Chapter 9. Rutherford Scattering, Radioactive Decay, Energetic Atomic Collisions

1. Rutherford Scattering

We reexamine Rutherford scattering, (Reference 9.1) with in the context of neutral solid mass atoms and particles under the influence of the central force field \( \Psi(r) \), and central force per unit mass \( f(r) = - \frac{d\Psi(r)}{dr} \). \( \Psi(r) \) is given by 3.8 with \( h_0 = r_0 \). i.e.

\[ \Psi(r) = - \frac{mH}{r_0} \left( \frac{2}{r_0} \right)^{d+1}, \quad r \leq r_0 \]
\[ |\Psi(r)| << \frac{mH}{r_0}, \quad r > r_0 \]

\[ f(r) = - \frac{mH}{r_0} \left( \frac{2}{r_0} \right)^{d+1}, \quad r \leq r_0 \]
\[ |f(r)| << \frac{mH}{r_0}, \quad r > r_0 \]

A neutral particle with the mass of He and incident kinetic energy \( \frac{1}{2}m_{He}v_0^2 \), hereafter called an \( \alpha \) particle, approaches a gold atom on a line passing a distance \( p \) from the center of the atom. Figure 9.1a.

Let \( r \) and \( \theta \) be polar coordinates for \( m_{He} \) with origin at the center of mass of the Au atom. The particle has components of velocity \( \frac{dr}{dt} \) and \( r \frac{d\theta}{dt} \). The field force, \( f_\alpha = -m_{He} \frac{d\Psi}{dr} \hat{r} \) for \( r \leq r_0 \) and \( |f_\alpha| << \frac{mH}{r_0^2} \), for \( r > r_0 \), has no component in the \( \hat{r} \) direction and

\[ |\hat{r} \times f_\alpha| = \hat{L} = I \cdot \alpha = I \frac{d^2\theta}{dt^2} = m_{He} r^2 \frac{d^2\theta}{dt^2} = 0. \]

Consequently the absolute value of the angular momentum \( I \omega \) is, \( I \omega = m_{He} r^2 \frac{d\theta}{dt} = \text{const} \). Evaluating \( I \omega \) at an arbitrary point for \( r > r_0 \), see fig. 9.1c: \( \sin \theta = \frac{V_\alpha}{V_0} = \frac{p}{r} \) and \( r \frac{d\theta}{dt} = V_0 p \) and \( I \omega = m_{He} r^2 \frac{d\theta}{dt} = m_{He} V_0 p \). The result also holds for \( r \leq r_0 \) as \( f_\alpha \) is radial.
FIGURE 9.1c

Writing down the laws for the conservation of energy and angular momentum:

9.1a \[
\frac{1}{2}m_{He}\left[\left(\frac{dr}{dt}\right)^2 + r^2(\frac{d\theta}{dt})^2\right] + m_{He}V(r) = \frac{1}{2}m_{He}V_0^2, \quad r < r_o
\]
9.1b \[
r^2\frac{d\theta}{dt} = V_o p, \quad p < r_o
\]
9.1c \[
\frac{1}{2}m_{He}\left[\left(\frac{dr}{dt}\right)^2 + r^2(\frac{d\theta}{dt})^2\right] = \frac{1}{2}m_{He}V_0^2, \quad r > r_o
\]
9.1d \[
r^2\frac{d\theta}{dt} = V_0 p, \quad p > r_o
\]

The problem is first examined assuming that \( m_{Au} \) and \( m_{He} \) are point particles and then re-examined assuming that \( m_{Au} \) is a solid mass atom of radius \( 1.28 \times 10^{-8} \text{cm} \) and \( m_{He} \) is a solid mass particle of a to be determined radius.

Eliminating \( \frac{d\theta}{dt} \) between 9.1 a and b and between 9.1 c and d and solving for \( \frac{dr}{dt} \) yields:

9.2a \[
\frac{dr}{dt} = \mp V_0 \left[ 1 - \frac{2\Psi(r)}{V_o^2} - \frac{P^2}{r^2} \right]^{\frac{1}{2}} = \mp V_0 \left[ 1 + \frac{2q}{r_0 (d+2)} \left[ (d+3) \cdot \left( \frac{r_0}{r} \right)^{d+2} \right] - \frac{p^2}{r^2} \right]^{\frac{1}{2}} \cdot q \equiv \frac{m_{Au}H}{V_o^2}, \quad r < r_o, \quad p < r_o
\]
9.2b \[
\frac{dr}{dt} = \mp V_0 \left[ 1 - \frac{p^2}{r^2} \right]^{\frac{1}{2}}, \quad r > r_o, \quad p > r_o
\]

The upper sign is used for \( dr < 0 \) and the lower sign is used for \( dr > 0 \). The \( \alpha \) particles from \( \text{Po}^{210} \) used in the Rutherford scattering experiment have \( V_o = 2.0 \times 10^9 \text{cm/sec} \) and translational kinetic energy \( KE_\alpha = \frac{1}{2}m_{He}V_0^2 = 1.36 \times 10^{-5} \text{ erg} = 8.4 \text{ Mev} \). As \( \frac{1}{8,000} \) of these particles back reflect from an Au foil (Ref. 9.1,9.2,9.3), the absolute value of the \( \alpha \) particle binding energy is: \( |B.E._\alpha| = \frac{m_{He}H}{r_\alpha^2} \geq 1.36 \times 10^{-5} \text{ erg} \). Solve for \( H \):

\[ H \geq \frac{r_\alpha^2}{m_{He}} = 0.30 \times 10^{-42}r_\alpha \] With \( r_\alpha \approx 10^{-13} \text{ cm} \); \( H \geq 0.30 \times 10^{-29} \text{ erg cm}^{-2} \).

At the point \( r_m \) of closest approach, \( \frac{dr}{dt} = 0 \). Solving 9.2 a and b for \( r_m \) yields:

9.3 \[
(r_m)^{(d+4)} - [(d+3) + r_0(d+2)(2q)^{-1}] (r_0)^{d+2} (r_m)^2 + (d+2)(2q)^{-1} (r_0)^{d+3} p^2 = 0, \quad r \leq r_o, \quad p \leq r_o, \quad d \neq 2
\]
9.4 \[
(r_m)^2 [1 - 2q(r_0)^{-1} \ln(r_0)^{-1}] \cdot p^2 = 0, \quad r \leq r_o, \quad p \leq r_o, \quad d = 2
\]
9.5 \[
r_m = p > r_0, \quad r > r_o
\]
Solutions for 9.3 are listed in table 9.2.
We wish to find angles $\theta_{es}$ and $\phi$ where $\theta_{es}$, the escape angle, is the angle from the +x axis to the point where the particle leaves the atom and $\phi$ is the angle the straight line trajectory of the alpha particle makes with the x-axis after it leaves the atom, where $\phi=\lim_{r \to \infty} (\theta-\pi)=\theta_{o}-\pi$ see fig. 9.1d and 9.2. The value of $\phi$ is derived below.

FIGURE 9.1d

Using $\frac{d\theta}{dt} = \frac{d\theta}{dr} \cdot \frac{dr}{dt}$ solve for $\frac{d\theta}{dr}$.

9.5 $\frac{d\theta}{dr} = \frac{p}{r^2} \left[ 1 + \frac{2q}{r_0(d+2)} \left( \frac{\hat{L}}{L_0} \right)^d \right] - \frac{p^2}{r^2} \right]^{-\frac{1}{2}}$

with

9.6 $\theta_1 = -p \int_{r_0}^{r_m} \left[ \frac{1 + \frac{2q}{r_0(d+2)} \left( \frac{\hat{L}}{L_0} \right)^d \right] \frac{1}{r^2} dr + \theta'_{r_0}$ where

$\theta'_{r_0} = \sin^{-1} \left( \frac{p}{r_0} \right)$. See fig. 9.2. $\theta_1$ is the angle between the +x axis and the point T on the circle of radius $r_m \leq P \leq r_0$ such that the trajectory of the $\alpha$ particle is tangent to the circle of radius $r_m$ at the point T. See fig. 9.3.
By symmetry, \( \theta_{ro} = 2\theta_1 \) where:

\[
9.7 \quad \theta_{ro} = 2\theta_1 = 2p \int_{r_0}^{r_m} \left[1 + \frac{2q}{r_0(d+2)} \left(\frac{d+2}{r_0^d} - \frac{p^2}{r^2}\right)\right]^{-\frac{1}{2}} dr + 2\theta_1
\]

The desired angle \( \phi \) is related to \( \theta_{ro} \) by:

\[
9.8 \quad \phi = \theta_{ro} - \pi
\]

Angle \( \theta_{es} \) is given by, \( \theta_{es} = (\theta_1 - \theta_1') + \theta_1 = 2\theta_1 - \theta_{ro}' = \theta_{ro} - \theta_0' \)

\[
9.9 \quad \theta_{es} = \theta_{ro} - \theta_0'
\]

All angles are in degrees. Note that \( \theta_0', \theta_{ro}, \) and \( \theta_{es} \) are measured from the +x axis and \( \phi \) is measured from the negative axis, \( r_m \) in cm. The integral in 9.7 is very sensitive to the value of \( r_m \), hence we look for another way to evaluate \( \phi \).
Experimentally, 99+% of all incoming α particles are deflected, \( \phi < 2^\circ \), by an Au foil 160 atoms thick. Ref. 9.1-9.3. and we choose H, H \( \geq 0.30 \cdot 10^{29} \text{erg cm}^{-2} \), to reflect this experimental fact. Using figure 9.3: \( \lim_{r_m \to P} \theta_1 = 90^\circ \) and \( \phi = 2\theta_1 = 180^\circ = 0 \) and the particle goes straight through the Au atom. We therefore require that \( r_m < P \), \( r_m = P \) and \( \phi < 2^\circ \). With these requirements, to the nearest order of magnitude, the best fit is \( H = 10^{30} \text{erg cm}^{-2} \). See Table 9.1A and B. Using Fig. 9.4 with \( P = \gamma r_0 \) and \( 0 \leq \gamma \leq r_0 \):

\[
\tan \phi_1 = (P - r_m)(x_1)^{-1} \pm P \left( \frac{r_m}{r_0^2} - P \right)^{-1/2} = \gamma(1 - \frac{r_m}{P})(1 - \gamma^2)^{-1/2}
\]

and \( \phi = 2\phi_1 = 2\tan^{-1}[\gamma(1 - \frac{r_m}{P})(1 - \gamma^2)^{-1/2}] \).

9.10 \( \phi = 2\phi_1 = 2\tan^{-1}[\gamma(1 - \frac{r_m}{P})(1 - \gamma^2)^{-1/2}] \)

A sample calculation follows. Using \( \gamma = 0.9 \), \( H = 10^{30} \text{erg cm}^{-2} \) and \( d = 0 \) where \( \rho(r) = \frac{(d+3)m}{4\pi r_0^3} r \), solve 9.3 for \( r_m \). Using 9.10: \( \phi = 2\tan^{-1}[\gamma(1 - \frac{r_m}{P})(1 - \gamma^2)^{-1/2}] = 2\tan^{-1}[0.9(1 - 1.2888)(1 - 0.9^2)^{-1/2}] = 1^\circ \).

**FIGURE 9.4**

**TABLE 9.1A**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( P )</th>
<th>( H )</th>
<th>( \phi )</th>
<th>( d )</th>
<th>( P )</th>
<th>( H )</th>
<th>( \phi )</th>
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<th>H</th>
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<td>0</td>
<td>( 10^{32} )</td>
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Using 9.3, 9.10 and \( H=1.0 \cdot 10^{30}\frac{erg\text{ cm}}{gm^2} \); Values of \( \theta'_0 \), \( \theta'_r \), \( \theta_{es} \) and \( \phi \) as a function of \( d \) and \( P \) are calculated below and tabulated in table 9.2. In the column labeled \( 10^8 r_m \), the upper number is calculated using 9.3 and the lower number is \( 10^8 P \), both numbers in cm.

TABLE 9.2

<table>
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<th>d</th>
<th>P</th>
<th>( \theta'_0 )</th>
<th>( \theta'_r )</th>
<th>( \theta_{es} )</th>
<th>( \phi )</th>
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<td>90</td>
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<td>117</td>
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<td>48</td>
<td>0.9682</td>
<td>180.9</td>
<td>132.9</td>
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<td>0.6449</td>
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<td>150.5</td>
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<tr>
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<td>166.2</td>
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<table>
<thead>
<tr>
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<th>P</th>
<th>( \theta'_0 )</th>
<th>( \theta'_r )</th>
<th>( \theta_{es} )</th>
<th>( \phi )</th>
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It must be remembered that tables 9.1A, 9.1B and 9.2 are the consequence of a field interaction between an assumed point mass Au atom and a point mass alpha particle. With this assumption, there are no contact forces.
We examine under what conditions an \( \alpha \) particle with radius \( r_\alpha = 1.3 \cdot 10^{-13} \text{cm} \) that enters a Au atom with radius \( r_0 = 1.3 \cdot 10^{-13} \text{cm} \) and travels along a diameter, can exit with 99+% of its incoming energy and with \( \phi < 2^\circ \).

Using 6.25, the mass \( m_{\text{cyl}} \) of a cylinder with radius \( r_\alpha \) and length \( 2r_0 \) (The diameter of a Au atom) to one significant digit is:

\[
m_{\text{cyl}} = m_{\text{Au}} \left( \frac{r_\alpha + 9}{5} \right) \left( \frac{r_\alpha}{r_0} \right)^{(p+3)} \cdot \left( \frac{1 - \left( \frac{r_\alpha}{r_0} \right)^{(p+1)}}{2(p+1)} \right). \text{, } r_\alpha < < r_0, -3 < p \leq 0
\]

The computed ratios \( \frac{m_{\text{cyl}}}{m_{\text{Au}}} \) and \( \frac{m_{\text{bly}}}{m_{\alpha}} \cdot 49 \cdot \frac{m_{\text{bly}}}{m_{\text{Au}}} \) are given to 1 significant figure in table 9.3 as a function of \( p \) using \( r_\alpha = 1.0 \cdot 10^{-13} \) and \( r_0 = 1.3 \cdot 10^{-8} \)

| \( p \) | \( \frac{m_{\text{cyl}}}{m_{\text{Au}}} \) | \( \frac{m_{\text{bly}}}{m_{\alpha}} \) | \( \frac{\text{KE}_{\text{bly}}}{\text{KE}_{\alpha}} \) | \( |\text{BE}_{\text{cyl}}| \) (ev) | \( \frac{|\text{BE}_{\text{cyl}}|}{\text{KE}_{\alpha}} \) | \( \text{KE}_{\text{cyl}} \) (ev) |
|----|----------------|----------------|----------------|----------------|----------------|----------------|
| 0  | 0.9\cdot 10^{-10} | 4\cdot 10^{-9} | 2\cdot 10^{-8} | 2 | 2\cdot 10^{-7} | 2\cdot 10^{-7} |
| -1 | 1\cdot 10^{-9}  | 5\cdot 10^{-8} | 2\cdot 10^{-7} | 20 | 2\cdot 10^{-6} | 2\cdot 10^{-6} |
| -2 | 0.8\cdot 10^{-5} | 4\cdot 10^{-4} | 2\cdot 10^{-3} | 1\cdot 10^{5} | 1\cdot 10^{-2} | 1\cdot 10^{-2} |
| -2.99 | 0.41 | 20 | 0.95 | 7\cdot 10^{9} | 8\cdot 10^{2} | 8\cdot 10^{2} |

At \( t=0 \), an alpha particle with kinetic energy \( \text{KE}_{\alpha} = \frac{1}{2} m_{\alpha}(V_\alpha)^2 = 1.4 \cdot 10^{-5} \text{erg} = 8.7 \cdot 10^{6} \text{ ev} \) and momentum \( m_{\alpha} V_{\alpha} (r_0) = -m_{\alpha} V_{\alpha} (r_0) \hat{x} \), \( V_{\alpha}(r_0) = 2 \cdot 10^{9} \text{cm} \text{s}^{-1} \), strikes an atom (Figure 9.5).

For \( m_{\text{bly}} < < m_{\alpha} \), the energy transferred to the cylinder is \( \text{KE}_{\text{cyl}} = \frac{1}{2} m_{\text{cyl}} (2V_\alpha)^2 + |\text{BE}_{\text{cyl}}| \) and

\[
\frac{\text{KE}_{\text{bly}}}{\text{KE}_{\alpha}} = \frac{\frac{1}{2} m_{\text{cyl}} (2V_\alpha)^2}{\frac{1}{2} m_{\alpha} V_{\alpha}^2} = 4 \cdot \frac{m_{\text{cyl}}}{m_{\alpha}}. \text{Table 9.2 lists computed values of } \frac{\text{KE}_{\text{bly}}}{\text{KE}_{\alpha}} \text{ to one significant figure.}
\]

\( \text{BE}_{\text{cyl}} \) is computed using 6.26 (With \( r_\alpha \) in place of \( r_{\text{ph}} \)) and 9.11.

\[
\text{BE}_{\text{cyl}} = -\frac{m_{\text{Au}} H}{r_0} \cdot m_{\text{cyl}} = -\frac{m_{\text{Au}}}{r_0} \cdot m_{\text{cyl}} = -2.7 \cdot 10^{-2} \cdot \frac{m_{\text{cyl}}}{m_{\text{Au}}} \text{ erg} = -1.7 \cdot 10^{40} \cdot \frac{m_{\text{cyl}}}{m_{\text{Au}}} \text{ ev}
\]

\[
\text{BE}_{\text{cyl}} = -1.7 \cdot 10^{10} \cdot \frac{m_{\text{cyl}}}{m_{\text{Au}}} \text{ ev}
\]
Using 9.12 and $KE_{\alpha}=8.7 \cdot 10^6 \text{ ev}$, one can compute $|BE_{\text{e}y1}|$ and $E_{\text{e}y1}$. Using table 9.2:

For $-2 \leq p \leq 0$, the $E_{\text{e}y1} \frac{K}{KE_{\alpha}} < 1$ and the alpha particle goes right through the Au atom in a straight line with essentially undiminished translational kinetic energy. Experimentally, approximately 1 in 8,000 alpha particles are reflected with $\phi > 90^\circ$. Ref. 9.3.

Now consider for $p = -2.99$: $E_{\text{e}y1} \frac{K}{KE_{\alpha}} = 8 \cdot 10^2$ and in general for $p$, $0 < p + 3 << 1$, the alpha particle does not have enough translational kinetic energy to go through the center of the Au atom and will either be adsorbed by the atom or be reflected with $\phi > 90^\circ$. It is therefore conjectured that those alpha particles that have $\phi > 90^\circ$ have struck an atom with $0 < p + 3 << 1$, and with $p$ sufficiently small so that $E_{\text{e}y1} \frac{K}{KE_{\alpha}} >> 1$.

2. Radioactive Decay

Within the confines of quantum mechanics, radioactive decay remains an enigma in so far as there is no fundamental answer to the question, “What determines at what instant a given atom will decay”. It is currently believed that an internal clock in each atom determines the instant at which the atom decays. We investigate the assumption that all radioactive decay is stimulated radioactive decay, that is, small mass particles adsorbed by the atom cause the atom to emit energetic $\alpha$ or $\beta$ or $\gamma$ or etc. particles. The problem then becomes one of accounting for the formation of the mass and energy of the stimulating small mass particles and the mass and energy of the resultant decay product and to account for the difference between an atom that decays and one that does not. If our assumption is correct, then the decay rate and half-life of an atom in elemental form is not necessarily the decay rate and half life of an atom in compound form and the decay rate and half-life may depend on the geometry of the radioactive sample. It is interesting to note that the assumption that all atoms currently on earth were created at the same time may be false, see chapter 11, section 11. The experimentally determined decay rate of $^{235}\text{U}$ mined in Canada may therefore not be the same as the decay rate of $^{235}\text{U}$ mined in India.
The $^{238}$U series starting with $^{238}$U and ending with stable $^{206}$Pb is of interest here. Naturally occurring $^{238}$U decays by α emission with a half-life of $t_1 = 4.5 \times 10^9$ years with $\lambda = 4.88 \times 10^{-18 \frac{1}{\text{sec}}}$ Consider 1 gm of $^{238}$U containing $N_1 = 2.5 \times 10^{21}$ atoms, with decay rate at t=0: $\frac{dN}{dt} = \lambda N_1 = 1.2 \times 10^4 \frac{\alpha}{\text{gm sec}}$. In general with $N = N_0 e^{-\lambda t}$ and $\frac{1}{N} \frac{dN}{dt} = \lambda$, the probability that a given atom will decay in the next second is $\lambda$. A major thrust of this text is to rid physics of the concept of charge and to create new models for “charge effects”. The α particle is here considered to be charge (q) neutral. See chapter 7 and 8. A basic assumption of the following model is that naturally occurring $^{238}$U atoms emits charge neutral 4.2Mev alpha particles.

From chapter 4, section 3: Let $\Delta t$ represent the average time interval measured from the instant that atom #1 is in contact with atom #2 until the instant that atom #1 is again in contact with atom #2: $\Delta t = 2 \frac{R(t,T)}{u_k(t)} = 2 \frac{R(t,T)}{2 \pi m \sqrt{K_T}}$. From table 4.1, $R(t,T)$ for U is $R(t,T) = 7.07 \times 10^{-11}$ cm and $\Delta t = 3.5 \times 10^{-14}$ sec. From the instant that atom #1 is in contact with atom #2 until the instant that atom #1 is again in contact with atom #2, atom #1 collides with its six neighbors at the rate of, $n_o = 10 / \Delta t = 2.9 \times 10^{14}$. Let $n_E$ represent the average number of times $N(t_0)$ atoms have $\geq E$ per unit time at $t_0$ where $K$ is measured between collisions. $N_0 n_o$ represents the total number of collisions of $N_0$ per unit time at $t_0$. Using 6.12:

$$n(t_0) = \frac{dN}{dt} = \lambda N(t_0) = \frac{2}{\sqrt{\pi}} N_0 n_0 \int_{-\infty}^{\infty} S e^{-S} dS,$$

and $1 = \frac{2}{\sqrt{\pi}} \frac{n_0}{\lambda} \int_{-\infty}^{\infty} S e^{-S} dS$. Where E is the translational K.E. of the U$^{238}$ atom. With $n_0 = 5.9 \times 10^{31}$, by direct computation $E_K = 75.4$ and at $T = 293^0K$, $E = 1.9\text{ev} = 3.0 \times 10^{-12}$ erg. Before collision, let atom #1 have initial translational kinetic energy $KE_{1,U_i} = \frac{1}{2} m U_i^2 = 1.9\text{ev}$ and let atom #2 have initial translational kinetic energy $KE_{2,U_i} = \frac{1}{2} m U_i^2$. If the collision were an elastic collision, the two atoms would exchange translational kinetic energies and the final translational kinetic energies would be: $KE_{1,U_f} = KE_{2,U_i}$ and $KE_{2,U_f} = KE_{1,U_i}$. In the present case let $t=0$ represent the instant during the collision at which the c.m. of #1 and the c.m. of #2 are at rest wrt one another. 1.9ev is in the form of percussive and shear wave energy in #2 and $\frac{1}{2} m U_i^2$ is in the form of percussive and shear wave energy in #1.

By the method of Chapter 6, Section 5, #2 produces and ejects a photon in the $\hat{x}$ direction. Let $W_E$ represent wave energy. At $t=0$, $W_{E,1,U_i} = KE_{2,U_i}$ and $W_{E,2,U_i} = KE_{1,U_i}$
Assuming that \( \text{WE}_{2,U_i} \) goes into creating a photon plus K.E. \( \text{WE}_{2,U_f} \), see fig. 9.6, at separation:

\[
\text{WE}_{2,U_i} = \text{K.E.}_{ph} + |\text{B.E.}_{ph}| + \text{K.E.}_{2,U_f} = \text{K.E.}_{ph} + |\text{B.E.}_{ph}| + \frac{m_{ph}}{m_U} \text{K.E.}_{ph} = \text{K.E.}_{ph} + |\text{B.E.}_{ph}| \leq 1.9\text{ev}
\]

and \( \text{KE}_{1,U_f} = \text{KE}_{2,U_i} \) with photon binding energy \( \text{B.E.}_{ph} = \frac{m_{ph}^2 U_{ph,f}}{r_{ph,f}} \).

Figure 9.6

Atom #2 creating a photon

9.13 \[
\text{WE}_{2,U_i} = \text{K.E.}_{ph} + |\text{B.E.}_{ph}| \leq 1.9\text{ev} \\
\text{KE}_{1,U_f} = \text{KE}_{2,U_i} \\
\text{KE}_{2,U_f} = \frac{m_{ph}}{m_U} \text{K.E.}_{ph}
\]

From 3.17, \( \frac{1}{2} U_{rms}^2 \Psi(h) = C_1 = \frac{1}{2} U_{rms}^2 (h_0) + \Psi(h_0) \) and the binding energy of the atom is:

\[
\text{BE} = T + V = m C_1 = \frac{1}{2} m U_{rms}^2 (h_0) + m \Psi(h_0), \text{ where } T + V = 2 \pi \int_0^{h_0} \Psi(h) U_{rms}^2 (h) dh + 4 \pi \int_0^{h_0} \Psi(h) \Psi(h) dh
\]

and using 3.8, \( \Psi(h) = - \frac{m H}{h_0 (p+2)} \left[ 1 - \frac{1}{p+3} \right] \left( \frac{h}{h_0} \right)^{p+2} \) With \( \Psi(h_0) = \frac{m H}{h_0} \), the binding energy becomes \( \text{BE} = \frac{1}{2} m U_{rms}^2 (h_0) - \frac{m H}{h_0} \). With density \( \rho(h) = \frac{(p+3) m}{4 \pi h_0^3} \left( \frac{h}{h_0} \right)^p \), and by direct computation:

9.14 \[
V = - \frac{m^2 H}{h_0 (p+2.5)}, \quad T = \frac{m^2 H}{h_0 (p+2.5)}, \quad \frac{1}{2} m U_{rms}^2 (h_0) = 2.5 \leq p \leq 0, \quad T + V = \frac{1}{2} m U_{rms}^2 (h_0) - \frac{m^2 H}{h_0} < 0 \\
V = - \infty, \quad T = + \infty, \quad - 3 < p < - 2.5, \quad T + V = \frac{1}{2} m U_{rms}^2 (h_0) - \frac{m^2 H}{h_0} < 0
\]

Note that \( T(\varepsilon) \) and \( V(\varepsilon) \) are \( 0 < T(\varepsilon) < \infty \) and \( - \infty < V(\varepsilon) < 0 \) where \( T(\varepsilon) = 2 \pi \int_0^{\infty} \rho(h) U_{rms}^2 (h) dh \)
\[ V(\varepsilon) = 4\pi \int_{-\varepsilon}^{\varepsilon} h^2 \Phi(h) dh \] where \( 0 < \varepsilon < h_0 \) and \( -3 < p \leq 0 \).

The potential energy, \( P.E_{ph,At} \), between a photon of mass \( m_{ph} \) and radius \( h_{0,ph} \) and an atom of mass \( m_{At} \) and radius \( h_{0,At} \) for \( h_{0,ph} \ll r_{0,At} \) is given by:

\[ P.E.(h)_{ph,At} = -m_{ph}m_{At} \frac{h}{h_{0,At}} \left[ \frac{1}{(p+2)} \left[ p+3 - \left( \frac{h}{h_{0,At}} \right)^{p+2} \right] \right], \quad h \leq h_{0,At} \]

With \( \Delta P.E.(h)_{ph,At} = P.E.(h)_{ph,At} - P.E.(h_{0,At})_{ph,At} \) \( \Delta P.E.(h)_{ph,At} \) is given by:

\[ \Delta P.E.(h)_{ph,At} = m_{ph}m_{At} \frac{h}{h_{0,At}} \left[ 1 - \frac{1}{(p+2)} \left[ p+3 - \left( \frac{h}{h_{0,At}} \right)^{p+2} \right] \right], \quad h \leq h_{0,At} \]

Where \( h(\ell,t) = \ell + \xi(\ell,t) \) with \( \xi(\ell,0) = 0 \) and \( \xi \) independent of time \( t \). With \( <> \) representing the space average value of \( |h(\ell,t)| \) on the surface of the sphere of radius \( r \) at time \( t \); \( h \) is defined as \( h(r,t) = <> |h(\ell,t)| = <> |\ell + \xi(\ell,t)| >. \)

We are interested in the case \( p = -3 + \varepsilon \), \( 0 < \varepsilon < 1 \) for which \( \rho(h_{0,-3+\varepsilon}) = 0 \) and consequently the oscillation amplitude of the atom surface will be appreciable i.e. \( \sim 0.1 < \frac{a}{h_0} < \sim 1 \). Although ignored here, this will alter the B.E. of the atom. See Chapter 10, Section 4. It can be shown for \( 1 \leq \varepsilon \leq 3 \), that \( m_{ph} > 10^{-24} \) gm which is non-physical.

\[ -\Delta P.E.(h)_{ph,U} \] becomes:

\[ -\Delta P.E.(h)_{ph,U} = 2.9 \cdot 10^{-16} m_{ph} \frac{h_{0,U}}{h} \left( 1 - \varepsilon \right). \]

From 9.13:

\[ K.E. + |B.E._{ph}| = K.E. + |\Omega_{ph}| = 1.99 eV = 3.0 \cdot 10^{-12} \text{erg} \]

and with \( K.E._{ph} \leq \Omega_{ph}^{2} \) then,

\[ r_{ph,f} \leq 6.7 \cdot 10^{41} m_{ph}^{2}. \]

We require \( -\Delta P.E.(h)_{ph,U} = |\Omega_{\alpha,Th}^{h} + 2KE_{\alpha} + KE_{Th} \) where \( B.E._{\alpha,Th} \) is the energy binding the alpha particle to Thorium while in the Uranium atom and \( K.E_{Th} = \frac{4}{234} K.E._{\alpha} = 0.02 K.E._{\alpha} \)

where it is assumed that \( |\Omega_{\alpha}| = K.E._{\alpha}. \)

\[ -\Delta P.E.(h)_{ph,U} = 2.9 \cdot 10^{-16} m_{ph} \left( \frac{h_{0,U}}{h} \right) \left( 1 - \varepsilon \right) = |\Omega_{\alpha,Th}^{h}| + 2.02 K.E._{\alpha} \]

with \( |\Omega_{\alpha,Th}^{h}| = \Omega_{Th}^{h} = 1.6 \cdot 10^{-7} \text{erg} = 1.0 \cdot 10^{-5} \text{ev} \) and \( 2.02 K.E._{\alpha} = 1.3 \cdot 10^{-5} \text{erg} \) and

\( -\Delta P.E.(h)_{ph,U} \) becomes:

\[ -\Delta P.E.(h)_{ph,U} = 1.3 \cdot 10^{-5} \text{erg} = 2.9 \cdot 10^{-16} m_{ph} \left( \frac{h_{0,U}}{h} \right) \left( 1 - \varepsilon \right). \]

Solving for...
\[ m_{ph} \text{ and } r_{ph,f} \text{ yields:} \]

\[ 9.18 \quad m_{ph} = 4.5 \cdot 10^{-22} \left( \frac{h}{h_{b,U}} \right)^{(1-\varepsilon)}, \quad r_{ph,f} \leq 0.13 \left( \frac{h}{h_{b,U}} \right)^{2(1-\varepsilon)}, \quad -\Delta \text{P.E.}(h)_{ph,U} = 1.3 \cdot 10^{-5} \text{erg}. \]

With \(0 < \varepsilon < 1\).

For \(0 < \varepsilon << 1\). By direction computation using the density function \( \rho(h) = \left( \frac{p+3}{4 \pi h_{0}^{3}} \right) \rho_{h} \),

the mass \( M(h) \) of the \(^{238}\text{U}\) atom between 0 and \( h \) is \( M(h) = m_{U}(\frac{h}{h_{0}})^{\varepsilon} \), with ratio: \( \frac{M(h)}{m_{U}} = (\frac{h}{h_{0}})^{\varepsilon} \).

With \(0 < \varepsilon < 10^{-2}\), 99\% of the mass lies within radius \( h \) where \( h = 0.99 \cdot r_{o} < 0.10 \), \( r_{o} = 0.37 \cdot r_{o} \), i.e. \( h < 0.37 \cdot r_{o} \), where \( r_{o} \) is the radius of the atom.

This is not physical as a sample of pure \(^{238}\text{U}\) would have a bulk modulus much smaller than the experimentally determined value. A naturally occurring sample of pure \(^{238}\text{U}\) with \(0 < \varepsilon << 1\) does not exist.

For \(10^{-2} < \varepsilon < 1\). It can be shown that the absolute value of the binding energy \( |\text{B.E.}_{ph}| \) of the volume \( V_{cyl} \) reamed out by the photon on its passage from \( r_{o} \) to \( h \) is larger than the kinetic energy of the photon where \( r_{o} \) - \( h \) is the distance the photon must travel in order to have the kinetic energy necessary to create a 4.2 Mev particle. Therefore for \(10^{-2} < \varepsilon < 1\), the incoming photon will not create a 4.2 Mev particle.

The volume \( V_{cyl} \) is: \( V_{cyl} = \pi \left( r_{o}^{2} - h \right) + \frac{2}{3} \pi r_{ph}^{3} \), where \( h \) is at the center of mass of the photon, \( 0 < h < r_{o} \).

A basic assumption for the above model is that naturally occurring \(^{238}\text{U}\) atoms emit 4.2Mev alpha particles. It is the author’s conclusion that the range energy relation is not true due to misidentification of the mass, speed, kinetic energy, radius and charge (Amplitude of radial oscillation) of the particle emitted by \(^{238}\text{U}\) and that the KE of the emitted particle is \( KE << 4.2\text{Mev} \).

An interesting solution to the problem is to assume that the alpha particle is actually the photon described below with \( m_{ph} << m_{\alpha} <^{238}\text{U} \) and \( KE_{ph} + |\text{B.E.}_{ph}| = \frac{1}{2} m_{ph} V_{ph}^{2} + |\text{B.E.}_{ph}| = 1.9 \text{ev} = 3.0 \cdot 10^{-12} \text{erg} \) with \(1.5 \cdot 10^{-12} < |\text{B.E.}_{ph}| < 3.0 \cdot 10^{-12} \text{erg} \). Assume the \( KE_{ph} \) is thermal, i.e.

\[ KE_{ph} = 1.5KT = 6.1 \cdot 10^{-14} \text{erg}, \quad \frac{3}{2} m_{ph} V_{ph}^{2} \text{ with } |\text{B.E.}_{ph}| = 3.0 \cdot 10^{-12} (1-2 \cdot 10^{-2}) \text{erg} = \frac{m_{ph}^{2} H}{r_{ph}}. \]

Solve for \( V_{ph} \) and \( m_{ph} \). This yields, \( m_{ph} = \frac{4.2 \cdot 10^{-13}}{V_{ph}^{2}} \) and \( r_{ph} = \frac{4}{3} \cdot 10^{42} m_{ph}^{2} = 0.5 \cdot 10^{16} \).

Computed values of \( m_{ph} \) and \( r_{ph} \) as functions of \( V_{ph} \) are given in Table 9.4.
The above model for particle emission from $^{238}\text{U}$ correctly derives:

1. The decay rate $|\frac{dN}{dt}|$ of $^{238}\text{U}$: $|\frac{dN}{dt}| = 1.2 \times 10^{-4}$ particles/gmsec

2. Assumes the KE of the emitted particle is thermal i.e. $\text{KE} = 6.1 \times 10^{-14}$ erg = 0.038ev

3. Derives $m_{\text{ph}}$ and $r_{\text{ph}}$ as functions of $V_{\text{ph}}$ as tabulated in table 9.4.

Experimentally for $^{238}\text{U}$, $\frac{dN}{dt} = 1.2 \times 10^{-4}$ particles/gmsec is independent of $T$ for $300 \leq T \leq 600^0\text{K}$ and therefore $I = (\frac{2}{\sqrt{\pi}}) \int_{\frac{E}{kT}}^{\infty} S \frac{1}{2} e^{-S} dS$ must be independent of $T$ over the same range. i.e. For those atoms producing a 1.9ev photon, i.e. $\text{KE}_{\text{ph}} + |\text{BE}_{\text{ph}}| = 1.9ev$

9.21

For example, with $T_1 = 293^0\text{K}$ and $T_2 = 600^0\text{K}$: $n(T_1) = 2.9 \times 10^{14} \text{col/sec}$ and $n(T_2) = 1.0 \times 10^{14} \text{col/sec}$. By direct calculation, $E_1 = 1.9ev$ and $E_2 = 3.8ev$. i.e. It requires more energy, $E_2 > E_1$ at $T_2 = 600^0\text{K}$, to cause $^{238}\text{U}$ to create and emit a particle with thermal kinetic energy $\text{KE}_{\text{ph}}$, $0.078ev$ and decay rate $1.2 \times 10^{-4}$ particles/gmsec and $|\text{BE}_{\text{ph}}| = 3.8 - 0.078 \frac{m_{\text{ph}}^2}{r_0^2} \Delta r_0$. 

TABLE 9.4

<table>
<thead>
<tr>
<th>$V_{\text{ph}}$ cm/sec</th>
<th>$m_{\text{ph}}$ gm</th>
<th>$r_{\text{ph}}$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^5$</td>
<td>$1.2 \times 10^{-23}$</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$10^6$</td>
<td>$1.2 \times 10^{-25}$</td>
<td>$1.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>$10^7$</td>
<td>$1.2 \times 10^{-27}$</td>
<td>$1.2 \times 10^{-12}$</td>
</tr>
<tr>
<td>$10^8$</td>
<td>$1.2 \times 10^{-29}$</td>
<td>$1.2 \times 10^{-16}$</td>
</tr>
<tr>
<td>$10^9$</td>
<td>$1.2 \times 10^{-31}$</td>
<td>$1.2 \times 10^{-20}$</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>$1.2 \times 10^{-33}$</td>
<td>$1.2 \times 10^{-24}$</td>
</tr>
<tr>
<td>$10^{11}$</td>
<td>$1.2 \times 10^{-35}$</td>
<td>$1.2 \times 10^{-28}$</td>
</tr>
</tbody>
</table>
The change in energy \( E_2 - E_1 = 1.9 \text{ev} = 3.0 \times 10^{-12} \text{erg} = \frac{m_0^2}{r_0^2} \Delta r_0 \) goes into increasing the internal energy of the emitting \(^{238}\text{U}\) atom with resultant computed increase in atomic radius of \( \Delta r_0 = 3.6 \times 10^{-15} \text{cm} \).
3. Very Energetic Atomic Collisions and the Higgs Particle

Consider two solid mass constant density atoms with equal energy and mass in a head on center of mass on center of mass elastic collision at $\chi=0$, $t=0$, where the initial translational kinetic energy is less than the absolute value of the binding energy. i.e. $KE_1<|m_0C_1|$ and $KE_2<|m_0C_1|$. All translational velocities measured from an inertial frame at rest at the center of mass of the 2 atom system. Figure 9.7.

![Figure 9.7](image)

Writing down the conservation of energy and momentum equations:

$$2KE_{1i}+2m_0C_{1i}=2KE_{1f}+2m_0C_{1f}<0, \ C_{1i}<0, \ C_{1f}<0$$

$$m_0V_{1i}\cdot\hat{x}+m_0V_{2i}\cdot\hat{x}+m_0(2|C_{1i}|)^{1/2}\cdot\hat{x}=\frac{1}{2}m_0(2|C_{1i}|)^{1/2}\cdot\hat{x}$$

$$m_0V_{1f}\cdot\hat{x}+m_0V_{2f}\cdot\hat{x}+m_0(2|C_{1f}|)^{1/2}\cdot\hat{x}=\frac{1}{2}m_0(2|C_{1f}|)^{1/2}\cdot\hat{x}$$

with $V_{1i}+V_{2i}=0$ and $V_{1f}+V_{2f}=0$. $V_{1i}$ is the initial velocity of the center of mass of atom #1 and $V_{2i}$ is the initial velocity of the center of mass of atom #2 and $V_{1f}$ is the final velocity of the center of mass of atom #1 and $V_{2f}$ is the final velocity of the center of mass of atom #2. $KE$ is the translational kinetic energy, $C_1$ is given by 3.17. Using 9.22a,

$$|C_{1i}|=|C_{1f}|=|C_{2i}|=|C_{2f}|$$

From symmetry considerations, $C_{1i}=C_{2i}$ and $C_{1f}=C_{2f}$. If the collision is elastic, then $|V_{1i}|=|V_{2i}|=|V_{1f}|=|V_{2f}|$ and $C_{1i}=C_{2i}=C_{1f}=C_{2f}$ and using 9.22b, 0=0. If the collision is inelastic then $|V_{1i}|=|V_{2i}|>|V_{1f}|=|V_{2f}|$ and $|C_{1i}|=|C_{2i}|>|C_{1f}|=|C_{2f}|$ and using 9.22b, again 0=0.

If the collision is not a center of mass on center of mass collision, fig 9.8, let $\chi_{1f}=m_0C_{1f}\cdot\frac{\chi_1^2}{2}$ with $l=\frac{2}{5}m_0C_1^2$, the conservation of energy and momentum equations become:
FIGURE 9.8

\[ \begin{array}{c}
\begin{array}{c}
\text{m}_0 \\
#2
\end{array} \\
\downarrow \quad \downarrow \quad \downarrow \\
\begin{array}{c}
\text{m}_0 \\
#1
\end{array}
\end{array} \]

9.22c \quad 2KE_{1i} + 2m_0C_{1i} = 2KE_{1f} + 2(\chi_{1i} + \frac{1}{2}i\omega_{1f}^2) < 0, \quad C_{1i} < 0, (\chi_{1i} + \frac{1}{2}i\omega_{1f}^2) < 0

\begin{align*}
& m_0V_{1i} \cdot \hat{\chi} + m_0V_{2i} \cdot \hat{\chi} + m_0(2|C_{1i}|) \frac{1}{2} \hat{\chi} \mp m_0(2|C_{2i}|) \frac{1}{2} \hat{\chi} = \\
& m_0V_{1f} + m_0V_{2f} \mp |\omega_{1f}^2| \pm(2|m_0\chi_{1i}|) \frac{1}{2} \hat{\chi} \mp(2|m_0\chi_{2i}|) \frac{1}{2} \hat{\chi}
\end{align*}

with \( V_{1i} + V_{2i} = 0 \) and \( V_{1f} + V_{2f} = 0 \) and \( |\omega_{1f}^2| + |\omega_{2f}^2| = 0 \), therefore:

9.22d \quad \pm \left( |m_0C_{1i}| \right)^{\frac{1}{2}} \hat{\chi} \pm \left( |m_0C_{2i}| \right)^{\frac{1}{2}} \hat{\chi} = \pm \left( |\chi_{1f}| \right)^{\frac{1}{2}} \hat{\chi} \pm \left( |\chi_{2f}| \right)^{\frac{1}{2}} \hat{\chi}

From symmetry considerations \( C_{1i} = C_{2i} \) and \( \chi_{1f} + \frac{1}{2}i\omega_{1f}^2 = \chi_{2f} + \frac{1}{2}i\omega_{2f}^2 \). If the collision is elastic, then \( |V_{1i}| = |V_{2i}| = |V_{1f}| = |V_{2f}| \) and \( m_0C_{1i} = m_0C_{2i} = \chi_{1f} + \frac{1}{2}i\omega_{1f}^2 = \chi_{2f} + \frac{1}{2}i\omega_{2f}^2 \)
and using 9.22d, \( 0 = 0 \). If the collision is inelastic then \( |V_{1i}| = |V_{2i}| \geq |V_{1f}| = |V_{2f}| \) and \( |C_{1i}| = |C_{2i}| \geq |\chi_{1f} + \frac{1}{2}i\omega_{1f}^2| = |\chi_{2f} + \frac{1}{2}i\omega_{2f}^2| \) and using 9.22d, again \( 0 = 0 \).

The straightforward generalizations of 9.22a and 9.22c for the case \( m_0V_{1i} \cdot \hat{\chi} + m_0V_{2i} \cdot \hat{\chi} \neq 0 \) may be carried out by the reader.

If the initial translational kinetic energy is much greater than the absolute value of the binding energy i.e. \( KE_{1i} = KE_{2i} \gg |m_0C_{1i}| \). The collision is no longer elastic and the 2 atoms are in the process of destroying one another and creating multiple subatomic particles. The 2 atoms elongate in the yz plane of collision at \( x=0, t=0^+ \) fig. 9.9 and the subatomic particles acquire a component of velocity in the yz plane at \( x=0 \).

9.22a becomes:

9.23 \quad 2KE_{1i} + 2m_0C_{1i} = \sum_{j=1}^{n} \{ \frac{1}{2}m_j^{2} + m_jC_{1i,jf} \} > 0, \quad KE_1 \gg |m_0C_{1i}|, KE_2 \gg |m_0C_{1i}|

\begin{align*}
& m_0V_{1i} \cdot \hat{\chi} + m_0V_{2i} \cdot \hat{\chi} + 2m_0(2|C_{1i}|) \frac{1}{2} \hat{\chi} = \sum_{j=1}^{n} \{ m_j^{2} \hat{\chi} + m_j(2|C_{1i,jf}|) \frac{1}{2} \hat{\chi} \}, \quad n \gg 2
\end{align*}
$V_{1i} + V_{2i} = 0$ and $U_{jf}$ is the final speed of the $j^{th}$ collision product. Using 9.23 note that, 
$$\sum_{j=1}^{n} (m_j U_{jf}(\hat{y} + \hat{z})) = 0$$
although in general $m_j U_{jf}(\hat{y} + \hat{z}) \neq 0$. We wish to show the conditions under which $U_{jf} \hat{x} = 0$.

The two atoms collide at $t=0$. Figure 9.9A. Let $h(r_{1}, t)$ represent the position of any point in atom #1 as a function of time and let $h(r_{2}, t)$ represent the position of the mirror image point of $r_{1} = x_{1} \hat{x} + y_{1} \hat{y} + z_{1} \hat{z}$ where $r_{2} = -x_{1} \hat{x} + y_{1} \hat{y} + z_{1} \hat{z}$. As usual, $h(r_{1}, t) = r_{1} + \chi(r_{1}, t)$, $\chi(r_{1}, 0) = 0$ with $r_{1}$ restricted to the undeformed atom #1 at $t=0$ and $h(r_{2}, t) = r_{2} + \chi(r_{2}, t)$, $\chi(r_{2}, 0) = 0$. $r_{1}$ and $r_{2}$ are independent of time and $h_{1}, h_{2}, r_{1}$ and $r_{2}$ are measured from an inertial frame at rest w.r.t. the center of mass of atoms 1 and 2.

FIGURE 9.9

![Diagram showing the positions of atoms before and after collision](image)
Considering both atoms to have zero spin about the x,y,z axis; \( h(\mathbf{r}_1, t) \) and \( h(\mathbf{r}_2, t) \) are composed of two parts. \( h(\mathbf{r}_1, t) = h_{1,\text{tr}}(\mathbf{r}_1, t) + h_{1,\text{vl}}(\mathbf{r}_1, t) \) and \( h(\mathbf{r}_2, t) = h_{2,\text{tr}}(\mathbf{r}_2, t) + h_{2,\text{vl}}(\mathbf{r}_2, t) \). \( h_{1,\text{tr}}, \) the translational motion, is measured from the center of mass of the two atoms to the center of mass of atom #1 and \( h_{2,\text{tr}}, \) is measured from the center of mass of the two atoms to the center of mass of atom #2.

(i) \( h(-\mathbf{r}_0, \mathbf{t}), t)_{1,\text{tr}} = -r_0 \hat{x} + \chi(-r_0 \hat{x}, 0)_{1,\text{tr}} \cdot \hat{x}, \chi(-r_0 \hat{x}, 0)_{1,\text{tr}} \cdot \hat{x} = 0 \) and \( h(-r_0 \hat{x}, 0)_{1,\text{tr}} = -r_0 \hat{x} \)

(ii) \( h(r_0 \hat{x}, t)_{2,\text{tr}} = r_0 \hat{x} + \chi(r_0 \hat{x}, 0)_{2,\text{tr}} \cdot \hat{x}, \chi(r_0 \hat{x}, 0)_{2,\text{tr}} \cdot \hat{x} = 0 \) and \( h(r_0 \hat{x}, 0)_{2,\text{tr}} = r_0 \hat{x} \)

\( h_{1,\text{vl}}, \) the vibrational motion, is measured from the center of mass of atom #1 to the point \( r_1 \) in atom #1 and \( h_{2,\text{vl}} \), the vibrational motion, is measured from the center of mass of atom #2 to the point \( r_2 \) in atom #2.

(iii) \( h(r_0 \hat{x} + r_1, t)_{1,\text{vl}} = r_0 \hat{x} + r_1 + \chi(r_0 \hat{x} + r_1, 0)_{1,\text{vl}} \cdot \hat{x}, \chi(r_0 \hat{x} + r_1, 0)_{1,\text{vl}} \cdot \hat{x} = 0 \) and \( h(r_0 \hat{x} + r_1, 0)_{1,\text{vl}} = r_0 \hat{x} + r_1 \)

(iv) \( h(-r_0 \hat{x} + r_2, t)_{2,\text{vl}} = -r_0 \hat{x} + r_2 + \chi(-r_0 \hat{x} + r_2, 0)_{2,\text{vl}} \cdot \hat{x}, \chi(-r_0 \hat{x} + r_2, 0)_{2,\text{vl}} \cdot \hat{x} = 0 \) and \( h(-r_0 \hat{x} + r_2, 0)_{2,\text{vl}} = -r_0 \hat{x} + r_2 \)

A mathematical model for the formation of the jet of particles follows. We wish to explain how the collision energy transforms the two atoms into a jet of particles traveling outward in the x=0 plane, figure 9.9.

Define \( \delta m_1 \) as the point like mass at \( \mathbf{r}_1 \) and define \( \delta m_2 \) as the point like mass at \( \mathbf{r}_2 \). Before the collision for \( t<0 \) and using 3.15,

\[ 9.24a \]
\[
\frac{1}{2} \delta m_1 \cdot \left[ h(\mathbf{r}_1, t)_{1,\text{vl}} \right]^2 + \delta m_1 \cdot \psi[h(\mathbf{r}_1, t)_{1,\text{vl}}] = \delta m_1 \cdot C_1 < 0
\]
\[
\frac{1}{2} \delta m_2 \cdot \left[ h(\mathbf{r}_2, t)_{2,\text{vl}} \right]^2 + \delta m_2 \cdot \psi[h(\mathbf{r}_2, t)_{2,\text{vl}}] = \delta m_2 \cdot C_1 < 0.
\]

In both cases, 9.23 written for \( \delta m_1 \) and \( \delta m_2 \) still hold. i.e.

\[ 9.24b \]
\[
\frac{1}{2} \delta m_1 \cdot \left[ h(\mathbf{r}_1, t)_{1,\text{tr}} \right]^2 + \frac{1}{2} \delta m_1 \cdot \left[ h(\mathbf{r}_1, t)_{1,\text{vl}} \right]^2 + \delta m_1 \cdot \psi[h(\mathbf{r}_1, t)_{1,\text{tr}}] > 0
\]
\[
\frac{1}{2} \delta m_2 \cdot \left[ h(\mathbf{r}_2, t)_{2,\text{tr}} \right]^2 + \frac{1}{2} \delta m_2 \cdot \left[ h(\mathbf{r}_2, t)_{2,\text{vl}} \right]^2 + \delta m_2 \cdot \psi[h(\mathbf{r}_2, t)_{2,\text{tr}}] > 0.
\]

After the collision for times \( t_1, t_2 \) with \( t_1, t_2 > t_1, t_2 > 0 \), and after the collision for times \( t_1, t_2 \) with \( t_1, t_2 > t_1, t_2 > 0 \):

\[ 9.24c \]
\[
\frac{1}{2} \delta m_1 \cdot \left[ h(\mathbf{r}_1, t)_{1,\text{vl}} \right]^2 + \delta m_1 \cdot \psi[h(\mathbf{r}_1, t)_{1,\text{tr}}] > 0, \text{ for } t_1, t_2 > t_1, t_2 > 0
\]
\[
\frac{1}{2} \delta m_2 \cdot \left[ h(\mathbf{r}_2, t)_{2,\text{vl}} \right]^2 + \delta m_2 \cdot \psi[h(\mathbf{r}_2, t)_{2,\text{tr}}] > 0, \text{ for } t_1, t_2 > t_1, t_2 > 0
\]
It is understood that \( \dot{h}(r_1, t)_{1, Vi} \) and \( \dot{h}(r_2, t)_{2, Vi} \) are the rms speeds.

We consider the case where \( \delta m_1 \) and \( \delta m_2 \) do not explode for \( t_{e_2} > t > t_{e_1} > 0 \) because the pressure forces created by the collision acting on the mass surrounding \( \delta m_1 \) and \( \delta m_2 \), prevent \( \delta m_1 \) and \( \delta m_2 \) from exploding. \( \delta m_1 \) and \( \delta m_2 \) are traveling in opposite x-directions with the same translational speed.

With \( \delta m_1 = \delta m_2 \) and due to the collision at \( t=0 \), at \( t=t_{e_2} \): \( \dot{h}(r_1, t)_{1, 1, Tr} = \dot{h}(r_2, t)_{2, 2, Tr} = 0 \), and \( \dot{h}(r_1, t)_{1, Vi} = \dot{h}(r_2, t)_{2, Vi} \neq 0 \). \( \delta m_1 \) and \( \delta m_2 \) are now on the solid planar surfaces of the two colliding atoms, and using 9.24c \( \delta m_1 \) and \( \delta m_2 \) escape from the solid planar surface and add their mass to the vapor like particle mass forming between the two colliding atoms. The unstable particles are constrained by the solid planar surfaces and escape with velocities in the yz plane: \( \dot{h}(r_1, t)_{1} = (0, \dot{h}(r_1, t)_{1} \cdot \hat{y}, \dot{h}(r_1, t)_{1} \cdot \hat{z}) \) and \( \dot{h}(r_2, t)_{2} = (0, \dot{h}(r_2, t)_{2} \cdot \hat{y}, \dot{h}(r_2, t)_{2} \cdot \hat{z}) \).

As regards the Higg's experiment, it is hypothesized that among the unstable particles escaping from the colliding solid mass atoms are to be found the 2 particles called tau leptons the existence of which is thought to provide evidence for the existence of the Higg's Boson. Although in no way agreeing with the Higg's theory, the author feels the existence of the unstable particles emitted by the colliding protons in the CERN Super Collider deserve the explanation given above.

From the above: 2 energetic atoms of the same species and translational kinetic energy traveling down the x axis in opposite directions collide at \( x=0 \) emitting a shower of particles in the yz plane at \( x=0 \). If now 4 atoms of the same species and translational kinetic energy traveling in the opposite \( \hat{x}, \cdot \hat{x}, \cdot \hat{z}, \cdot \hat{z} \) directions, fig. 9.9, collide at \( x=0 \), the resultant jet of particles is in the \( \hat{y}, \cdot \hat{y} \) along the y axis.

Returning to the two colliding atoms.

Let \( N \) equal the total number of particles emitted in the \( x=0 \) plane and let \( \bar{m} \) be the average mass of the \( N \) particles. Assuming \( N \bar{m} = 2m_1 \), then the total self binding energy of the \( N \) particles is, \( BE_T = N \bar{m} \sum_{1}^{N} \bar{m} \psi(s_0) = N(\bar{m} \bar{m}) \frac{2}{\bar{s}_0} \). The total final energy of the \( N \) particles is \( E_{T,i} = \frac{1}{2} N \bar{m} \cdot \bar{v}^2 \cdot N(\bar{m} \bar{m}) \frac{2}{\bar{s}_0} \). \( \bar{v}^2 \) is the average value of \( V^2 \), averaged over the \( N \) collision products particles.

The initial total energy is, \( E_{T,i} = 2\left[ \frac{1}{2} m_1 V_0^2 \cdot (m_1) \frac{2}{\bar{r}_0} \right] \) where \( m_1 \) and \( V_0 \) are the mass and translational speed of each atom at collision. Equating the two energies yields:
9.25a \[ E_{T,i} = 2 \left[ \frac{1}{2} m_1 V_0^2 - \frac{1}{2} \left( \frac{m_1}{r_0} \right)^2 H \right] = \frac{1}{2} N \delta m \cdot \delta E - N(\delta m)^2 \frac{H}{\delta m} = E_{T,f} > 0 \]

\[ BE_{T,i} = N(\delta m)^2 \frac{H}{\delta m} < 0 \]

Valid for \( \frac{1}{2} m_1 V_0^2 > (\frac{m_1}{r_0})^2 H \). Solving 9.25a for \( \delta m_0 \) yields:

9.25b \[ \delta m_0 = -N(\delta m)^2 H (E_{T,i} - \frac{1}{2} N \delta m \cdot \delta E)^{-1} \]

References


9.4 T. Ludlam and L. McLerran, “What Have We Learned From the Relativistic Heavy Ion Collider”, Physics Today, October 2003, p 48