Chapter 9. Rutherford Scattering, Radioactive Decay, Energetic Atomic Collisions

1. Rutherford Scattering

We reexamine Rutherford scattering, (Reference 9.1) with in the context of neutral solid mass atoms and particles under the influence of the central force field $\Psi(r)$, and central force per unit mass $\underset{\sim}{f}(r)=-\frac{d\Psi(r)}{dr}\hat{r}$. $\Psi(r)$ is given by 3.8 with $h_o=r_o$. i.e.

9.1
$$\Psi(r) = -\frac{mH}{r_o}\frac{1}{(d+2)}\left[(d+3)-\left(\frac{r}{r_o}\right)^{d+2}\right],\ -3<d\leq 0,\ r\leq r_o$$
$$|\Psi(r)| << \frac{mH}{r_o},\ r>r_o$$
$$\underset{\sim}{f}(r) = -\frac{mH}{r_o^2}\left(\frac{r}{r_o}\right)^{d+1}\hat{r},\ r\leq r_o$$
$$|\underset{\sim}{f}(r)| << \frac{mH}{r_o^2},\ r>r_o$$

A neutral particle with the mass of He and incident kinetic energy $\frac{1}{2}m_{He}V_o^2$, hereafter called an α particle, approaches a gold atom on a line passing a distance p from the center of the atom. Figure 9.1a.

FIGURE 9.1a

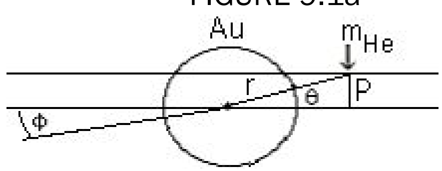
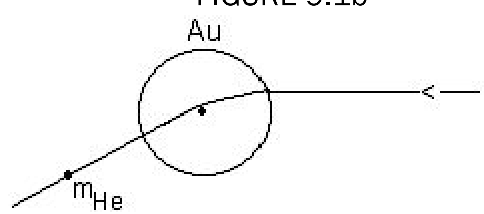


FIGURE 9.1b

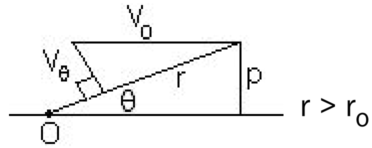


Rutherford Scattering

Let r and θ be polar coordinates for m_{He} with origin at the center of mass of the Au atom. The particle has components of velocity $\frac{dr}{dt}$ and $r\frac{d\theta}{dt}$. The field force, $\underset{\sim}{f}_\alpha=-m_{He}\frac{d\Psi}{dr}\hat{r}$ for $r\leq r_o$ and $|\underset{\sim}{f}_\alpha| << \frac{mH}{r_o^2}$, for $r>r_o$, has no component in the $\hat{\theta}$ direction and

$|\underset{\sim}{r}\times\underset{\sim}{f}_\alpha| = \dot{L} = I\cdot\alpha = I\cdot\frac{d^2\theta}{dt^2} = m_{He}r^2\cdot\frac{d^2\theta}{dt^2}=0$. Consequently the absolute value of the angular momentum $I\omega$ is, $I\omega=m_{He}r^2\frac{d\theta}{dt}=\text{const}$. Evaluating $I\omega$ at an arbitrary point for $r > r_o$, see fig. 9.1c: $\sin\theta=\frac{V_\theta}{V_o}=\frac{p}{r}$ and $rV_\theta=r^2\frac{d\theta}{dt}=V_o p$ and $I\omega=m_{He}r^2\frac{d\theta}{dt}=m_{He}V_o p$. The result also holds for $r\leq r_o$ as $\underset{\sim}{f}_\alpha$ is radial.

FIGURE 9.1c



Writing down the laws for the conservation of energy and angular momentum:

9.1a $\frac{1}{2}m_{He}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\theta}{dt}\right)^2\right]+m_{He}\Psi(r)=\frac{1}{2}m_{He}V_o^2$, $r < r_o$

9.1b $r^2\frac{d\theta}{dt}=V_op$, $p < r_o$

9.1c $\frac{1}{2}m_{He}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\theta}{dt}\right)^2\right]=\frac{1}{2}m_{He}V_o^2$, $r > r_o$

9.1d $r^2\frac{d\theta}{dt}=V_op$, $p > r_o$

The problem is first examined assuming that m_{Au} and m_{He} are point particles and then re-examined assuming that m_{Au} is a solid mass atom of radius $1.28\cdot10^{-8}$cm and m_{He} is a solid mass particle of a to be determined radius.

Eliminating $\frac{d\theta}{dt}$ between 9.1 a and b and between 9.1 c and d and solving for $\frac{dr}{dt}$ yields:

9.2a $\frac{dr}{dt}=\mp V_o\left[1-\frac{2\Psi(r)}{V_o^2}-\frac{p^2}{r^2}\right]^{\frac{1}{2}}=\mp V_o\left[1+\frac{2q}{r_o(d+2)}\left[(d+3)-\left(\frac{r}{r_o}\right)^{d+2}\right]-\frac{p^2}{r^2}\right]^{\frac{1}{2}}$: $q\equiv\frac{m_{Au}H}{V_o^2}$, $r < r_o$, $p < r_o$

9.2b $\frac{dr}{dt}=\mp V_o\left[1-\frac{p^2}{r^2}\right]^{\frac{1}{2}}$, $r > r_o$, $p > r_o$

The upper sign is used for dr<0 and the lower sign is used for dr>0. The α particles from Po^{210} used in the Rutherford scattering experiment have $V_o=2.0\cdot10^9\frac{cm}{sec}$ and translational kinetic energy $KE_\alpha=\frac{1}{2}m_{He}V_o^2=1.36\cdot10^{-5}$ erg=8.4Mev. As $\frac{1}{8,000}$ of these particles back reflect from an Au foil (Ref. 9.1,9.2,9.3), the absolute value of the α particle binding energy is: $|B.E._\alpha|=\frac{m_\alpha^2H}{r_\alpha}\geq 1.36\cdot10^{-5}$ erg. Solve for H: $H\geq\frac{r_\alpha}{m_\alpha^2}=0.30\cdot10^{42}r_\alpha$. With $r_\alpha\approx10^{-13}$cm; $H\geq0.30\cdot10^{29}\frac{erg\ cm}{gm^2}$.

At the point r_m of closest approach, $\frac{dr}{dt}=0$. Solving 9.2 a and b for r_m yields:

9.3 $(r_m)^{d+4}-[(d+3)+r_o(d+2)(2q)^{-1}](r_o)^{d+2}(r_m)^2+(d+2)(2q)^{-1}(r_o)^{d+3}p^2=0$, $r\leq r_o$, $p\leq r_o$, $d\neq2$

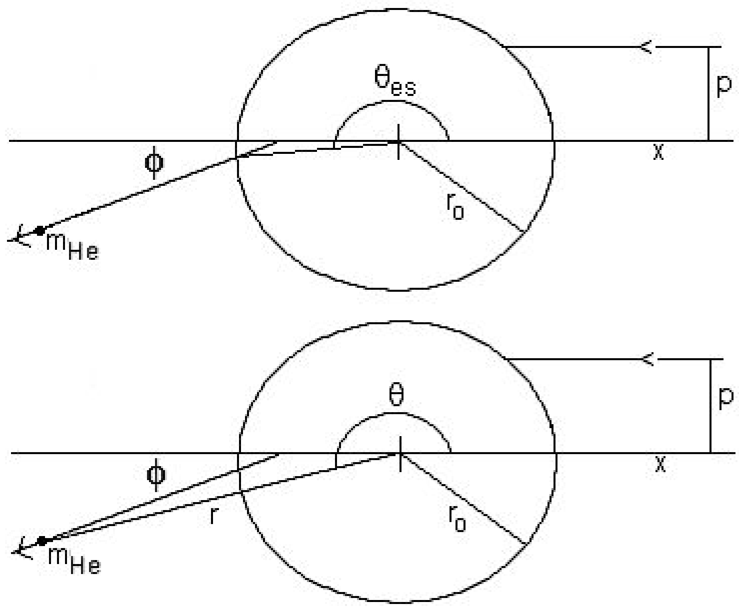
$(r_m)^2[1-2q(r_o)^{-1}\ln(r(r_o)^{-1})]-p^2=0$, $r\leq r_o$, $p\leq r_o$, d=2

9.4 $r_m=p>r_o$, $r>r_o$

Solutions for 9.3 are listed in table 9.2.

We wish to find angles θ_{es} and ϕ where θ_{es}, the escape angle, is the angle from the +x axis to the point where the particle leaves the atom and ϕ is the angle the straight line trajectory of the alpha particle makes with the x-axis after it leaves the atom, where $\phi=\lim_{r\to\infty}(\theta-\pi)\equiv\theta_{r_o}-\pi$ see fig. 9.1d and 9.2. The value of ϕ is derived below.

FIGURE 9.1d



Using $\frac{\frac{d\theta}{dt}}{\frac{dr}{dt}}=\frac{d\theta}{dr}$ solve for $\frac{d\theta}{dr}$.

9.5 $$\frac{d\theta}{dr}=\mp\frac{p}{r^2}\left[1+\frac{2q}{r_0(d+2)}[(d+3)-(\frac{r}{r_0})^{d+2}]-\frac{p^2}{r^2}\right]^{-\frac{1}{2}}$$

with

9.6 $$\theta_1=-p\int_{r_0}^{r_m}\frac{1}{r^2}\left[1+\frac{2q}{r_0(d+2)}[(d+3)-(\frac{r}{r_0})^{d+2}]-\frac{p^2}{r^2}\right]^{-\frac{1}{2}}dr+\theta'_{r_0}$$ where

$\theta'_{r_0}=\sin^{-1}(\frac{p}{r_0})$. See fig. 9.2. θ_1 is the angle between the +x-axis and the point T on the circle of radius $r_m\le P\le r_o$ such that the trajectory of the α particle is tangent to the circle of radius r_m at the point T. See fig. 9.3.

FIGURE 9.2

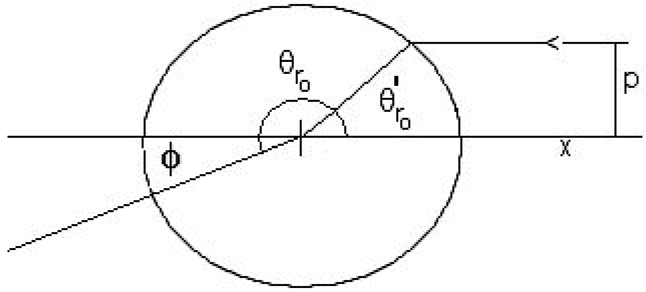
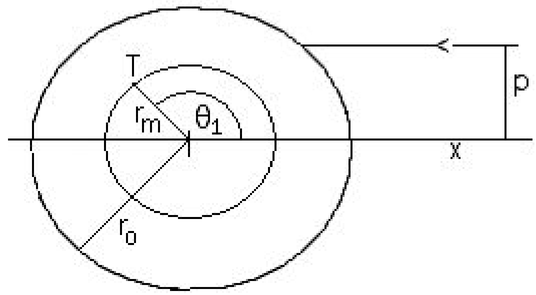


FIGURE 9.3



By symmetry, $\theta_{r_o}=2\theta_1$ where:

9.7 $$\theta_{r_o}=2\theta_1=-2p\int_{r_o}^{r_m}\frac{1}{r^2}\left[1+\frac{2q}{r_o(d+2)}\left[(d+3)-\left(\frac{r}{r_o}\right)^{d+2}\right]-\frac{p^2}{r^2}\right]^{-\frac{1}{2}}dr+2\theta'_{r_o}$$

The desired angle ϕ is related to θ_{r_o} by:

9.8 $$\phi=\theta_{r_o}-\pi$$

Angle θ_{es} is given by, $\theta_{es}=(\theta_1-\theta'_{r_o})+\theta_1=2\theta_1-\theta'_{r_o}=\theta_{r_o}-\theta'_{r_o}$

9.9 $$\theta_{es}=\theta_{r_o}-\theta'_{r_o}$$

All angles are in degrees. Note that θ'_{r_o}, θ_{r_o} and θ_{es} are measured from the +x axis and ϕ is measured from the negative axis, r_m in cm. The integral in 9.7 is very sensitive to the value of r_m, hence we look for another way to evaluate ϕ.

Experimentally, 99+% of all incoming α particles are deflected, $\phi < 2^o$, by a Au foil 160 atoms thick. Ref. 9.1-9.3. and we choose H, $H \ge 0.30\cdot 10^{29}\frac{erg\ cm}{gm^2}$, to reflect this experimental fact. Using figure 9.3: $\lim_{r_m \to P} \theta_1 = 90^o$ and $\phi = 2\theta_1 - 180^o = 0$ and the particle goes straight through the Au atom. We therefore require that $r_m < P$, $r_m \doteq P$ and $\phi < 2^o$. With these requirements, to the nearest order of magnitude, the best fit is $H = 10^{30}\frac{erg\ cm}{gm^2}$. See Table 9.1A and B. Using Fig. 9.4 with $P = \gamma r_o$ and $0 \le \gamma \le r_o$:

$$\tan\phi_1 \doteq (P - r_m)(x_1)^{-1} \doteq P(1-\frac{r_m}{P})(r_o^2 - P^2)^{-\frac{1}{2}} = \gamma(1-\frac{r_m}{P})\,(1-\gamma^2)^{-\frac{1}{2}} \text{ and } \phi = 2\phi_1 = 2\tan^{-1}[\gamma(1-\frac{r_m}{P})(1-\gamma^2)^{-\frac{1}{2}}].$$

9.10 $\quad \phi = 2\phi_1 = 2\tan^{-1}[\gamma(1-\frac{r_m}{P})\,(1-\gamma^2)^{-\frac{1}{2}}]$

A sample calculation follows. Using $\gamma = 0.9$, H= $H = 10^{30}\frac{erg\ cm}{gm^2}$ and d=0 where $\rho(r) = \frac{(d+3)m}{4\pi r_o^3}(\frac{r}{r_o})^d$, solve 9.3 for r_m. Using 9.10: $\phi = 2\tan^{-1}[\gamma(1-\frac{r_m}{P})\,(1-\gamma^2)^{-\frac{1}{2}}] = 2\tan^{-1}[.9(1-\frac{1.1628}{1.1700})(1-.9^2)^{-\frac{1}{2}}] = 1^o$.

FIGURE 9.4

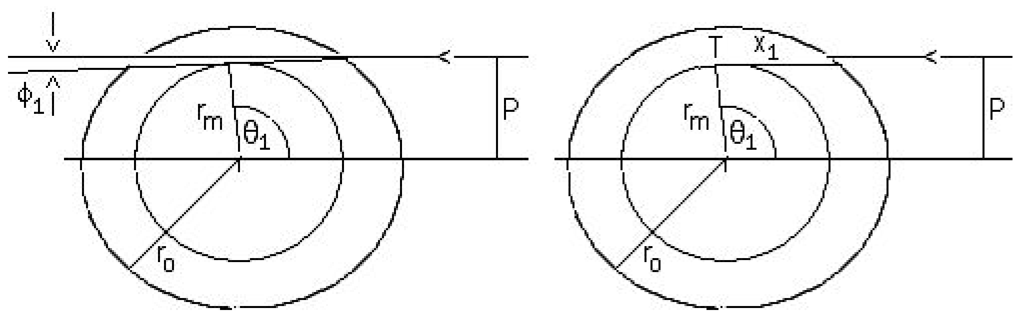


TABLE 9.1A

d	P	H	ϕ	d	P	H	ϕ
0	r_o	10^{29}	0	0	r_o	10^{30}	0
0	$0.9r_o$	10^{29}	.002	0	$0.9r_o$	10^{30}	1
0	$0.75r_o$	10^{29}	.01	0	$0.75r_o$	10^{30}	.9
0	$0.5r_o$	10^{29}	.01	0	$0.5r_o$	10^{30}	.5
0	$0.25r_o$	10^{29}	.009	0	$0.25r_o$	10^{30}	.2
0	$0.1r_o$	10^{29}	.003	0	$0.1r_o$	10^{30}	.1
0	0	10^{29}	0	0	0	10^{30}	0

TABLE 9.1B

d	P	H	ϕ	d	P	H	ϕ
0	r_o	10^{31}	0	0	r_o	10^{32}	0
0	$0.9r_o$	10^{31}	15	0	$0.9r_o$	10^{32}	78
0	$0.75r_o$	10^{31}	9	0	$0.75r_o$	10^{32}	48
0	$0.5r_o$	10^{31}	5	0	$0.5r_o$	10^{32}	26
0	$0.25r_o$	10^{31}	2	0	$0.25r_o$	10^{32}	12
0	$0.1r_o$	10^{31}	1	0	$0.1r_o$	10^{32}	5
0	0	10^{31}	0	0	0	10^{32}	0

Using 9.3, 9.10 and $H=1.0\cdot10^{30}\frac{\text{erg cm}}{\text{gm}^2}$; Values of θ'_{r_o}, θ_{r_o}, θ_{es} and ϕ as a function of d and P are calculated below and tabulated in table 9.2. In the column labeled $10^8 r_m$, the upper number is calculated using 9.3 and the lower number is 10^8P, both numbers in cm.

TABLE 9.2

d	P	θ'_{r_o}	$10^8 r_m$	θ_{r_o}	θ_{es}	ϕ
0	r_o	90	1.30	180	90	0
0	$0.9r_o$	64	1.1628 1.1700	181	117	1
0	$0.75r_o$	48	0.9682 0.9750	180.9	132.9	0.9
0	$0.5r_o$	30	0.6449 0.6500	180.5	150.5	0.5
0	$0.25r_o$	14	0.3222 0.3250	180.2	166.2	0.2
0	$0.1r_o$	5.7	0.1289 0.1300	180.1	174.4	0.1
0	0	0	0	180	180	0

d	P	θ'_{r_o}	$10^8 r_m$	θ_{r_o}	θ_{es}	ϕ
-1	r_o	90	1.30	180	90	0
-1	$0.9r_o$	64	1.1619 1.1700	181.6	117.6	1.6
-1	$0.75r_o$	48	0.9674 0.9750	181	133	1.0
-1	$0.5r_o$	30	0.6439 0.6500	180.6	150.6	0.6
-1	$0.25r_o$	14	0.3215 0.3250	180.3	166.3	0.3
-1	$0.1r_o$	5.7	0.1285 0.1300	180.1	174.4	0.1
-1	0	0	0	180	180	0

d	P	θ'_{r_o}	$10^8 r_m$	θ_{r_o}	θ_{es}	ϕ
-2	r_o	90	1.30	180	90	0
-2	$0.9r_o$	64	1.1619 1.1700	180.16	116.16	0.16
-2	$0.75r_o$	48	0.9674 0.9750	180.24	132.24	0.24
-2	$0.5r_o$	30	0.6439 0.6500	180.29	150.29	0.29
-2	$0.25r_o$	14	0.3215 0.3250	180.26	166.26	0.26
-2	$0.1r_o$	5.7	0.1285 0.1300	180.16	174.46	0.16
-2	0	0	0	180	180	0

d	P	θ'_{r_o}	$10^8 r_m$	θ_{r_o}	θ_{es}	ϕ
-2.5	r_o	90	1.3	180	90	0
-2.5	$0.9r_o$	64	1.1619 1.1700	181.6	117.6	1.6
-2.5	$0.75r_o$	48	0.9670 0.9750	181.1	133.1	1.1
-2.5	$0.5r_o$	30	0.6426 0.6500	180.75	140.75	0.75
-2.5	$0.25r_o$	14	0.3190 0.3250	180.55	166.55	0.55
-2.5	$0.1r_o$	5.7	0.1258 0.1300	180.37	174.67	0.37
-2.5	0	0	0	180	180	0

d	P	θ'_{r_o}	$10^8 r_m$	θ_{r_o}	θ_{es}	ϕ
-3	r_o	90	1.30	180	90	0
-3	$0.9r_o$	64	1.1618 1.1700	181.6	117.6	1.6
-3	$0.75r_o$	48	0.9668 0.9750	181.1	133.1	1.1
-3	$0.5r_o$	30	0.6419 0.6500	180.82	150.82	.82
-3	$0.25r_o$	14	0.3169 0.3250	180.74	166.74	.74
-3	$0.1r_o$	5.7	0.1221 0.1300	180.70	175	.70
-3	0	0	0	180	180	0

It must be remembered that table 9.1A, 9.1B and 9.2 are the consequence of a field interaction between an assumed point mass Au atom and a point mass alpha particle. With this assumption, there are no contact forces.

We examine under what conditions an α particle with radius $r_\alpha=1.3\cdot10^{-13}$cm that enters a Au atom with radius $r_o=1.3\cdot10^{-13}$cm and travels along a diameter, can exit with 99+% of its incoming energy and with $\phi<2^o$.
Using 6.25, the mass m_{cyl} of a cylinder with radius r_α and length $2r_o$ (The diameter of a Au atom) to one significant digit is:

9.11 $$m_{cyl}\doteq m_{Au}\left\{\frac{p+9}{6}\left(\frac{r_\alpha}{r_o}\right)^{(p+3)}+\frac{(p+3)}{2(p+1)}\left[1-\left(\frac{r_\alpha}{r_o}\right)^{p+1}\right]\left(\frac{r_\alpha}{r_o}\right)^2\right\},\ r_\alpha<<r_o,\ -3<p\leq 0$$

The computed ratios $\frac{m_{cyl}}{m_{Au}}$ and $\frac{m_{cyl}}{m_\alpha}=49\frac{m_{cyl}}{m_{Au}}$ are given to 1 significant figure in table 9.3 as a function of p using $r_\alpha=1.0\cdot10^{-13}$ and $r_o=1.3\cdot10^{-8}$

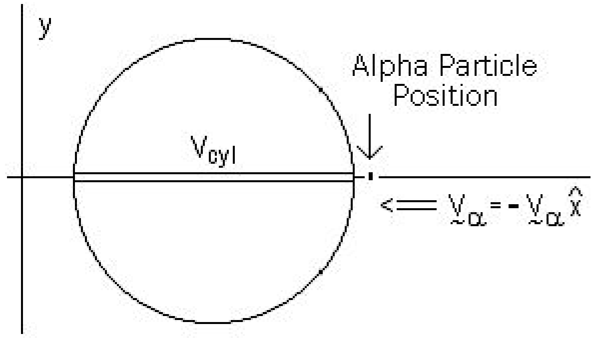
Table 9.3

p	$\frac{m_{cyl}}{m_{Au}}$	$\frac{m_{cyl}}{m_\alpha}$	$\frac{KE_{cyl}}{KE_\alpha}$	$\lvert BE_{cyl}\rvert$(ev)	$\frac{\lvert BE_{cyl}\rvert}{KE_\alpha}$	$\frac{E_{cyl}}{KE_\alpha}$
0	$0.9\cdot10^{-10}$	$4\cdot10^{-9}$	$2\cdot10^{-8}$	2	$2\cdot10^{-7}$	$2\cdot10^{-7}$
-1	$1\cdot10^{-9}$	$5\cdot10^{-8}$	$2\cdot10^{-7}$	20	$2\cdot10^{-6}$	$2\cdot10^{-6}$
-2	$0.8\cdot10^{-5}$	$4\cdot10^{-4}$	$2\cdot10^{-3}$	$1\cdot10^{5}$	$1\cdot10^{-2}$	$1\cdot10^{-2}$
-2.99	0.41	20	0.95	$7\cdot10^{9}$	$8\cdot10^{2}$	$8\cdot10^{2}$

At t=0, an alpha particle with kinetic energy $KE_\alpha=\frac{1}{2}m_\alpha(V_\alpha)^2=1.4\cdot10^{-5}$erg.$=8.7\cdot10^{6}$ev and momentum $m_\alpha \underset{\sim}{V}_\alpha(r_o)=-m_\alpha V_\alpha(r_o)\hat{x}$, $V_\alpha(r_o)=2\cdot10^{9}\frac{cm}{sec}$, strikes an atom (Figure 9.5).
For $m_{cyl}<<m_\alpha$, the energy transferred to the cylinder is $E_{cyl}=\frac{1}{2}m_{cyl}(2V_\alpha)^2+\lvert BE_{cyl}\rvert$ and $\frac{KE_{cyl}}{KE_\alpha}=\frac{\frac{1}{2}m_{cyl}[2V_\alpha]^2}{\frac{1}{2}m_\alpha[V_\alpha]^2}=4\frac{m_{cyl}}{m_\alpha}$. Table 9.2 lists computed values of $\frac{KE_{cyl}}{KE_\alpha}$ to one significant figure.
BE_{cyl} is computed using 6.26 (With r_α in place of r_{ph}) and 9.11.

$$BE_{cyl}=-\frac{m_{Au}H}{r_o}\cdot m_{cyl}=-\frac{m_{Au}^2H}{r_o}\cdot\frac{m_{cyl}}{m_{Au}}=-2.7\cdot10^{-2}\cdot\frac{m_{cyl}}{m_{Au}}\text{ erg}=-1.7\cdot10^{10}\cdot\frac{m_{cyl}}{m_{Au}}\text{ ev}$$

9.12 $$BE_{cyl}=-1.7\cdot10^{10}\cdot\frac{m_{cyl}}{m_{Au}}\text{ ev}$$



Using 9.12 and $KE_\alpha=8.7\cdot10^6$ev, one can compute $\frac{|BE_{cyl}|}{KE_\alpha}$ and $\frac{E_{cyl}}{KE_\alpha}$. Using table 9.2:

For $-2 \le p \le 0$, the $\frac{E_{cyl}}{KE_\alpha}$ is: $\frac{E_{cyl}}{KE_\alpha} << 1$ and the alpha particle goes right through the Au atom in a straight line with essentially undiminished translational kinetic energy. Experimentally, approximately 1 in 8,000 alpha particles are reflected with $\phi>90^o$. Ref. 9.3.

Now consider for $p=-2.99$: $(\frac{E_{cyl}}{KE_\alpha}) = 8\cdot10^2$ and in general for p, $0<p+3<<1$, the alpha particle does not have enough translational kinetic energy to go through the center of the Au atom and will either be adsorbed by the atom or be reflected with $\phi>90^o$. It is therefore conjectured that those alpha particles that have a $\phi>90^o$ have struck an atom with $0<p+3<<1$, and with p sufficiently small so that $\frac{E_{cyl}}{KE_\alpha} >> 1$.

2. Radioactive Decay

Within the confines of quantum mechanics, radioactive decay remains an enigma in so far as there is no fundamental answer to the question, “What determines at what instant a given atom will decay”. It is currently believed that an internal clock in each atom determines the instant at which the atom decays. We investigate the assumption that all radioactive decay is stimulated radioactive decay, that is, small mass particles adsorbed by the atom cause the atom to emit energetic α or β or γ or etc. particles. The problem then becomes one of accounting for the formation of the mass and energy of the stimulating small mass particles and the mass and energy of the resultant decay product and to account for the difference between an atom that decays and one that does not. If our assumption is correct, then the decay rate and half-life of an atom in elemental form is not necessarily the decay rate and half life of an atom in compound form and the decay rate and half-life may depend on the geometry of the radioactive sample. It is interesting to note that the assumption that all atoms currently on earth were created at the same time may be false, see chapter 11, section 11. The experimentally determined decay rate of ^{235}U mined in Canada may therefore not be the same as the decay rate of ^{235}U mined in India.

The ^{238}U series starting with ^{238}U and ending with stable ^{206}Pb is of interest here. Naturally occurring ^{238}U decays by α emission with a half-life of $t_{\frac{1}{2}}=4.5\cdot10^{9}$ years with $\lambda=4.88\cdot10^{-18}\frac{1}{sec}$. Consider 1 gm of ^{238}U containing $N_1=2.5\cdot10^{21}$ atoms, with decay rate at t=0: $|\frac{dN}{dt}|=\lambda N_1=1.2\cdot10^{4}\frac{\alpha}{gm\,sec}$. In general with $N=N_o e^{-\lambda t}$ and $\frac{1}{N}\frac{dN}{dt}=-\lambda$, the probability that a given atom will decay in the next second is λ. A major thrust of this text is to rid physics of the concept of charge and to create new models for "charge effects". The α particle is here considered to be charge (q) neutral. See chapter 7 and 8. A basic assumption of the following model is that naturally occurring ^{238}U atoms emits charge neutral 4.2Mev alpha particles.
From chapter 4, section 3: Let $\overline{\Delta t}$ represent the average time interval measured from the instant that atom #1 is in contact with atom #2 until the instant that atom #1 is again in contact with atom #2: $\overline{\Delta t}=2\frac{\overline{R(t,T)}}{\overline{U_x(t)}}=2\overline{R(t,T)}[2\pi m]^{1/2}/[KT]^{1/2}$. From table 4.1, $\overline{R(t,T)}$ for U is $\overline{R(t,T)}\Big|_{293^oK}=7.07\cdot10^{-11}$cm and $\overline{\Delta t}=3.5\cdot10^{-14}$sec. From the instant that atom #1 is in contact with atom #2 until the instant that atom #1 is again in contact with atom #2, atom #1 collides with its six neighbors at the rate of, $n_o=10/\overline{\Delta t}=2.9\cdot10^{14}\frac{col}{sec}$.
Let n_E represent the average number of times $N(t_o)_o$ atoms have $\overline{KE}\geq E$ per unit time at t_o where $\overline{KE}$ is measured between collisions. $N_o n_o$ represents the total number of collisions of N_o per unit time at t_o. Using 6.12:

$$n(t_o)_E=|\frac{dN}{dt}|=\lambda N(t_o)_o=\frac{2}{\sqrt{\pi}}N_o n_o\int_{\frac{E}{KT}}^{\infty}S^{\frac{1}{2}}e^{-S}dS \text{ , and } 1=\frac{2}{\sqrt{\pi}}\frac{n_o}{\lambda}\int_{\frac{E}{KT}}^{\infty}S^{\frac{1}{2}}e^{-S}dS$$

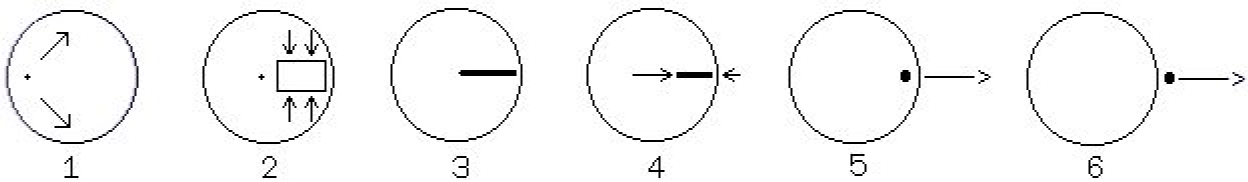
: Where E is the translational K.E. of the U^{238} atom. With $\frac{n_o}{\lambda}=5.9\cdot10^{31}$, by direct computation $\frac{E}{KT}=75.4$ and at $T=293^oK$, $E=1.9ev=3.0\cdot10^{-12}$erg. Before collision, let atom #1 have initial translational kinetic energy $KE_{1,U_i}=\frac{1}{2}m_U W_i^2=1.9ev$ and let atom #2 have initial translational kinetic energy $KE_{2,U_i}=\frac{1}{2}m_U W_i^2$. If the collision were an elastic collision, the two atoms would exchange translational kinetic energies and the final translational kinetic energies would be: $KE_{1,U_f}=KE_{2,U_i}$ and $KE_{2,U_f}=KE_{1,U_i}$. In the present case let t=0 represent the instant during the collision at which the c.m. of #1 and the c.m. of #2 are at rest wrt one another. 1.9ev is in the form of percussive and shear wave energy in #2 and $\frac{1}{2}m_U W_i^2$ is in the form of percussive and shear wave energy in #1.
By the method of Chapter 6, Section 5, #2 produces and ejects a photon in the $\hat{x}$ direction. Let WE represent wave energy. At t=0, $WE_{1,U_i}=KE_{2,U_i}$ and $WE_{2,U_i}=KE_{1,U_i}$

Assuming that WE_{2,U_i} goes into creating a photon plus $K.E._{2,U_f}$, see fig. 9.6, at separation:

$WE_{2,U_i}=K.E._{ph}+|B.E._{ph}|+K.E._{2,U_f}=K.E._{ph}+|B.E._{ph}|+(\frac{m_{ph}}{m_U})K.E._{ph}\doteq K.E._{ph}+|B.E._{ph}|\doteq 1.9ev$

and $KE_{1,U_f}=KE_{2,U_i}$ with photon binding energy $B.E._{ph}\doteq -\frac{m_{ph}^2 H}{r_{ph,f}}$.

Figure 9.6



Atom #2 creating a photon

9.13 $$WE_{2,U_i}\doteq K.E._{ph}+|B.E._{ph}|\doteq 1.9ev$$
$$KE_{1,U_f}= KE_{2,U_i}$$
$$KE_{2,U_f}=(\frac{m_{ph}}{m_U})K.E._{ph}$$

From 3.17, $\frac{1}{2}U_{rms}^2(h)+\Psi(h)=C_1=\frac{1}{2}U_{rms}^2(h_0)+\Psi(h_0)$ and the binding energy of the atom is:

$BE=T+V=mC_1=\frac{1}{2}mU_{rms}^2(h_0)+m\Psi(h_0)$, where $T+V=2\pi\int_0^{h_0}h^2\rho(h)U_{rms}^2(h)dh+4\pi\int_0^{h_0}h^2\Psi(h)\rho(h)dh$

and using 3.8, $\Psi(h)=-\frac{mH}{h_0}\frac{(p+3)}{(p+2)}[1-\frac{1}{(p+3)}(\frac{h}{h_0})^{p+2}]$. With $\Psi(h_0)=-\frac{mH}{h_0}$, the binding energy becomes $BE=\frac{1}{2}mU_{rms}^2(h_0)-\frac{m^2H}{h_0}$. With density $\rho(h)=\frac{(p+3)m}{4\pi h_0^3}(\frac{h}{h_0})^p$, and by direct computation:

9.14 $$V=-\frac{m^2H}{h_0}\frac{(p+3)}{(p+2.5)},\ T=\frac{m^2H}{h_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(h_0),\ -2.5\le p\le 0,\ T+V=\frac{1}{2}mU_{rms}^2(h_0)-\frac{m^2H}{h_0}<0$$
$$V=-\infty,\quad T=+\infty,\quad -3<p\le -2.5,\ T+V=\frac{1}{2}mU_{rms}^2(h_0)-\frac{m^2H}{h_0}<0$$

Note that $T(\varepsilon)$ and $V(\varepsilon)$ are $0<T(\varepsilon)<\infty$ and $-\infty<V(\varepsilon)<0$ where $T(\varepsilon)=2\pi\int_\varepsilon^{h_0}h^2\rho(h)U_{rms}^2(h)dh$

and $V(\varepsilon)=4\pi\int_{\varepsilon}^{h_o} h^2\Psi(h)\rho(h)dh$ where $0<\varepsilon<h_o$ and $-3<p\leq 0$.

The potential energy, $P.E._{ph,At}$, between a photon of mass m_{ph} and radius $h_{o,ph}$ and an atom of mass m_{At} and radius $h_{o,At}$ for $h_{o,ph}<<r_{o,At}$ is given by:

9.15 $\quad P.E.(h)_{ph,At} \doteq -m_{ph}m_{At}\frac{H}{h_{o,At}}\left[\frac{1}{(p+2)}[p+3-(\frac{h}{h_{o,At}})^{p+2}]\right]$, $h \leq h_{o,At}$

With $\Delta P.E.(h)_{ph,At} = P.E.(h)_{ph,At} - P.E.(h_{o,At})_{ph,At}$: $\Delta P.E.(h)_{ph,At}$ is given by:

9.16 $\quad \Delta P.E.(h)_{ph,At} \doteq m_{ph}m_{At}\frac{H}{h_{o,At}}\left[1-\frac{1}{(p+2)}[p+3-(\frac{h}{h_{o,At}})^{p+2}]\right]$, $h \leq h_{o,At}$

Where $\underset{\sim}{h}(\underset{\sim}{r},t)=\underset{\sim}{r}+\underset{\sim}{\chi}(\underset{\sim}{r},t)$ with $\underset{\sim}{\chi}(\underset{\sim}{r},0)=0$ and $\underset{\sim}{r}$ independent of time t. With <> representing the space average value of $|\underset{\sim}{h}(\underset{\sim}{r},t)|$ on the surface of the sphere of radius r at time t: h is defined as $h(r,t)=<|\underset{\sim}{h}(\underset{\sim}{r},t)|>=<|\underset{\sim}{r}+\underset{\sim}{\chi}(\underset{\sim}{r},t)|>$.
We are interested in the case $p=-3+\varepsilon$, $0<\varepsilon<<1$ for which $\rho(h_o,-3+\varepsilon)\underset{\varepsilon\to 0}{\longrightarrow}>0$ and consequently the oscillation amplitude of the atom surface a will be appreciable i.e. $\sim 0.1<\frac{a}{h_o}<\sim 1$. Although ignored here, this will alter the B.E. of the atom. See Chapter 10, Section 4. It can be shown for $1\leq\varepsilon\leq 3$, that $m_{ph}>10^{-24}$gm which is non-physical.
$-\Delta P.E.(h)_{ph,U}$ becomes: $-\Delta P.E.(h)_{ph,U} \doteq 2.9\cdot 10^{16}m_{ph}[\frac{h_{o,U}}{h}]^{(1-\varepsilon)}$. From 9.13:
$K.E._{ph}+|B.E._{ph}|=K.E._{ph}+\frac{m_{ph}^2 H}{r_{ph,f}}=1.9ev=3.0\cdot 10^{-12}erg$ and with $K.E._{ph}\leq\frac{m_{ph}^2 H}{r_{ph,f}}$ then,
$r_{ph,f}\leq 6.7\cdot 10^{41}m_{ph}^2$.
We require $-\Delta P.E.(h)_{ph,U}=|BE_{\alpha,Th}|+2KE_{\alpha}+KE_{Th}$ where $BE_{\alpha,Th}$ is the energy binding the alpha particle to Thorium while in the Uranium atom and $KE_{Th}=\frac{4}{234}KE_{\alpha}=0.02KE_{\alpha}$ where it is assumed that $|BE_{\alpha}|=KE_{\alpha}$.

9.17 $\quad -\Delta P.E.(h)_{ph,U}=2.9\cdot 10^{16}m_{ph}[\frac{h_{o,U}}{h}]^{(1-\varepsilon)}=|BE_{\alpha,Th}|+2.02KE_{\alpha}$

with $|BE_{\alpha,Th}|\approx\frac{m_{He}m_{Th}H}{r_{o,Th}}=1.6\cdot 10^{-7}erg=1.0\cdot 10^{5}ev.$ and $2.02KE_{\alpha}=1.3\cdot 10^{-5}erg$ and

$-\Delta P.E.(h)_{ph,U}$ becomes: $-\Delta P.E.(h)_{ph,U}=1.3\cdot 10^{-5}erg=2.9\cdot 10^{16}m_{ph}[\frac{h_{o,U}}{h}]^{(1-\varepsilon)}$. Solving for

m_{ph} and $r_{ph,f}$ yields:

9.18 $m_{ph}=4.5\cdot 10^{-22}(\frac{h}{h_{o,U}})^{(1-\varepsilon)}$, $r_{ph,f} \le 0.13(\frac{h}{h_{o,U}})^{2(1-\varepsilon)}$, $-\Delta P.E.(h)_{ph,U}=1.3\cdot 10^{-5}$erg.

With $0<\varepsilon<1$.

For $0<\varepsilon<<1$. By direction computation using the density function $\rho(h)=\frac{(p+3)m}{4\pi h_o^3}(\frac{h}{h_o})^p$,

the mass M(h) of the ^{238}U atom between 0 and h is $M(h)=m_U(\frac{h}{r_0})^{\varepsilon}$, with ratio: $\frac{M(h)}{m_U}=(\frac{h}{r_0})^{\varepsilon}$.

With $0<\varepsilon<10^{-2}$, 99% of the mass lies within radius h where $h=.99^{\frac{1}{\varepsilon}}\cdot r_o<.99^{100}\cdot r_o=0.37\cdot r_o$

i.e. $h<0.37\cdot r_o$, where r_o is the radius of the atom.

This is not physical as a sample of pure ^{238}U would have a bulk modulus much smaller than the experimentally determined value. A naturally occurring sample of pure ^{238}U with $0<\varepsilon<<1$ does not exist.

For $10^{-2}<\varepsilon<1$. It can be shown that the absolute value of the binding energy $|B.E._{ph}|$ of the volume V_{cyl} reamed out by the photon on its passage from r_o to h is larger than the kinetic energy of the photon where r_o-h is the distance the photon must travel in order to have the kinetic energy necessary to create a 4.2 Mev particle. Therefore for $10^{-2}<\varepsilon<1$, the incoming photon will not create a 4.2 Mev particle.

The volume V_{cyl} is: $V_{cyl}=\pi r_{ph}^2(r_o-h)+\frac{2}{3}\pi r_{ph}^3$ where h is at the center of mass of the photon, $0<h<r_o$.

A basic assumption for the above model is that naturally occurring ^{238}U atoms emit 4.2Mev alpha particles. It is the author's conclusion that the range energy relation is not true due to misidentification of the mass, speed, kinetic energy, radius and charge (Amplitude of radial oscillation) of the particle emitted by ^{238}U and that the KE of the emitted particle is KE<<4.2Mev.

An interesting solution to the problem is to assume that the alpha particle is actually the photon described below with $m_{ph}<<m_{\alpha}<^{238}U$ and $KE_{ph}+|BE_{ph}|=\frac{1}{2}m_{ph}V_{ph}^2+|BE_{ph}|=1.9ev=3.0\cdot 10^{-12}$erg with $1.5\cdot 10^{-12}\le|BE_{ph}|<3.0\cdot 10^{-12}$erg. Assume the KE_{ph} is thermal, i.e.

$KE_{ph}=1.5KT=6.1\cdot 10^{-14}erg=\frac{1}{2}m_{ph}V_{ph}^2$ with $|BE_{ph}|=3.0\cdot 10^{-12}(1-2.0\cdot 10^{-2})erg=\frac{m_{ph}^2H}{r_{ph}}$.

Solve for V_{ph} and m_{ph}. This yields, $m_{ph}=\frac{1.2\cdot 10^{-13}}{V_{ph}^2}$ and $r_{ph}\doteq\frac{1}{3}\cdot 10^{42}m_{ph}^2=\frac{0.5\cdot 10^{16}}{V_{ph}^4}$

Computed values of m_{ph} and r_{ph} as functions of V_{ph} are given in Table 9.4

TABLE 9.4

$V_{ph}\ \frac{cm}{sec}$	m_{ph} gm	r_{ph} cm
10^5	$1.2\cdot10^{-23}$	$1.2\cdot10^{-4}$
10^6	$1.2\cdot10^{-25}$	$1.2\cdot10^{-8}$
10^7	$1.2\cdot10^{-27}$	$1.2\cdot10^{-12}$
10^8	$1.2\cdot10^{-29}$	$1.2\cdot10^{-16}$
10^9	$1.2\cdot10^{-31}$	$1.2\cdot10^{-20}$
10^{10}	$1.2\cdot10^{-33}$	$1.2\cdot10^{-24}$
10^{11}	$1.2\cdot10^{-35}$	$1.2\cdot10^{-28}$

The above model for particle emission from ^{238}U correctly derives:

1. The decay rate $|\frac{dN}{dt}|$ of ^{238}U: $|\frac{dN}{dt}|=1.2\cdot10^4\ \frac{particles}{gm\,sec}$.
2. Assumes the KE of the emitted particle is thermal i.e. $KE=6.1\cdot10^{-14}erg=0.038ev$
3. Derives m_{ph} and r_{ph} as functions of V_{ph} as tabulated in table 9.4.

Experimentally for ^{238}U, $\frac{dN}{dt}=1.2\cdot10^4\ \frac{particles}{gm\,sec}$ is independent of T for $300\lesssim T\lesssim 600^oK$

and therefore $1=(\frac{2}{\sqrt{\pi}})(\frac{n}{\lambda})\int_{\frac{E}{KT}}^{\infty} S^{\frac{1}{2}}e^{-S}dS$ must be independent of T over the same

range. i.e. For those atoms producing a 1.9ev photon, i.e. $KE_{ph}+|BE_{ph}|=1.9ev$

9.21 $$n(T_1)\int_{\frac{1}{K}(\frac{E_1}{T_1})}^{\infty} S^{\frac{1}{2}}e^{-S}dS=n(T_2)\int_{\frac{1}{K}(\frac{E_2}{T_2})}^{\infty} S^{\frac{1}{2}}e^{-S}dS$$

For example, with $T_1=293\ ^oK$ and $T_2=600\ ^oK$: $n(T_1)=2.9\cdot10^{14}\frac{col}{sec}$ and $n(T_2)=1.0\cdot10^{14}\frac{col}{sec}$. By direct calculation, $E_1=1.9ev$ and $E_2=3.8ev$. i.e. It requires more energy, $E_2>E_1$ at $T_2=600\ ^oK$, to cause ^{238}U to create and emit a particle with thermal kinetic energy KE_{ph}.0.078ev and decay rate $1.2\cdot10^4\ \frac{particles}{gm\,sec}$ and

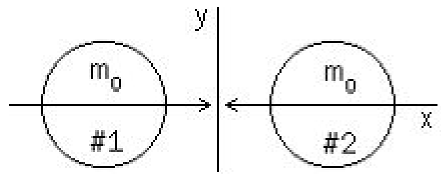
$|BE_{ph}|=3.8-0.078-\frac{m_u^2 H}{r_o^2}\Delta r_o$.

The change in energy $E_2 - E_1 = 1.9 ev = 3.0 \cdot 10^{-12} erg = \frac{m_u^2 H}{r_o^2} \Delta r_o$ goes into increasing the internal energy of the emitting ^{238}U atom with resultant computed increase in atomic radius of $\Delta r_o = 3.6 \cdot 10^{-15} cm$.

3. Very Energetic Atomic Collisions and the Higgs Particle

Consider two solid mass constant density atoms with equal energy and mass in a head on center of mass on center of mass elastic collision at $\underset{\sim}{x}=0$, t=0, where the initial translational kinetic energy is less than the absolute value of the binding energy. i.e. $KE_1<|m_oC_1|$ and $KE_2<|m_oC_1|$. All translational velocities measured from an inertial frame at rest at the center of mass of the 2 atom system. Figure 9.7.

FIGURE 9.7



Writing down the conservation of energy and momentum equations:

9.22a $$2KE_{1_i}+2m_oC_{1_i}=2KE_{1_f}+2m_oC_{1_f}<0,\quad C_{1_i}<0,\ C_{1_f}<0$$

$$m_oV_{1_i}\cdot\hat{x}+m_oV_{2_i}\cdot\hat{x}\pm m_o(2|C_{1_i}|)^{\frac{1}{2}}\cdot\hat{x}\mp m_o(2|C_{2_i}|)^{\frac{1}{2}}\cdot\hat{x}=$$

$$m_oV_{1_f}\cdot\hat{x}+m_oV_{2_f}\cdot\hat{x}\pm m_o(2|C_{1_f}|)^{\frac{1}{2}}\cdot\hat{x}\mp m_o(2|C_{2_f}|)^{\frac{1}{2}}\cdot\hat{x}$$

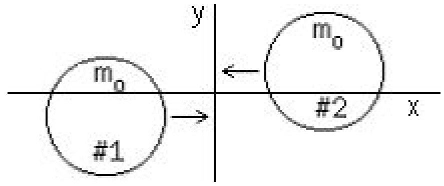
with $V_{1_i}+V_{2_i}=0$ and $V_{1_f}+V_{2_f}=0$. $\underset{\sim}{V}_{1_i}$ is the initial velocity of the center of mass of atom #1 and $\underset{\sim}{V}_{2_i}$ is the initial velocity of the center of mass of atom #2 and $\underset{\sim}{V}_{1_f}$ is the final velocity of the center of mass of atom #1 and $\underset{\sim}{V}_{2_f}$ is the final velocity of the center of mass of atom #2. KE is the translational kinetic energy, C_1 is given by 3.17. Using 9.22a,

9.22b $$\pm|C_{1_i}|\mp|C_{2_i}|=\pm|C_{1_f}|\mp|C_{2_f}|$$

From symmetry considerations, $C_{1_i}=C_{2_i}$ and $C_{1_f}=C_{2_f}$. If the collision is elastic, then $|V_{1_i}|=|V_{2_i}|=|V_{1_f}|=|V_{2_f}|$and $C_{1_i}=C_{2_i}=C_{1_f}=C_{2_f}$ and using 9.22b, 0=0. If the collision is inelastic then $|V_{1_i}|=|V_{2_i}|\gtrless|V_{1_f}|=|V_{2_f}|$ and $|C_{1_i}|=|C_{2_i}|\gtrless|C_{1_f}|=|C_{2_f}|$ and using 9.22b, again 0=0.

If the collision is not a center of mass on center of mass collision, fig 9.8, let $\chi_{1_f}=m_oC_{1_f}-\frac{1}{2}I\omega_{1_f}^2$ with $I=\frac{2}{5}m_or_o^2$, the conservation of energy and momentum equations become:

FIGURE 9.8



9.22c $$2KE_{1_i}+2m_oC_{1_i}=2KE_{1_f}+2(\chi_{1_f}+\tfrac{1}{2}I\omega^2_{1_f})<0,\ C_{1_i}<0,\ (\chi_{1_f}+\tfrac{1}{2}I\omega^2_{1_f})<0$$

$$m_oV_{1_i}\cdot\hat{x}+m_oV_{2_i}\cdot\hat{x}\pm m_o(2|C_{1_i}|)^{\frac{1}{2}}\cdot\hat{x}\mp m_o(2|C_{2_i}|)^{\frac{1}{2}}\cdot\hat{x}=$$

$$m_o\underset{\sim}{V}_{1_f}+m_o\underset{\sim}{V}_{2_f}+I\underset{\sim}{\omega}_{1_f}+I\underset{\sim}{\omega}_{2_f}\pm(2|m_o\chi_{1_f}|)^{\frac{1}{2}}\cdot\hat{x}\mp(2|m_o\chi_{2_f}|)^{\frac{1}{2}}\cdot\hat{x}$$

with $V_{1_i}+V_{2_i}=0$ and $V_{1_f}+V_{2_f}=0$ and $I\underset{\sim}{\omega}_{1_f}+I\underset{\sim}{\omega}_{2_f}=0$, therefore:

9.22d $$\pm(|m_oC_{1_i}|)^{\frac{1}{2}}\cdot\hat{x}\mp(|m_oC_{2_i}|)^{\frac{1}{2}}\cdot\hat{x}=\pm(|\chi_{1_f}|)^{\frac{1}{2}}\cdot\hat{x}\mp(|\chi_{2_f}|)^{\frac{1}{2}}\cdot\hat{x}$$

From symmetry considerations $C_{1_i}=C_{2_i}$ and $\chi_{1_f}+\frac{1}{2}I\omega^2_{1_f}=\chi_{2_f}+\frac{1}{2}I\omega^2_{2_f}$. If the collision is elastic, then $|V_{1_i}|=|V_{2_i}|=|V_{1_f}|=|V_{2_f}|$ and $m_oC_{1_i}=m_oC_{2_i}=\chi_{1_f}+\frac{1}{2}I\omega^2_{1_f}=\chi_{2_f}+\frac{1}{2}I\omega^2_{2_f}$ and using 9.22d, 0=0. If the collision is inelastic then $|V_{1_i}|=|V_{2_i}|\gtrless|V_{1_f}|=|V_{2_f}|$ and $|C_{1_i}|=|C_{2_i}|\gtrless|\chi_{1_f}+\frac{1}{2}I\omega^2_{1_f}|=|\frac{1}{2}I\omega^2_{2_f}|$ and using 9.22d, again 0=0.

The straightforward generalizations of 9.22a and 9.22c for the case $m_oV_{1_i}\cdot\hat{x}+m_oV_{2_i}\cdot\hat{x}\neq 0$ may be carried out by the reader

If the initial translational kinetic energy is much greater than the absolute value of the binding energy i.e. $KE_{1_i}=KE_{2_i}>>|m_oC_1|$. The collision is no longer elastic and the 2 atoms are in the process of destroying one another and creating multiple subatomic particles. The 2 atoms elongate in the yz plane of collision at x=0, $t=0^+$ fig. 9.9 and the subatomic particles acquire a component of velocity in the yz plane at x=0. 9.22a becomes:

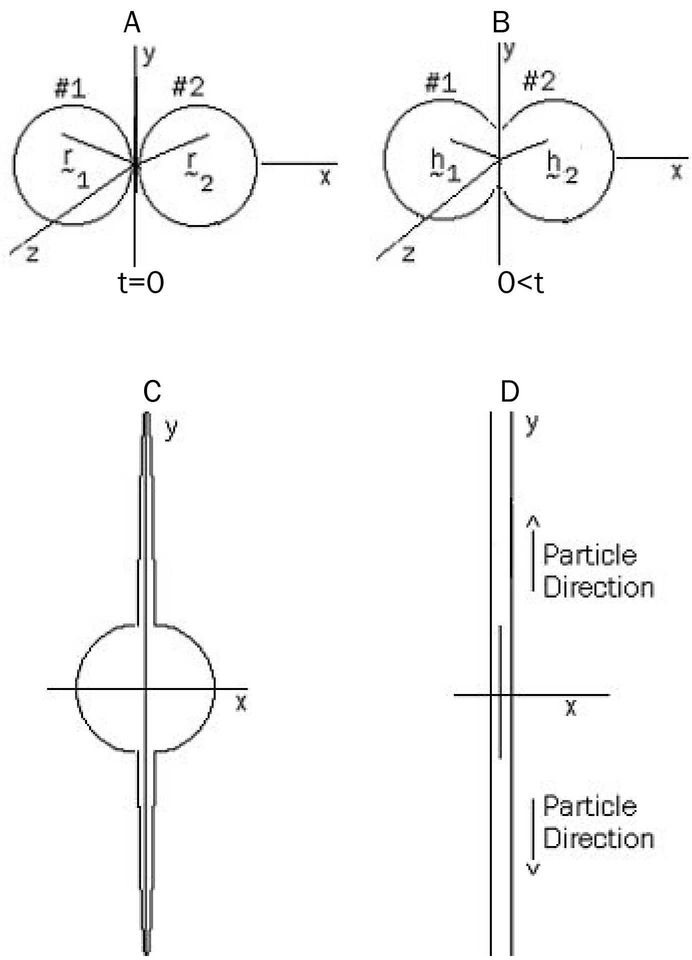
9.23 $$2\,KE_{1_i}+2m_oC_1=\sum_{j=1}^{n}\{\tfrac{1}{2}m_jU^2_{j\,f}+m_jC_{1,j_f}\}>0,\ KE_1>>|m_oC_1|,\ KE_2>>|m_oC_1|$$

$$m_oV_{1_i}\cdot\hat{x}+m_oV_{2_i}\cdot\hat{x}+2m_o(2|C_{1_i}|)^{\frac{1}{2}}\cdot\hat{x}=\sum_{j=1}^{n}\{m_j\underset{\sim}{U}_{j_f}+m_j(2|C_{1,j_f}|)^{\frac{1}{2}}\cdot\hat{x}\},\ n>>2$$

$V_{1_i}+V_{2_i}=0$ and U_{j_f} is the final speed of the j^{th} collision product. Using 9.23 note that, $\sum_{j=1}^{n}\left(m_j\mathbf{U}_{j_f}(\hat{y}+\hat{z})\right)=0$ although in general $m_j\mathbf{U}_{j_f}(\hat{y}+\hat{z})\neq 0$. We wish to show the conditions under which $\mathbf{U}_{j_f}\cdot\hat{x}=0$.

The two atoms collide at t=0. Figure 9.9A. Let $\mathbf{h}(\mathbf{r}_1,t)_1$ represent the position of any point in atom #1 as a function of time and let $\mathbf{h}(\mathbf{r}_2,t)_2$ represent the position of the mirror image point of $\mathbf{r}_1=x_1\hat{x}+y_1\hat{y}+z_1\hat{z})$ where $\mathbf{r}_2=-x_1\hat{x}+y_1\hat{y}+z_1\hat{z}$. As usual, $\mathbf{h}(\mathbf{r}_1,t)_1=\mathbf{r}_1+\boldsymbol{\chi}(\mathbf{r}_1,t)_1$, $\boldsymbol{\chi}(\mathbf{r}_1,0)_1=0$ with $\mathbf{r}_1$ restricted to the undeformed atom #1 at t=0 and $\mathbf{h}(\mathbf{r}_2,t)_2=\mathbf{r}_2+\boldsymbol{\chi}(\mathbf{r}_2,t)_2$, $\boldsymbol{\chi}(\mathbf{r}_2,0)_2=0$. $\mathbf{r}_1$ and $\mathbf{r}_2$ are independent of time and $\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{r}_1$ and $\mathbf{r}_2$ are measured from an inertial frame at rest w.r.t. the center of mass of atoms 1 and 2.

FIGURE 9.9



Considering both atoms to have zero spin about the x,y,z axis; $\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1$ and $\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2$ are compose of two parts, $\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1=\underset{\sim}{h}_{1,tr}+\underset{\sim}{h}_{1,vi}$ and $\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2=\underset{\sim}{h}_{2,tr}+\underset{\sim}{h}_{2,vi}$. $\underset{\sim}{h}_{1,tr}$, the translational motion, is measured from the center of mass of the two atoms to the center of mass of atom #1 and $\underset{\sim}{h}_{2,tr}$ is measured from the center of mass of the two atoms to the center of mass of atom #2.

(i) $\underset{\sim}{h}(-r_o\hat{x},t)_{1,tr}=-r_o\hat{x}+\chi(-r_o\hat{x},t)_{1,tr}\cdot\hat{x}$, $\chi(-r_o\hat{x},0)_{1,tr}\cdot\hat{x}=0$ and $\underset{\sim}{h}(-r_o\hat{x},0)_{1,tr}=-r_o\hat{x}$

(ii) $\underset{\sim}{h}(r_o\hat{x},t)_{2,tr}=r_o\hat{x}+\chi(r_o\hat{x},t)_{2,tr}\cdot\hat{x}$, $\chi(r_o\hat{x},0)_{2,tr}\cdot\hat{x}=0$ and $\underset{\sim}{h}(r_o\hat{x},0)_{2,tr}=r_o\hat{x}$

$\underset{\sim}{h}_{1,vi}$, the vibrational motion, is measured from the center of mass of atom #1 to the point $\underset{\sim}{r}_1$ in atom #1 and $\underset{\sim}{h}_{2,vi}$, the vibrational motion, is measured from the center of mass of atom #2 to the point $\underset{\sim}{r}_2$ in atom #2.

(iii) $\underset{\sim}{h}(r_o\hat{x}+\underset{\sim}{r}_1,t)_{1,vi}=r_o\hat{x}+\underset{\sim}{r}_1+\underset{\sim}{\chi}(r_o\hat{x}+\underset{\sim}{r}_1,t)_{1,vi}$, $\underset{\sim}{\chi}(r_o\hat{x}+\underset{\sim}{r}_1,0)_{1,vi}=0$ and $\underset{\sim}{h}(r_o\hat{x}+\underset{\sim}{r}_1,0)_{1,vi}=r_o\hat{x}+\underset{\sim}{r}_1$

(iv) $\underset{\sim}{h}(-r_o\hat{x}+\underset{\sim}{r}_2,t)_{2,vi}=-r_o\hat{x}+\underset{\sim}{r}_2+\underset{\sim}{\chi}(-r_o\hat{x}+\underset{\sim}{r}_2,t)_{2,vi}$, $\underset{\sim}{\chi}(-r_o\hat{x}+\underset{\sim}{r}_2,0)_{2,vi}=0$ and $\underset{\sim}{h}(-r_o\hat{x}+\underset{\sim}{r}_2,0)_{2,vi}=-r_o\hat{x}+\underset{\sim}{r}_2$

A mathematical model for the formation of the jet of particles follows. We wish to explain how the collision energy transforms the two atoms into a jet of particles traveling outward in the x=0 plane, figure 9.9.

Define δm_1 as the point like mass at $\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1$ and define δm_2 as the point like mass at $\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2$. Before the collision for t<0 and using 3.15,

9.24a $\quad \frac{1}{2}\delta m_1\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_{1,vi}]^2+\delta m_1\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1]_1=\delta m_1\cdot C_1<0$

$\frac{1}{2}\delta m_2\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_{2,vi}]^2+\delta m_2\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2]_2=\delta m_2\cdot C_1<0.$

In both cases, 9.23 written for δm_1 and δm_2 still hold. i.e.

9.24b $\quad \frac{1}{2}\delta m_1\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_{1,tr}]^2+\frac{1}{2}\delta m_1\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_{1,vi}]^2+\delta m_1\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1]_1>0$

$\frac{1}{2}\delta m_2\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_{2,tr}]^2+\frac{1}{2}\delta m_2\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_{2,vi}]^2+\delta m_2\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2]_2>0.$

After the collision for times $t_{\underset{\sim}{r}_1,\varepsilon_2}$, $t_{\underset{\sim}{r}_1,\varepsilon_1}$ with $t_{\underset{\sim}{r}_1,\varepsilon_2}>t>t_{\underset{\sim}{r}_1,\varepsilon_1}>0$, and after the collision for times $t_{\underset{\sim}{r}_2,\varepsilon_2}$, $t_{\underset{\sim}{r}_2,\varepsilon_1}$ with $t_{\underset{\sim}{r}_2,\varepsilon_2}>t>t_{\underset{\sim}{r}_2,\varepsilon_1}>0$:

9.24c $\quad \frac{1}{2}\delta m_1\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_{1,vi}]^2+\delta m_1\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_1,t)_1]_1>0$, for $t_{\underset{\sim}{r}_1,\varepsilon_2}>t>t_{\underset{\sim}{r}_1,\varepsilon_1}>0$

$\frac{1}{2}\delta m_2\cdot[\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_{2,vi}]^2+\delta m_2\cdot\psi[\underset{\sim}{h}(\underset{\sim}{r}_2,t)_2]_2>0$, for $t_{\underset{\sim}{r}_2,\varepsilon_2}>t>t_{\underset{\sim}{r}_2,\varepsilon_1}>0$

It is understood that $\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_{1,Vi}$ and $\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_{2,Vi}$ are the rms speeds.
We consider the case where δm_1 and δm_2 do not explode for $t_{\varepsilon_2}>t>t_{\varepsilon_1}>0$ because the pressure forces created by the collision acting on the mass surrounding δm_1 and δm_2, prevent δm_1 and δm_2 from exploding.
δm_1 and δm_2 are traveling in opposite x-directions with the same translational speed. With $\delta m_1=\delta m_2$ and due to the collision at t=0, at $t=t_{\varepsilon_2}$: $\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t_{\varepsilon_2})_{1,Tr}=-\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t_{\varepsilon_2})_{2,Tr}=0$, and $\overline{\dot{\underset{\sim}{h}}}(\underset{\sim}{r}_1,t_{\varepsilon_2})_{1,Vi}=\overline{\dot{\underset{\sim}{h}}}(\underset{\sim}{r}_2,t_{\varepsilon_2})_{2,Vi}\neq 0$. δm_1 and δm_2 are now on the solid planar surfaces of the two colliding atoms, and using 9.24c δm_1 and δm_2 escape from the solid planar surface and add their mass to the vapor like particle mass forming between the two colliding atoms. The unstable particles are constrained by the solid planar surfaces and escape with velocities in the yz plane: $\dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_1=(0, \dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_1\cdot\hat{y}, \dot{\underset{\sim}{h}}(\underset{\sim}{r}_1,t)_1\cdot\hat{z})$ and $\dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_2=(0, \dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_2\cdot\hat{y}, \dot{\underset{\sim}{h}}(\underset{\sim}{r}_2,t)_2\cdot\hat{z})$.

As regards the Higg's experiment, it is hypothesized that among the unstable particles escaping from the colliding solid mass atoms are to be found the 2 particles called tau leptons the existence of which is thought to provide evidence for the existence of the Higg's Boson. Although in no way agreeing with the Higg's theory, the author feels the existence of the unstable particles emitted by the colliding protons in the CERN Super Collider deserve the explanation given above.

From the above: 2 energetic atoms of the same species and translational kinetic energy traveling down the x axis in opposite directions collide at x=0 emitting a shower of particles in the yz plane at x=0. If now 4 atoms of the same species and translational kinetic energy traveling in the opposite $\hat{x},-\hat{x}, \hat{z},-\hat{z}$ directions, fig. 9.9, collide at $\underset{\sim}{x}=0$, the resultant jet of particles is in the $\hat{y},-\hat{y}$ along the y axis.

Returning to the two colliding atoms.
Let N equal the total number of particles emitted in the x=0 plane and let $\overline{\delta m}$ be the average mass of the N particles. Assuming $N\overline{\delta m}\doteq 2m_1$, then the total self binding energy of the N particles is, $BE_T=N\overline{C}_1\doteq N\overline{\Psi(s_0)}=-N\overline{(\delta m)^2\frac{H}{s_0}}$ and assuming statistical independence, $BE_T=-N(\overline{\delta m})^2\frac{H}{\overline{s}_0}$. The total final energy of the N particles is $E_{T,f}=\frac{1}{2}N\overline{\delta m}\cdot\overline{V^2}-N(\overline{\delta m})^2\frac{H}{\overline{s}_0}$. $\overline{V^2}$ is the average value of V^2, averaged over the N collision products particles.
The initial total energy is, $E_{T,i}=2[\frac{1}{2}m_1V_0^2-(m_1)^2\frac{H}{r_0}]$ where m_1 and V_o are the mass and translational speed of each atom at collision. Equating the two energies yields:

9.25a $E_{T,i}=2[\frac{1}{2}m_1V_0^2-(m_1)^2\frac{H}{r_0}]=\frac{1}{2}N\overline{\delta m}\cdot\overline{V^2}-N(\overline{\delta m})^2\frac{H}{\overline{s}_0}=E_{T,f}>0$

$BE_T=-N(\overline{\delta m})^2\frac{H}{\overline{s}_0}<0$

Valid for $\frac{1}{2}m_1V_0^2>(m_1)^2\frac{H}{r_0}$. Solving 9.25a for $\overline{s}_0$ yields:

9.25b $\overline{s}_0=-N(\overline{\delta m})^2H(E_{T,i}-\frac{1}{2}N\overline{\delta m}\cdot\overline{V^2})^{-1}$

References

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