with respect to the ground. With no water droplets,  $n_w=0$ ,  $C_2$  is:

$C_2=(\gamma \frac{2n_0KT_0}{2\rho_0})^{\frac{1}{2}}+MC_0=(\gamma \frac{n_0KT_0}{\rho_0})^{\frac{1}{2}}+MC_0=(1+M)C_0$ . With the advent of the breakup of water droplets, the speed of sound  $C_f$  at fig. 12.9 H directly in front of the moving object is:

$$C_f=(\gamma \frac{n_fKT_0}{\rho_f})^{\frac{1}{2}}+MC_0=(\gamma \frac{2n_1KT_0}{2\rho_0})^{\frac{1}{2}}+MC_0=(\gamma \frac{\bar{m}_fKT_0}{\bar{m}_0})^{\frac{1}{2}}+MC_0=(\gamma \frac{n_0(1+\frac{n_{rw}}{n_0})KT_0}{\rho_0})^{\frac{1}{2}}+MC_0=[(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M]C_0. \text{ For future use:}$$

$$12.20 \quad C_2=(1+M)C_0$$

$$C_f=[(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M]C_0$$

The time interval  $\Delta t$  that it takes the trailing edge of the nascent shock beginning at  $x=L_0$  at  $t=0$  with wave length  $\lambda_0$  (Fig. 12.8) to overtake the leading edge of the nascent shock (i.e. The leading edge separating  $\rho=\rho_0$  from  $\rho=2\rho_0=\rho_f$ , traveling with speed  $2MC_0$ ) is:  $\Delta t=(L_0-\lambda_0)(C_2)^{-1}=L_0(C_f)^{-1}=(L_0-\lambda_0)[(1+M)C_0]^{-1}$

$$1=L_0[(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M]C_0^{-1}$$

Solving the last two expressions for  $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M$  yields:  $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M=(1+M)(1-\frac{\lambda_0}{L_0})^{-1}$ .

$\Delta t$  and  $\frac{n_{rw}}{n_0}$  become:

$$12.21 \quad \Delta t=(1-\frac{\lambda_0}{L_0})\frac{L_0}{(M+1)C_0}$$

$$\frac{n_{rw}}{n_0}=\left\{\frac{1+\frac{\lambda_0}{L_0}\cdot M}{1-\frac{\lambda_0}{L_0}}\right\}^2-1$$

Note that using  $MC_0=10(N_{rW})^{\frac{1}{2}} \frac{cm}{sec}$  (See sec 10), it follows that:  $\lim_{M \rightarrow 0} N_{rW}=0$  and

$\lim_{M \rightarrow 0} \frac{n_{rW}}{n_0}=0$  and using 12.20,  $\lim_{M \rightarrow 0} C_f=C_2$ . Using  $\Delta t=(L_0-\lambda_0)(C_2)^{-1}=L_0(C_f)^{-1}$ :

$\lim_{M \rightarrow 0} L_0(C_f)^{-1}=L_0(C_2)^{-1}=(L_0-\lambda_0)(C_2)^{-1}$  and consequently,  $\lim_{M \rightarrow 0} \lambda_0=0$ . Observationally

$\lim_{M \rightarrow 0} L_0=0$  and using 12.21,  $\lim_{M \rightarrow 0} \Delta t=0$ . Summing up:

$$12.22 \quad \lim_{M \rightarrow 0} \frac{n_{rW}}{n_0}=0, \quad \lim_{M \rightarrow 0} \lambda_0=0, \quad \lim_{M \rightarrow 0} L_0=0 \quad \text{and} \quad \lim_{M \rightarrow 0} \Delta t=0$$

If the total volume of floating water droplets is 1% of the total volume of air through

which the solid object passes and using  $MC_0=10(N_{rW})^{\frac{1}{2}} \frac{cm}{sec}$  (See sec 10),

$n_{rW}=7.8 \cdot 10^8 \cdot N_{rW}$  and  $\Delta P=n_{rW}KT$  as developed for table 2.5 and 12.6 yields  $N_{rW}$  (The number of water molecules released per floating droplet after 1 collision with the moving object) and  $n_{rW}$  (The

number of water molecules released per  $cm^3$  after 1 collision with the moving object) and  $\Delta P$  (The change in pressure as measured from an inertial frame at rest with respect to the center of mass of the moving air. The change in pressure is due to the release of water molecules from droplets thereby increasing  $n_f$ ).  $C_0$  as

experimentally determined at sea level for  $T=20^{\circ}C$  is,  $C_0=3.4 \cdot 10^4 (\frac{cm}{sec})=760\text{mph}$ .

Using  $MC_0=2C_0$ , a table of values of  $N_{rW}$ ,  $n_{rW}$  and  $\Delta P$  computed for Mach2 is presented

TABLE 12.7, Sea Level, Mach2

$N_{rW}$	$2C_0(\text{mph})$	$n_{rW}(\frac{1}{cm^3})$	$\Delta P(\frac{dy}{cm^2})$	$\Delta P(\frac{lb}{ft^2})$
$4.6 \cdot 10^7$	1520	$3.6 \cdot 10^{16}$	$1.5 \cdot 10^3$	0.54
$4.6 \cdot 10^7 N_C$	1520	$3.6 \cdot 10^{16} N_C$	$1.5 \cdot 10^3 N_C$	$0.54 N_C$
$7.8 \cdot 10^8$	1520	$6.1 \cdot 10^{17}$	$2.55 \cdot 10^4$	9.2

in table 12.7. If the total number of collisions is  $N_C$ , then the  $N_{rW}$ ,  $n_{rW}$  and  $\Delta P$  columns are multiplied by  $N_C$  where  $0 \leq N_C \leq 17$ .

The change in pressure  $\Delta P_{M2}$  due to the collision of air molecules with the moving

object at Mach2 is:  $\Delta P_{M2}=\rho(\text{Mach2})^2=(2\rho_0)(6.8 \cdot 10^4)^2=1.1 \cdot 10^7 \frac{dy}{cm^2}$ .

Using 12.21,  $\Delta t$  and  $\frac{n_{rw}}{n_0}$  are evaluated and compiled in table 12.8 for a solid of cross section 2cmX2cm and  $L_0 \approx 0.6$ cm traveling at Mach 2 as in fig. 12.8. At  $T=20^\circ\text{C}$  at sea level,  $n_0=2.5 \cdot 10^{19}$  and for the droplet under consideration, fig 12.6,  $0 \leq \frac{n_{rw}}{n_0} \leq 2.4 \cdot 10^{-2}$

2

Table 12.8, Sea Level, Mach2

$\frac{n_{rw}}{n_0}$	$\frac{\lambda_0}{L_0}$	$\Delta t(\text{sec})$
0	0	$5.9 \cdot 10^{-6}$
$1.0 \cdot 10^{-3}$	$1.67 \cdot 10^{-4}$	$5.9 \cdot 10^{-6} (1 - 1.67 \cdot 10^{-4})$
$1.0 \cdot 10^{-2}$	$1.67 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 1.67 \cdot 10^{-3})$
$2.0 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 3.3 \cdot 10^{-3})$
$2.4 \cdot 10^{-2}$	$6.6 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 6.6 \cdot 10^{-3})$

As  $t^- \rightarrow \Delta t = t_0 = 0$ , the trailing edge overtakes the leading edge and the energy density, mass density and pressure between the leading and trailing edge increase forming a shock. Observationally, the shock is traveling with group speed  $MC_0$  at approximately constant distance  $L_0$  in front of the moving object. This means that water droplets in front of the shock are being broken up by collision with the molecules making up the shock and the shock is entering an atmosphere where the speed of the shock is  $C_S = MC_0 = (1 + \frac{n_{w,s}}{n_0})^{\frac{1}{2}} C_0$  and  $(M^2 - 1)n_0 = n_{w,s}$  where  $n_{w,s}$  is the number of water molecules per  $\text{cm}^3$  that are released from water droplets by collision with the shock and reflect in front of the shock.

How is it that the speed of the shock remains constant at  $MC_0$ ? If  $n_{w,s} > (M^2 - 1)n_0$  the shock starts to travel away from the moving object, loses energy and ceases to exist and if  $n_{w,s} < (M^2 - 1)n_0$ , the moving object overtakes the shock and again the shock ceases to exist. If  $n_{w,s} = (M^2 - 1)n_0$  the shock remains at a ~fixed distance in front of the moving object and energy is constantly pumped into it by the moving object.

The shock acts as a shield and depending on the mass density and thickness of the shock, a certain percentage  $P$  of the incident air molecules in front of the shock reflect off the shock and  $100 - P$  percent go through the shock. Among those that reflect off of the shock are the water droplets that are broken up and release  $n_{w,s}$  water molecules per  $\text{cm}^3$  in front of the shock.

Of the  $100-P$  percent that go through the shock,  $n(100-P)$  percent reflect off the moving body and make up the molecules that are pumped back into the shock and  $(1-n)(100-P)$  move around and behind the moving body where  $0 < n < 1$ .

## 11. Surface Tension

The surface tension field force,  $\frac{\bar{f}_f}{(2r_o + \bar{R}_w)}$ , is the force holding two clathrates together

on the surface of the drop. As  $P(T)_{ex} \rightarrow P(T)_v^+$  the water drop rapidly evaporates and the surface tension field force (the clathrate-clathrate field force) goes to zero,  $f_{cl-cl} = 0$ . The result is that the surface tension field force is a function of the external pressure  $P(T)_{ex}$  and as will be shown, the creation of infrared radiation.

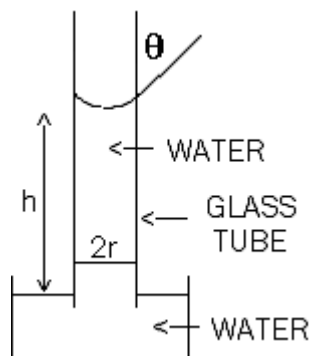
Consider the meniscus of water in a glass tube of radius  $r$ . The meniscus is due to the attraction of the water molecules on the water surface in contact with the glass graduate, to the glass graduate. Figure 12.4. The pressure at the bottom of the water column of height  $h$  is  $P = \rho_w g h$  with force  $-F_P \hat{z} = -\pi r^2 P \hat{z} = -\pi r^2 \rho_w g h \hat{z}$ . The origin of the surface

tension force is the attraction between the monomolecular layer of water molecules on the surface of the water in contact with the glass tube, to the glass tube. The total force due to surface tension in the  $+\hat{z}$  direction is:  $F_T \hat{z} = 2\pi r \sigma_T \cos \theta$

where  $\sigma_T$  is the coefficient of surface tension. The forces are in equal and opposite

directions so that  $F_T \hat{z} = F_P$  and solving for  $h$ :  $h = 2\sigma_T \cos \theta (\rho_w g r)^{-1}$ . Experimentally  $\sigma_T = 72.8 \frac{dy}{cm}$  and  $h$  becomes;  $h = 0.149 \frac{\cos \theta}{r}$ . The maximum value of the force  $f_M$  that one surface water molecule exerts on one of its neighbors in the radial direction due to surface tension is:  $f_M = 2r_w \sigma_T$  and using 12.12,  $f_M = 1.8 \cdot 10^{-6} dy$ .

FIGURE 12.4





The work  $W_M$  to pull apart 2 water molecules held together by surface tension forces at  $20^\circ\text{C}$  is  $W_M \approx f_M \cdot \Delta s \approx f_M \cdot 10^{-10} = 1.8 \cdot 10^{-16} \text{ erg} = 1.1 \cdot 10^{-4} \text{ eV}$  with a  $\Delta T$  equivalence of,  $\Delta T = 0.85^\circ\text{K}$ . i.e. raising the temperature by  $0.85^\circ\text{K}$ . destroys the surface tension. This is not observed and consequently the actual force  $f_A$  holding two water molecules together must be  $f_A > f_M$ . Note that assuming that  $\Delta s \geq 10^{-9} \text{ cm}$  yields  $|\frac{\Delta \rho}{\rho}| \geq 15\%$  as the water climbs up the capillary tube. This is not observed.

## 12.

## Reference

12.1. Milton Van Dyke, "An Album of Fluid Motion", The Parabolic Press, Stanford California, 1982