On the Absolute Constancy of the Speed of Electromagnetic Radiation

1. Prologue

Central to all twentieth century atomic physics is the expression, derived from Special Relativity Theory (SRT), $E=mc_0^2$. This formula was first derived in 1905 by Albert Einstein, from the Lorentz Transform and the conservation of energy and momentum laws: See reference 1.1 The Lorentz Transform is derived from the I. and II. postulates of relativity theory where the first and second postulates are:

I. All inertial frames are equivalent for the performance of physical experiments.
II. The speed of light is an absolute constant.

It is understood that the speed of light referred to in postulate II is finite.
It is proved in this chapter from Postulate I that:

(i) The speed of light is not an absolute constant.
(ii) The velocity of light is vector ally additive
(iii) Postulates I and II are inconsistent with one another.
(iv) The Galilean Transform is the physically correct transform between inertial frame S and S'.
(v) The negative results of the Michelson-Morley Experiment do not prove that the speed of light is an absolute constant.
(vi) $E \neq mc_0^2$

Historically, in order to preserve the consistency of Postulates I and II, a rule has been invoked called the "Relativity of simultaneity". References 1.2 through 1.5. It is proved from Postulate I that:

(vii) The "Relativity of simultaneity" rule is false. See 1.41.

Simultaneity is defined below.

2. The I and II Postulates of Special Relativity Theory

The I and II postulates contain terms, e.g. inertial frame, speed, absolute constant, which require operational definitions. In addition, in order to understand postulate I, it is necessary to operationally define in frame clock synchrony, i.e. clocks in inertial frame S with clocks in S and clocks in inertial frame S' with clocks in S' and cross frame clock synchrony, clocks in S with clocks in S'. It is assumed that the reader is familiar with the operational definitions of the above terms.
There are as well certain facts extrapolated from observation pertaining to postulates I and II that will be useful in deriving the physical consequences of postulates I and II e.g. A given point $\underset{\sim}{x}_1$ at time t_1 in S is coincident with one and only one point $\underset{\sim}{x}'_1$ at

t'_1 in S' as determined by observer $O_{\underset{\sim}{x}_1}$ at rest at $\underset{\sim}{x}_1$ in S at time t_1, and the given point $\underset{\sim}{x}'_1$ at time t'_1 in S' is coincident with one and only one point $\underset{\sim}{x}_1$ at time t_1 in S as determined by observer $O'_{\underset{\sim}{x}'_1}$ at rest at $\underset{\sim}{x}'_1$ in S' at time t'_1.

For future use define 1.1 and 1.2 as:

1.1 O.D.-1
O.F.-1
Postulate I

1.2 O.D.-2
O.F.-2
Postulate I
Postulate II

O.D.-1 stands for the operational definitions of the undefined terms contained in Postulates I and O.F.-1 stands for observational facts pertaining to Postulate I. O.D.-2 stands for the operational definitions of the undefined terms contained in Postulates I and II and O.F.-2 stands for observational facts pertaining to Postulates I and II.

An inertial frame is a non accelerating coordinate frame as measured by on board accelerometers.

The first postulate means that any experiment performed in inertial frame S, will have the same result as the same experiment performed in inertial frame S'.

Consider any experiment screwed down to the frame of a black box and let the numerical output of the experiment be typed on a piece of paper. A consequence of the I postulate is that if the black box is first placed in inertial frame S and the experiment performed, and then placed in inertial frame S' and the experiment again performed, the numerical result as typed on the two pieces of paper will be the same.

Notice that it is possible to distinguish between inertial frames by performing experiments that are not screwed down in one black box and transported from inertial frame S to inertial frame S'. e.g. the constant velocity $\underset{\sim}{V}$ of an object O measured from inertial frame S and then measured from inertial frame S' where S' is moving with respect to S with constant velocity $\underset{\sim}{U}$. The velocity of the object O as measured from S i.e. $\underset{\sim}{V}$, does not equal the velocity of O as measured from S' i.e. $\underset{\sim}{V}-\underset{\sim}{U}$ in the non relativistic theory, and makes it possible to distinguish S from S'. In this example object O is not screwed down to a black box.

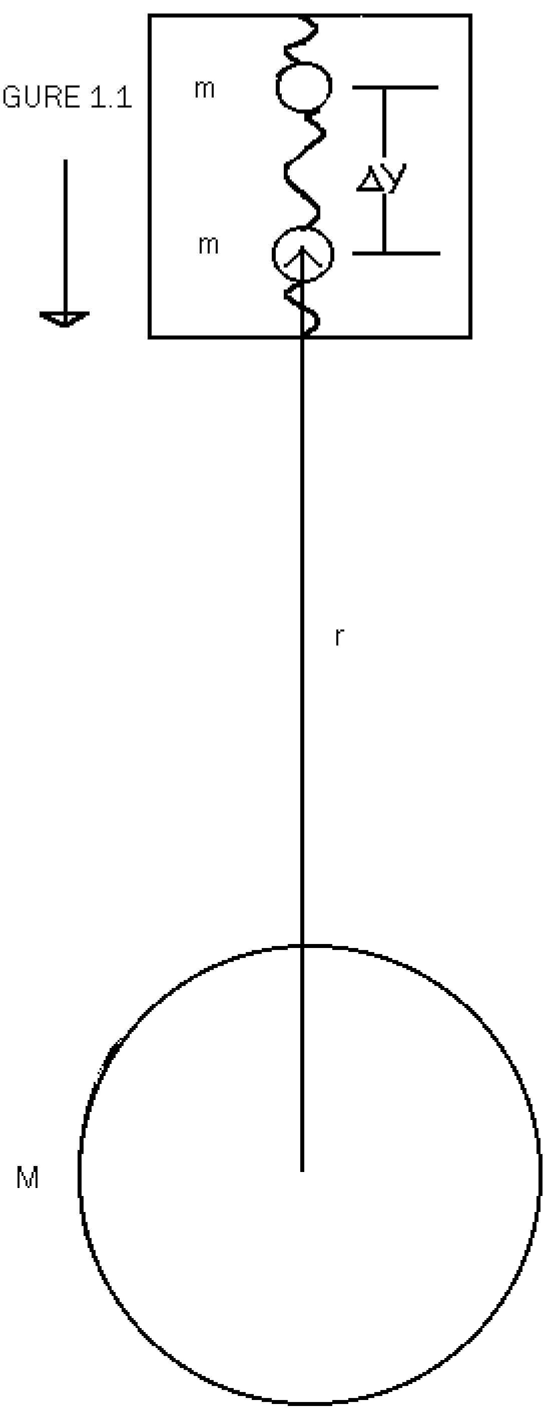
If the frames S and S' are not inertial, then the frames will be accelerating as measured by on board accelerometers. Forces proportional to the acceleration will be acting on the experiment in the black box, and if the acceleration of S and S' is unequal, the typed results on the two pieces of paper representing the results of the experiment performed in S and S', may not be the same.

In principle one can even differentiate, by performing on board experiments, between an inertial frame in the presence of no external field and a freely falling frame in a gravitational field. Consider two masses, figure 1.1, held to a frame with three Hook's Law springs. The frame is then screwed down to a freely falling coordinate frame.
The two masses will experience tidal forces which will tend to increase or decrease the distance Δy between the two masses depending on the orientation of the two masses with respect to the gravitational field lines. If the gravitational force is a $1/r^2$ force, and if the two masses are aligned in the $\hat{r}$ direction, and if $\Delta y << r$, then the absolute magnitude of the difference between the forces tending to separate the two masses is proportional to $2\Delta y/r^3$. This experiment in principle allows one to differentiate between an inertial frame and a freely falling frame in a gravitational force field.

Consider inertial frame S with observer $O_{\underset{\sim}{x}_i}$ stationary at $\underset{\sim}{x}_i$ in S and observer $O_{\underset{\sim}{x}_j}$ stationary at $\underset{\sim}{x}_j$ in S. Let clock $K_{\underset{\sim}{x}_i}$ stationary at $\underset{\sim}{x}_i$ and clock $K_{\underset{\sim}{x}_j}$ stationary at $\underset{\sim}{x}_j$ be synchronized with one another in the usual way and let LS represent a light source at arbitrary point x in S: See figure 1.2
Inertial frame S, observers and light source are in vacuum.
The light source is turned on and the leading edge of the light beam from LS crosses $\underset{\sim}{x}_j$ at time t_j as determined by $O_{\underset{\sim}{x}_j}$ and then crosses $\underset{\sim}{x}_i$ at time t_i as determined by $O_{\underset{\sim}{x}_i}$.
The speed of light from the source is:

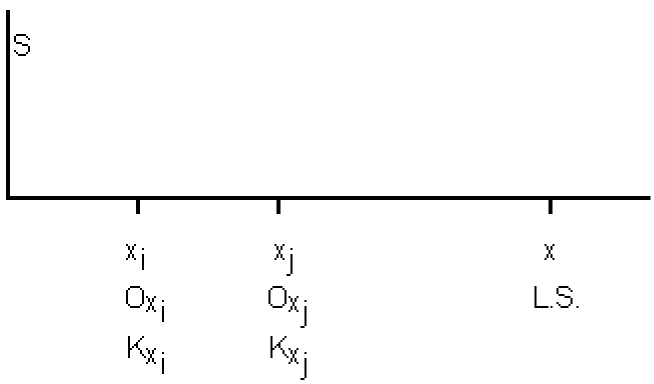
1.3 $$c_o = \frac{(x_j - x_i)}{(t_i - t_j)}$$

FIGURE 1.1



Now let the light source move with constant speed U in the $\hat{\mathbf{x}}$ direction, $\mathbf{U}=\pm U\hat{\mathbf{x}}$. If the II postulate is physically true, then the speed of light c_o as determined by 1.3 will not depend on the speed of the light source ± U with respect to $O_{\mathbf{x}_j}$ or $O_{\mathbf{x}_i}$ stationary in S. The numerical answer c_o produced using 1.3 will always be the same regardless of the speed of the light source ± U with respect to $O_{\mathbf{x}_j}$ or $O_{\mathbf{x}_i}$.

FIGURE 1.2



In the following it will be proved that:

Theorem 1.0 If postulational system 1.1 and its consequences are physically true, then the speed of light is not an absolute constant, and the velocity of light is vectorally additive. See 1.52

A consequence of theorem 1.0 is that any consequence of postulational system 1.2 that uses the second postulate, such as the Lorentz Transform and $E=mc_0^2$, is physically false and does not represent physical reality.

Let S and S' be inertial frames, with the x,y,z, axis of S, parallel respectively to the x',y',z', axis of S', as determined by observers in S and S'. Let the x axis of S be coincident with the x' axis of S'. S' moves with respect to S with constant velocity $\mathbf{U}$ so that the x' axis of S' is always coincident with the x axis of S.
Clocks in S are synchronized with clocks in S, as determined by observers in S, by using a light source at rest at the origin of S. Clocks in S' are synchronized with clocks in S', as determined by observers in S', by using a light source at rest at the origin of S'. Clocks in S are synchronized with clocks in S' by turning the lights at the origin of S and S' on, at the instant for which the origins of S and S' are coincident as determined by observers at the origins of S and S'.

The operational definition of the length of a moving object is as follows. Let the length of a linear object at rest on the x' axis of S', be $\Delta x' = x'_2 - x'_1$ as determined by observers at rest at x'_2 and x'_1. Let observer $O_{\underset{\sim}{x}_2}$ at rest at x_2 in S be coincident with x'_2 at time t_o as measured by $O_{\underset{\sim}{x}_2}$ using clock $K_{\underset{\sim}{x}_2}$ at rest at x_2. Let observer $O_{\underset{\sim}{x}_1}$ at rest at x_1 in S be coincident with x'_1 at time $t_o$as measured by $O_{\underset{\sim}{x}_1}$ using clock $K_{\underset{\sim}{x}_1}$ at rest at x_1. The length $\Delta x'$ as measured in S is operationally defined as $\Delta x = x_2 - x_1$.

To derive:

Corollary I. The transform relating $\underset{\sim}{x}$,t as measured from rectilinear inertial frame S, And $\underset{\sim}{x}$',t' as measured from rectilinear inertia frame S', is a linear transform.

Derivation: With the orientation of S and S' as above and from symmetry thought Experiments, Reference 1.6, y=y' and z=z'.
It is inferred by extrapolating from observation that ∃ functions f and g of the following functional form, that map each x in S at time t into one and only one x' in S' at time t' as determined y observer $O_{\underset{\sim}{x}}$ stationary at point $\underset{\sim}{x}$ in S, i.e.

x'=f(x,t), y'=y, z'=z, and t'=g(t): And that the inverse $f^{-1}$and g^{-1} are also functions that map each $\underset{\sim}{x}$' in S' at time t' into one and only one $\underset{\sim}{x}$ in S at time t as determined by observer $O'_{x'}$ stationary at point $\underset{\sim}{x}$' in S'. i.e.

$x=f^{-1}(x',t')$, y=y', z=z' and $t=g^{-1}(t')$. However as direct observations have only been done for U<<c, it is explicitly assumed that functions g and g^{-1} are functions of the coordinate in the direction of motion, as well as of time i.e. t'=g(x,t) and $t=g^{-1}(x',t')$ with t'=g(t) and $t=g^{-1}(t')$ for $U \ll c_o$.

Given the orientation of S and S' stated above, the transform relating $\underset{\sim}{x}$,t in S to $\underset{\sim}{x}$',t' in S' is of the following type.

1.4 $\quad x' = \alpha x + \beta t$, $\quad y' = y$

1.5 $\quad t' = \delta x + \gamma t$. $\quad z' = z$

$\alpha, \beta, \delta, \gamma$ are undetermined constants.

3. Evaluation of β, δ, γ .

Let $\underset{\sim}{U}$ be the vector velocity of S' with respect to S, where $\underset{\sim}{U}=U\hat{x}$ and $0 < U < \infty$. It is a standard problem to show, using the operational definitions of U and repeated use of postulational system 1.1 and 1.4 and 1.5 that:

1.6 $x'=\alpha(x-Ut)$, $y'=y$ $U>0$

1.7 $t'=[\frac{(1-\alpha^2)}{\alpha U}]x+\alpha t$, $z'=z$, $\alpha>0$

with inverse:

1.8 $x=\alpha(x'+Ut')$ $y'=y$ $U>0$

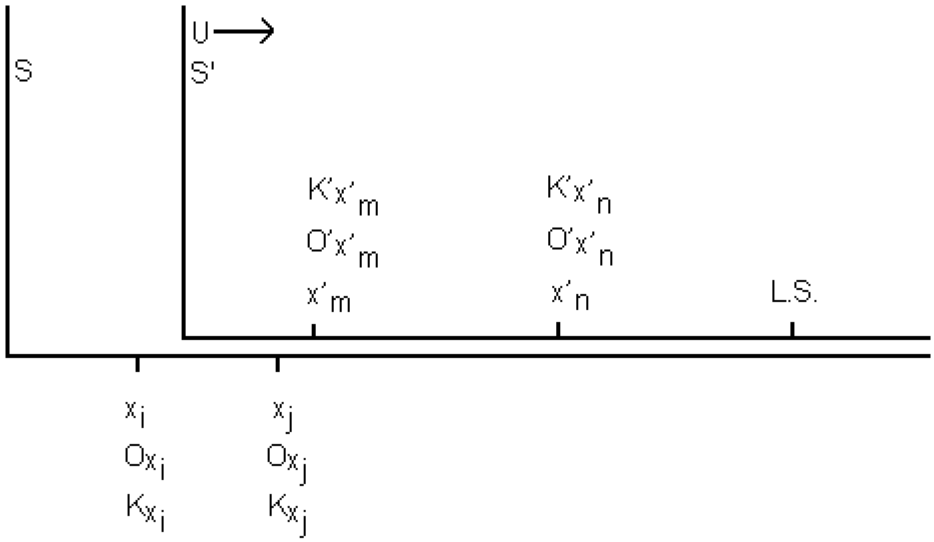
1.9 $t=-[\frac{(1-\alpha^2)}{\alpha U}]x'+\alpha t'$, $z'=z$, $\alpha>0$

Historically it has been assumed that it is necessary to make an additional assumption in order to determine α. A. Einstein in l905 assumed the speed of light to be a finite absolute constant and derived $\alpha= 1/(1-U^2/c_0^2)^{1/2}$, and the Lorentz Transform.

Alternatively, one can assume the velocity of light to be vector ally additive $\underset{\sim}{c}=\underset{\sim}{c}'_o+U\hat{x}$, and derive $\alpha=1$. See figure 1.3. The light source is at rest in S', S' is moving with respect to S with velocity $U\hat{x}$. $\underset{\sim}{c}'_o$ is the velocity of light as measured in inertial frame S' from a source at rest in S' and $\underset{\sim}{c}$ is the velocity of light as measured in inertial frame S from the source at rest in S'. For the experiment depicted in figure 1.3, $\underset{\sim}{c}'_o=[(x'_n-x'_m)/(t'_m-t'_n)]\,\hat{x}'$ and $\underset{\sim}{c}=[(x_j-x_i)/(t_i-t_j)]\,\hat{x}$ and $\underset{\sim}{c}'_o=-|\underset{\sim}{c}'_o|\,\hat{x}$, where $|\underset{\sim}{c}'_o|\equiv c'_o$: Consequently $\underset{\sim}{c}=(-c'_o+U)\,\hat{x}$. $\alpha=1$, when substituted into 1.6 through 1.9 generates the Galilean Transform.

In what follows it will be proved that no additional assumptions are necessary to uniquely determine α. α will be derived from 1.1, corollary 1, 1.6 and 1.7. The derivation yields $\alpha=1$, from which follows the Galilean Transform.

Figure-1.3



4. The Michelson-Morley Experiment

The Michelson-Morley interferometer experiment was first successfully performed at the Case Institute of Technology, Cleveland Ohio in l887. The interferometer was designed to detect the presence of the ether, a mass less medium pervading all space. The ether was hypothesized to exist by James C. Maxwell to provide a supporting medium for electromagnetic radiation in vacuum: Much as water is a supporting medium for surface water waves and air is a supporting medium for sound waves. Electromagnetic radiation was considered by Maxwell to be undulations of the ether. For an observer at rest in the ether, the speed of light c is independent of the speed of the source with respect to the ether, just as the speed of a surface water wave for an observer at rest with respect to the water is independent of the speed, with respect to the water, of the boat creating the water wave. For an observer moving with respect to the ether, the speed of light depends on the speed of the observer with respect to the ether $|\underset{\sim}{c}| = |\underset{\sim}{c}'_o - \underset{\sim}{W}|$, where $\underset{\sim}{c}$ in this instance, is the velocity of light with respect to an observer moving with velocity $\underset{\sim}{W}$ with respect to the ether, and $\underset{\sim}{c}'_o$ in this instance, is the velocity of light with respect to the ether.

Maxwell's Equations in their original form contained scalar terms representing the components of $\underset{\sim}{c}'_o - \underset{\sim}{W}$. If the ether did not exist, then one of the physical underpinnings of Maxwell's Equations would be wrong and Maxwell's Equations in

their original form would be wrong. The negative results of the Michelson-Morley experiment proved the non existence of the ether and plunged the then relatively small physics community into a crises.
With the publication of the Special Relativity Theory in 1905, Albert Einstein was able to resolve the crises by effectively substituting a new assumption, the absolute constancy of the speed of light, for the old assumption, the hypothesized existence of the ether. For it fortuitously turned out, that Maxwell's Equations with $\underset{\sim}{W}=0$, are invariant with respect to the Lorentz Transform. Thus by changing one of the basic physical postulates of electrodynamics, Einstein was able to preserve the mathematical form of Maxwell's Equations and derive the negative results of the Michelson-Morley experiment.
The main result of this chapter however is that the speed of light is not an absolute constant, and thus once again one of the physical underpinnings of Maxwell's Equations will be proved to be wrong.

As will be proved in Theorem 1.1, the assumption of the absolute constancy of the speed of light, or the assumption of the vectorial additivity of the velocity of light, may be used to derive the negative results of the Michelson-Morley experiment.

Theorem 1.1 The negative results of the Michelson-Morley experiment do not prove that the speed of light is an absolute constant.

Proof of Theorem 1.1:
Let $\underset{\sim}{V}'$ be the velocity of an object as measured from S' and let $\underset{\sim}{V}$ be the velocity of the same object as measured from S, where $\underset{\sim}{V}' = (V'_1, V'_2, 0')$ and $\underset{\sim}{V} = (V_1, V_2, 0)$.
From 1.6 with $V'_1dt'=dx'$, $V'_1dt'=\alpha dx-\alpha Udt$. From 1.7, $dt'=\frac{(1-\alpha^2)}{\alpha U}dx+\alpha dt$ and $V'_1dt'=V'_1[\frac{(1-\alpha^2)}{\alpha U}dx+\alpha dt]=\alpha dx-\alpha Udt$. Solving for dx/dt using the last equality yields:

1.10 $$V_1 \equiv dx/dt = \frac{\alpha^2 U(U+V'_1)}{\alpha^2 U+(\alpha^2-1)V'_1}$$

Similarly from 1.6 with $V'_2dt'=dy'=dy$. From 1.7, $dt'=\frac{(1-\alpha^2)}{\alpha U}dx+\alpha dt$ and $V'_2dt'=V'_2[\frac{(1-\alpha^2)}{\alpha U}dx+\alpha dt]=dy$. Solving for dy/dt using the last equality yields:

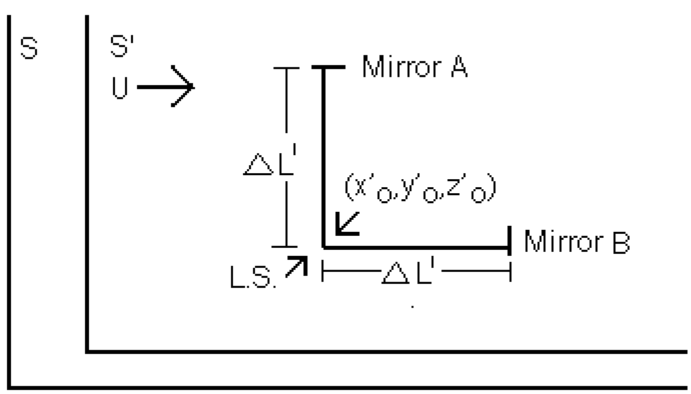
1.11 $$V_2 \equiv dy/dt = \alpha\left[1+\frac{(1-\alpha^2)(U+V'_1)}{\alpha^2 U+(\alpha^2-1)V'_1}\right]V'_2$$

Figure 1.4 is a schematic diagram of the Michelson-Morley interferometer experiment. The interferometer is at rest in inertial frame S' which is moving with constant velocity $U\hat{\mathbf{x}}$ with respect to inertial frame S. S is at rest in the hypothesized ether. The interferometer consists of two arms at right angles to each other, each arm is of length $\Delta L'$ as measured by observers at rest in S'. A light source L.S., at rest with respect to the interferometer, is placed at the union of the two arms and a mirror is placed at the end of each arm at A and B. The L.S. is at point $\mathbf{x}'_0$ at rest in S'. The L.S. is turned on at time t_0', as measured by observer $O'_{\mathbf{x}'_0}$ at rest at $\mathbf{x}'_0$, using clock $K'_{\mathbf{x}'_0}$, also at rest at $\mathbf{x}'_0$. The observer $O_{\mathbf{x}_0}$ at rest at $\mathbf{x}_0$ in inertial frame S, is coincident with $O'_{\mathbf{x}'_0}$ at time t_0 as measured by $O_{\mathbf{x}_0}$ using clock $K_{\mathbf{x}_0}$ at rest at $\mathbf{x}_0$ i.e.

1.12 $\quad x_0=\alpha(x'_0+Ut'_0),\quad y_0=y'_0,\quad z_0=z'_0,\quad t_0=-[\frac{(1-\alpha^2)}{\alpha U}]x'_0+\alpha t'_0.$

At the instant of coincidence, both observers $O_{\mathbf{x}_0}$ and $O'_{\mathbf{x}'_0}$ agree, by directly observing their own and each others position and time using their own and each others coordinates and clocks, as to the validity of 1.12.

FIGURE 1.4



For $O'_{\mathbf{x}'_0}$ the light from L.S. leaves at time t'_0. The beam in the $\hat{\mathbf{x}}'$ direction reflects from the mirror at B and returns to $O'_{\mathbf{x}'_0}$ at time $t'_0+2\frac{\Delta L'}{c'_0}$ and the beam in the $\hat{y}'$ direction reflects from the mirror at A and returns to O'$\underset{\sim}{x}'_o$ at time $t'_o+2\frac{\Delta L'}{c'_o}$. c'_o is the speed of light from the L.S. as measured by observers in S'. The total round trip time $\Delta t'_o$ for both beams is $\Delta t'_o \equiv 2\frac{\Delta L'}{c'_o}$. Thus both beams return to O'$\underset{\sim}{x}'_o$ at the same instant and generate a simultaneous event for O'$\underset{\sim}{x}'_o$ and for O$\underset{\sim}{x}_1$ at time t_1 where: $\underset{\sim}{x}_1=(x_1, y_o, z_o)$ and $x_1=\alpha[x'_o+U(t'_o+2\frac{\Delta L'}{c'_o})]$, $y_o=y'_o$, $z_o=z'_o$, and $t_1=-[\frac{(1-\alpha^2)}{\alpha U}]x'_o+\alpha[t'_o+2\frac{\Delta L'}{c'_o}]$.

Let $\underset{\sim}{V}'_{y'}$ be the velocity of the beam of light up the $\hat{y}'$ branch of the interferometer, $\underset{\sim}{V}'_{y'}=(0', c'_o, 0')$, as determined by observers in S'. Using 1.11, the y component of $\underset{\sim}{V}'_{y'}$ as determined by observers in S is: $V_2= c'_o/\alpha$. As y=y', then $\Delta y= \Delta y'$ and $\Delta L\,\hat{y}=\Delta L'\,\hat{y}'$

Let Δt_1 be the time taken for the beam of light from the L.S. in the $\hat{y}'$ direction to reach the mirror at A as determined by observers in S, i.e. $\Delta t_1=\Delta L/V_2=\Delta L'/V_2=\alpha\frac{\Delta L'}{c'_o}$. The total round trip time is:

1.13 $\quad 2\Delta t_1=2\alpha\frac{\Delta L'}{c'_o}$

Let $\underset{\sim}{V}'_d$ be the velocity of the beam of light down the $\hat{x}'$ branch of the interferometer, $\underset{\sim}{V}'_d=(c'_o, 0', 0')$ with $V'_{d_1}=c'_o$, as determined by observers in S'. As determined by observers in S, the velocity $\underset{\sim}{V}'_d$ is, using 1.10,

$\underset{\sim}{V}_d=\frac{(U+c'_o)\alpha^2 U}{\alpha^2 U+(\alpha^2-1)c'_o}\,\hat{x}$ with $V_{d_1}=\frac{(U+c'_o)\alpha^2 U}{\alpha^2 U+(\alpha^2-1)c'_o}$ From 1.6, as $x'=\alpha(x-Ut)$, then $\Delta x'=\alpha\Delta x$ and $\Delta L'\,\hat{x}'=\alpha\Delta L\,\hat{x}$. Let Δt_2 be the time taken for the beam of light from the L.S. in the $\hat{x}'$ direction to reach the mirror at B as determined by observers in S, i.e. $V_{d_1}\Delta t_2 = \Delta L+U\Delta t_2=\Delta L'/\alpha+U\Delta t_2$

Solving for Δt_2 using V_{d_1} yields:

1.14 $\quad \Delta t_2=\frac{[\alpha^2(U+c'_o)-c'_o]\Delta L'}{\alpha U c'_o}$

Let $\underset{\sim}{V}'_b$ be the velocity of the beam of light back down the $\hat{x}'$ branch of the interferometer after it has reflected from the mirror at B. $\underset{\sim}{V}'_b = (-c'_o, 0', 0')$ with $V'_{b_1} = -c'_o$, as determined by observers in S'. As determined by observers in S, the velocity $\underset{\sim}{V}'_b$ is, using 1.10, $\underset{\sim}{V}_b = \left(\frac{(U - c'_o)\alpha^2 U}{[\alpha^2 U - (\alpha^2 - 1)c'_o]}\right)\hat{x}$ with $V_{b_1} = \frac{(U - c'_o)\alpha^2 U}{[\alpha^2 U - (\alpha^2 - 1)c'_o]}$

Let Δt_3 be the time taken for the beam of light from the mirror at B to reach the L.S. as determined by observers in S, i.e. $V_{b_1}\Delta t_3 = -\Delta L + U\Delta t_3 = -(\Delta L'/\alpha) + U\Delta t_3$. The sign in front of ΔL is the negative of the sign in front of ΔL used in the derivation of Δt_2 as the direction of the velocity of light used in the derivation of Δt_2 is the opposite of the direction of the velocity of light used in the derivation of Δt_3. Solving for Δt_3 using V_{b_1} yields:

1.15 $$\Delta t_3 = \frac{[\alpha^2(U - c'_o) + c'_o]\Delta L'}{\alpha U c'_o}$$

The negative results of the Michelson-Morley experiment require $2\Delta t_1 = \Delta t_2 + \Delta t_3$. By direct computation:

1.16 $$\Delta t_2 + \Delta t_3 = 2\alpha \frac{\Delta L'}{c'_o}$$

and from 1.13: $2\,\Delta t_1 = 2\alpha \frac{\Delta L'}{c'_o}$, consequently $2\,\Delta t_1 = \Delta t_2 + \Delta t_3$. Q.E.D.

The negative results of the Michelson-Morley experiment thus hold for any $\alpha > 0$, and in particular for $\alpha=1$, the Galilean Transform and concomitant vectorial additivity of the velocity of light, and for $\alpha^2 = \frac{1}{1 - U^2/c_0^2}$ the Lorentz Transform and concomitant absolute constancy of the speed of light. This proves Theorem 1.1.

5. In Frame and Cross Frame Simultaneity

Consider the five events A,B,C,D,E diagrammed in figure 1.5. Event A is the instant for which observer O_{x_i}, stationary at x_i in S, reads t_g using clock K_{x_i} also stationary at x_i.

Event B is the instant for which observer O_{x_j} stationary at x_j in S, reads t_g using clock K_{x_j} also stationary at x_j.

FIGURE 1.5

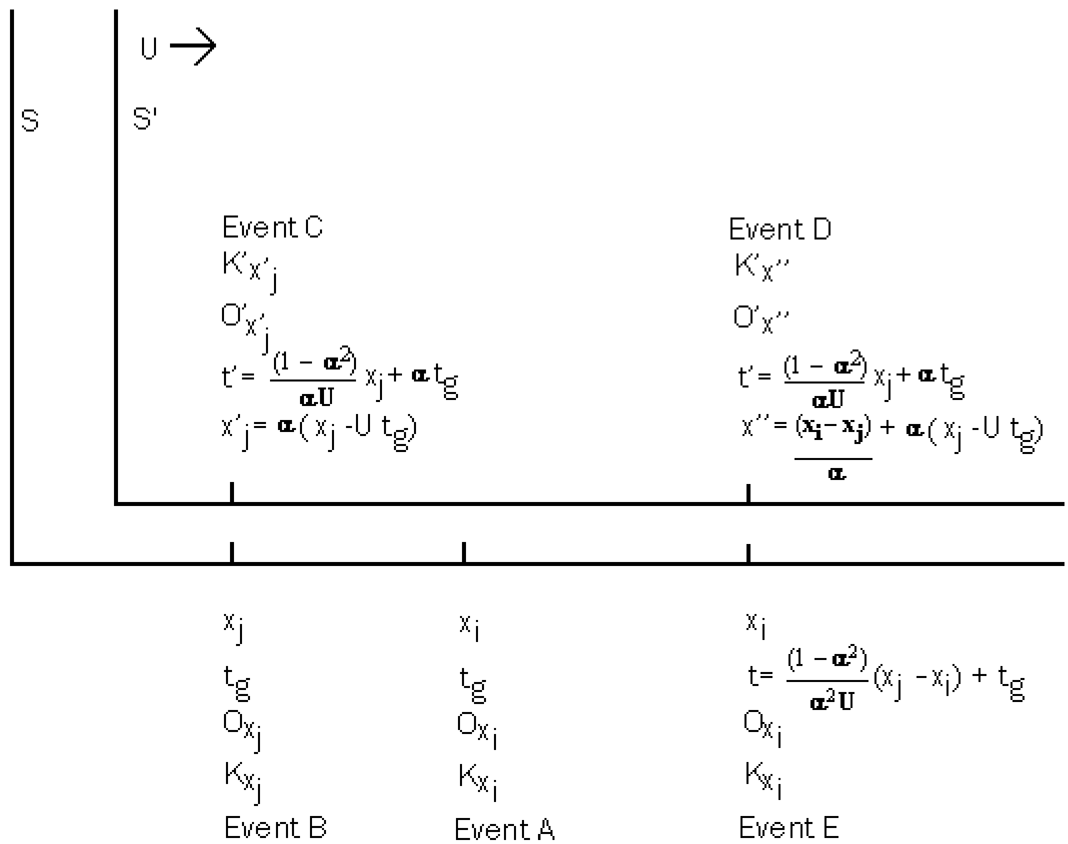


FIGURE 1.5 x_i, x_j, t_g are arbitrary but fixed. Event A and Event E occur at the same spatial point x_i. They are not drawn at the same spatial point to emphasize that in general $t_g \neq t$.

Event C is the instant for which observer $O'x_j'$, stationary at x_j', in S', reads t' using clock $K'x_j'$ also stationary at x_j'.

x_j and t_g of event B are related to x_j' and t' of event C by the linear transform 1.6 and 1.7. See figure 1.5.

Event D is the instant for which observer $O'_{x''}$ stationary at x'' in S', reads t' using clock $K'_{x''}$ also stationary at x''.
Event E is the instant for which observer O_{x_i}, stationary at x_i in S, reads t using clock K_{x_i} also stationary at x_i.
x'' and t have been chosen so that x_i and t of event E are related to x'' and t' of event D by the linear transform 1.6 and 1.7. See figure 1.5.
In what follows it is not assumed that t_g=t. Event A and event E occur at the same spatial point x_i; They are not drawn at the same spatial point to emphasize that in general $t_g \neq t$.
The event A, see fig. 1.5, has been defined to be: The instant for which observer O_{x_i}, stationary at x_i in S, reads t_g using clock K_{x_i} also stationary at x_i. Let this statement be shortened to "O_{x_i} reads t_g" and let the symbol $P_{x_i}(t_g)$ represent "O_{x_i} reads t_g". Let event A be defined by:

1.17 Event $A \equiv P_{x_i}(t_g) \equiv O_{x_i}$ reads t_g

Similarly, from the definitions of events B through E, one can define:

1.18 Event $B \equiv P_{x_j}(t_g) \equiv O_{x_j}$ reads t_g

1.19 Event $C \equiv P_{x_j}'(t') \equiv O'_{x_j}{}'$ reads t'

1.20 Event $D \equiv P_{x''}(t') \equiv O'_{x''}$ reads t'

1.21 Event $E \equiv P_{x_i}(t) \equiv O_{x_i}$ reads t

Where x_i,x_j,x_j',x'',t' and t are diagrammed in figure 1.5. Consider now the five events F,G,H,I,J diagrammed in figure 1.6.
Event F is the instant for which observer O_{x_i}, stationary at x_i in S reads t_f using clock K_{x_i} also stationary at x_i. t_f may be any fixed but arbitrary time such that $t_f \neq t_g$. Observer O_{x_i}, point x_i and time t_g are the same observer, spatial point and time as appear in figure 1.5 and figure 1.6.
Event G is the instant for which observer O_{x_j}, stationary at x_j in S, reads t_f using clock K_{x_j} also stationary at x_j. Observer O_{x_j} and point x_j are the same observer and spatial point as appear in figure 1.5 and figure 1.6.

FIGURE 1.6

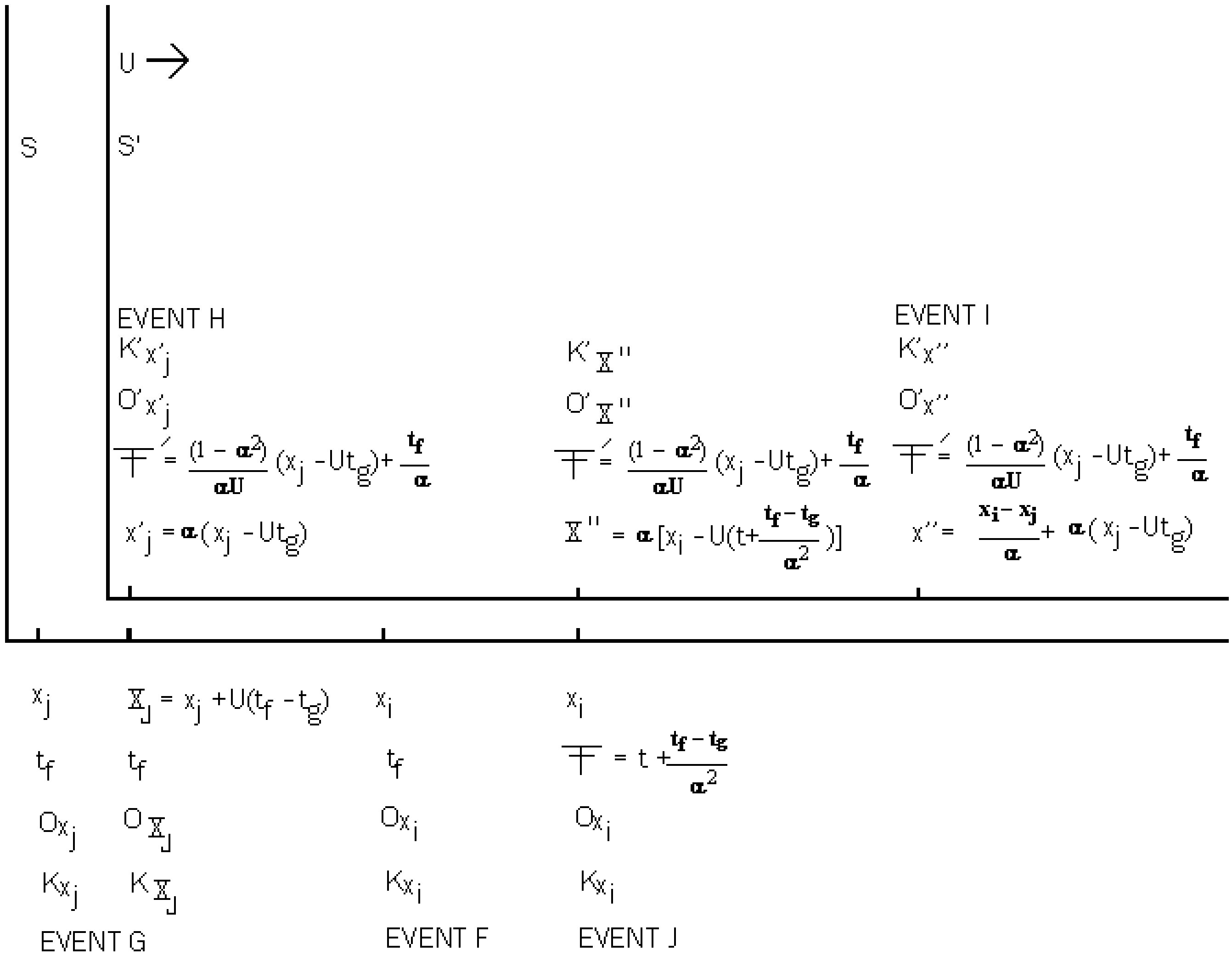


FIGURE 1.6 x_i, x_j, x_j', x'', t' and t_g are the same as in figure 1.5. t_f is arbitrary and fixed subject to $t_f \neq t_g$. Event F and Event J occur at the same spatial point x_i. They are not drawn at the same spatial point to emphasize that in general $t_f \neq$ Ŧ.

Event H is the instant for which observer $O'_{x_j'}$, stationary at x_j' in S', reads Ŧ' using clock $K'_{x_j'}$ also stationary at x_j'. Observer $O'_{x_j'}$ and point x_j' are the same observer and spatial point as appear in figure 1.5 and figure 1.6. The time Ŧ' however is derived as follows. Using x_j' of event H and t_f of event G one can determine $\mathbb{X}_J$

and Ŧ’ of figure 1.6, using 1.6 and 1.9. It is this Ŧ’ which is used in the definition of event H. See figure 1.6.

Event I is the instant for which observer $O'_{x''}$, stationary at x’’ in S’, reads Ŧ’ using clock $K'_{x''}$ also stationary at x’’. Observer $O'_{x''}$ and point x’’ are the same observer and spatial point as appear in figure 1.5. The time Ŧ’ is the same Ŧ’ as appears in event H in figure 1.6.

Event J is the instant for which observer O_{x_i} , stationary at x_i in S, reads Ŧ using clock K_{x_i} also stationary at x_i . Observer O_{x_i} and point x_i are the same observer and spatial point as appear in figure 1.5.

Time Ŧ is derived as follows. Using x_i of event J and Ŧ’ of event I, one can determine X'' and Ŧ of figure 1.6, using 1.8 and 1.9. It is this Ŧ which is used in the definition of event J.

In what follows it is not assumed that t_f= Ŧ.

Event F and event J occur at the same spatial point to emphasize that in general $t_f \neq$ Ŧ.

The event F, see figure 1.6, has been defined to be: The instant for which observer O_{x_i} , stationary at x_i is S, reads t_f, $t_f \neq t_g$, using clock K_{x_i} also stationary at x_i. Let this statement be shortened to “O_{x_i} reads t_f“ and let the symbol $P_{x_i}(t_f)$ represent “O_{x_i} reads t_f“. Let event F be defined by:

1.22 Event F≡ $P_{x_i}(t_f)$≡ O_{x_i} reads t_f

Similarly, from the definition of events G through J, one can define:

1.23 Event G≡$P_{x_j}(t_f)$≡ O_{x_j} reads t_f

1.24 Event H≡P_{x_j}’(Ŧ’)≡ O'_{x_j}’ reads Ŧ’

1.25 Event I≡$P_{x''}$(Ŧ’)≡ $O'_{x''}$ reads Ŧ’

1.26 Event J≡P_{x_i}(Ŧ)≡ O_{x_i} reads Ŧ

Where x_i,x_j,x_j’ are diagrammed in figure 1.5, and t_f, Ŧ and Ŧ’ are diagrammed in figure 1.6.

6. Time Dependent, Observer Dependent, Truth Tables

A time dependent truth value is operationally assigned to events A through E of figure

1.5, as follows. Event A is operationally true only at the instant for which O_{x_i} reads t_g as determined by O_{x_i}. Event A is operationally false at any instant for which O_{x_i} reads t_f as determined by O_{x_i} , where t_f is any time such that $t_f \neq t_g$ as in figure 1.6. This means that for every instant for which event A is false, there is a $t_{f,}$ $t_f \neq t_g$ such that O_{x_i} reads t_f as determined by O_{x_i} .

The operational definition of the time dependent truth value for event B is the same as that of event A with O_{x_j} substituted for O_{x_i} and event B substituted for event A.

Event C is operationally true only at the instant for which $O'_{x_j'}$ reads t', figure 1.5, as determined by $O'_{x_j'}$. Event C is operationally false at any instant for which $O'_{x_j'}$ reads Ŧ', figure 1.6, as determined by $O'_{x_j'}$, where Ŧ' is any time such that Ŧ'≠t', t' as defined in figure 1.5. This means that for every instant for which C is false, there is a Ŧ', Ŧ'≠t', such that O'_{x_j} reads Ŧ' as determined by $O'_{x_j'}$.

The operational definition of the time dependent truth value for event D is the same as that of event C with $O'_{x''}$ substituted for $O'_{x_j'}$ and event D substituted for event C.

Event E is operationally true only at the instant for which O_{x_i} reads t, figure 1.5, as as determined by O_{x_i} Event E is operationally false at any instant for which O_{x_i} reads Ŧ, figure 1.6, as determined by O_{x_i} where Ŧ is any time such that Ŧ≠t, t as defined in figure 1.5. This means that for every instant for which event E is false, there is a Ŧ, Ŧ≠t, such that O_{x_i} reads Ŧ as determined by O_{x_i}.

With these operational definitions of the time dependent truth values of event A through event E, and using figures 1.5 and 1.6 note that:

(i) For every instant for which event A is false, there is an event F, such that event F is true, is one instant for which event A is false.

(ii) For every instant for which event B is false, there is an event G, such that event G is true, is one instant for which event B is false.

(iii) For every instant for which event C is false, there is an event H, such that event H is true, is one instant for which event C is false.

(iv) For every instant for which event D is false, there is an event I, such that event I is true, is one instant for which event D is false.

(v) For every instant for which event E is false, there is an event J such that event J is true, is one instant for which event E is false.

The following two statements are physically true by virtue of in frame clock synchronization, clocks is S with clocks in S.

1.27 (i) If event A is operationally true, then event B is operationally true at the same instant, as determined by O_{x_i}, at which event A is operationally true. See figure 1.5.

(ii) If event A is operationally false, then event B is operationally false at the same instant, as determined by O_{x_i}, at which event A is operationally false. See figure 1.6.

Let [A] be the operational truth value of event A and [B] be the operational truth value of event B. Let T_1 be the functional whose domain is [A] and whose range is [B], where [A] T_1[B] is defined by 1.27 (i) and (ii). The operationally truth table of [A] T_1[B] is:

1.28

	[A]	T_1[B]
(i)	T	T
(ii)	F	F

In what follows the functionals T_2,T_3,T_4 and the truth tables for [B] T_2 [C], [C] T_3 [D], [D] T_4 [E] will be operationally defined.
The following two statements are physically true by virtue of equations 1.6 and 1.7.

1.29 (i) If event B is operationally true, then event C is operationally true at the same instant, as determined by O_{x_j}, at which event B is operationally true. See figure 1.5.

(ii) If event B is operationally false, then event C is operationally false at the same instant, as determined by O_{x_j}, at which event B is operationally false. See figure 1.6

Let [C] be the operational truth value of event C. Let T_2 be the functional whose domain is [B] and whose range is [C], where [B] T_2 [C] is defined by 1.29 (i) and (ii). The operational truth table of [B] T_2 [C] is:

1.30

	[B] T_2	[C]
(i)	T	T
(ii)	F	F

The following two statements are physically true by virtue of in frame clock synchronization, clocks in S' with clocks in S'.

1.31 (i) If event C is operationally true, then event D is operationally true at the same instant, as determined by $O'_{x_j'}$, at which event C is operationally true. See figure 1.5.

(ii) If event C is operationally false, then event D is operationally false at the same instant, as determined by $O'_{x_j'}$, at which event C is operationally false. See figure 1.6.

Let [D] be the operational truth value of event D. Let T_3 be the functional whose domain is [C] and whose range is [D], where [C] T_3 [D] is defined by 1.31 (i) and (ii). The operational truth table of [C] T_3 [D] is:

1.32

	[C] T_3	[D]
(i)	T	T
(ii)	F	F

The following two statements are physically true by virtue of equations 1.6 and 1.7.

1.33 (i) If event D is operationally true, then event E is operationally true at the same instant, as determined by $O'_{x''}$, at which event D is operationally true. See figure 1.5.

(ii) If event D is operationally false, then event E is operationally false at the same instant, as determined by $O'_{x''}$, at which event D is operationally false. See figure 1.6.

Let [E] be the operational truth value of event E. Let T_4 be the functional whose domain is [D] and whose range is [E], where [D] T_4 [E] is defined by 1.33 (i) and (ii). The operational truth table of [D] T_4 [E] is:

1.34

	[D] T_4	[E]
(i)	T	T
(ii)	F	F

7. Derivation of α

Consider the truth tables of [A] T_1[B] and [B] T_2 [C] defined above.

1.28

	[A] T_1	[B]
(i)	T	T
(ii)	F	F

1.30

	[B] T_2	[C]
(i)	T	T
(ii)	F	F

The unique instant for which [B] is T, i.e. time t_g , see figure 1.5, as determined by O_{x_j} reading clock K_{x_j} , is the same instant in both 1.28(i), [B] column and in 1.30(i), [B] column. [B] is F is not a unique instant, however for any instant for which [B] is F, there is a t_f , $t_f \neq t_g$, see figure 1.6, such that O_{x_j} reads t_f is the same instant in both 1.28(ii), [B] column and in 1.30(ii), [B] column, as determined by O_{x_j} reading clock K_{x_j}. Therefore, one can form the composite functional:

1.35

	[A] T_1	[B] T_2	[C]
(i)	T	T	T
(ii)	F	F	F

1.35(i) is generally interpreted to mean that event A and event C are simultaneous events as determined by O_{x_i} reading clock K_{x_i}. However if one assumes that event C and event A are simultaneous events as determined by $O'_{x_j'}$, reading clock $K'_{x_j'}$, see figure 1.5, then one can prove that it is a direct consequence of 1.1 that the speed of light is not an absolute constant and that the velocity of light is vector ally additive: See references 1.2, 1.3, 1.4. In order to prevent this, given that event A and event C are simultaneous events as determined by O_{x_i} reading clock K_{x_i}, event C and event A are assumed not simultaneous events as determined by $O'_{x_j'}$, reading clock $K'_{x_j'}$. That is, it is assumed that simultaneity of events like event A and event C is not symmetric. This is called the relativity of simultaneity.

In what follows it will be proved that it is a direct consequence of 1.1 that, event A and event C are simultaneous events as determined by O_{x_i} reading clock K_{x_i}, and that, event C and event A are simultaneous events as determined by $O'_{x_j'}$, reading clock $K'_{x_j'}$.

For a further general discussion of the simultaneity of events like event A and event C, see appendix 1A at the end of this chapter.

Next consider the truth tables 1.35 and 1.32 where 1.32 is:

1.32

	[C]	T_3	[D]
(i)	T		T
(ii)	F		F

The unique instant for which [C] is T, i.e. time t', see figure 1.5, as determined by $O'_{x_j'}$ reading $K'_{x_j'}$, is the same instant in both 1.35(i), [C] column and in 1.32(i), [C] column. [C] is F is not a unique instant, however for any instant for which [C] is F, there is a Ŧ', Ŧ'≠t', see figure 1.6, such that $O'_{x_j'}$ reads Ŧ' is the same instant in both 1.35 (ii), [C] column and in 1.32(ii), [C] column, as determined by $O'_{x_j'}$ reading $K'_{x_j'}$

Therefore one can form the composite functional:

1.36

	[A]	T_1	[B]	T_2	[C]	T_3	[D]
(i)	T		T		T		T
(ii)	F		F		F		F

Now consider the truth tables 1.36 and 1.34 where 1.34 is:

1.34

	[D]	T_4	[E]
(i)	T		T
(ii)	F		F

The unique instant for which [D] is T, i.e. t', see figure 1.5, as determined by $O'_{x''}$ reading $K'_{x''}$, is the same instant in both 1.36(i), [D] column and in 1.34(i), [D] column. [D] is F is not a unique instant, however for any instant for which [D] is F,there is a Ŧ', Ŧ'≠t', see figure 1.6, such that $O'_{x''}$ reads Ŧ' is the same instant in both 1.36(ii), [D] column and in 1.34(ii), [D] column, as determined by $O'_{x''}$ reading $K'_{x''}$. Therefore one can form the composite functional:

1.37

	[A]	T_1	[B]	T_2	[C]	T_3	[D]	T_4	[E]
(i)	T		T		T		T		T
(ii)	F		F		F		F		F

Given that event A and event C are simultaneous events as determined by O_{x_i} reading clock K_{x_i} and that event C and event E are simultaneous events as determined by O'_{x_j}' reading K'_{x_j}', see figure 1.5, it is tempting to conclude that event A and event E are simultaneous events as determined by O_{x_i} reading clock K_{x_i} . If this were so, then $t_g = \frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g$, and therefore $\alpha = 1$.
Historically to avoid this, it has been assumed that event A and event E are not simultaneous events. This too is called the relativity of simultaneity. In what follows it will be proved that it is a direct consequence of 1.1 that event A and event E are simultaneous events as determined by O_{x_i} reading clock K_{x_i} .

For convenience let [A]Φ[E] represent 1.37, i.e. $\Phi \equiv T_1[B]\, T_2\, [C]\, T_3\, [D]\, T_4$. 1.37 can be rewritten:

1.38

	[A]	Φ	[E]
(i)	T		T
(ii)	F		F

From 1.17, $A \equiv P_{x_i}(t_g)$ and from 1.21, $E \equiv P_{x_i}(t) = P_{x_i}\left(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g\right)$. The last step is taken from the definition of t in figure 1.5.
1.38 becomes:

1.39

$$[P_{x_i}(t_g)]\ \Phi\ [P_{x_i}\left(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g\right)]$$

	$[P_{x_i}(t_g)]$	Φ	$[P_{x_i}(\ldots)]$
(i)	T		T
(ii)	F		F

Consider 1.39 for the two following cases.

Case I. $\alpha > 0$, $\alpha \neq 1$ and Case II. $\alpha = 1$. Case I. is considered first.

Case I. $\alpha > 0$, $\alpha \neq 1$

If $\alpha \neq 1$, then table 1.39 remains as written. Table 1.39 is valid for any x_j in a sufficiently small neighborhood of x_i. However: $P_{x_i}(t_g)$ is independent of x_j and by changing the value of x_j, one does not change the instant for which $[P_{x_i}(t_g)]$ is T and the instants for which $[P_{x_i}(t_g)]$ is F. $P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)$ is dependent on x_j and by changing the value of x_j, one can change the instant for which $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$ is T and the instants for which $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$ is F, without changing the instant for which $[P_{x_i}(t_g)]$ is T, and the instants for which $[P_{x_i}(t_g)]$ is F. As $t = (\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)$, see figure 1.5, by changing the value of x_j, with x_i, t_g, α, U fixed, one can determine t to be any desired value.

In order to satisfy 1.39(i), x_j must be chosen so that for all x_j, $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$ is T. This means that x_j must be chosen so that the expression for t is always in agreement with the reading of clock K_{x_i} as determined by O_{x_i}.

In order to satisfy 1.39(ii), x_j must be chosen so that for all x_j, $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$ is F. This means that x_j must be chosen so that the expression for t is always not in agreement with the reading of clock K_{x_i} as determined by O_{x_i}.

In order therefore to satisfy table 1.39, $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$ must both be true for all time t and false for all time t, $t \geq t_g$, as determined by O_{x_i} reading clock K_{x_i}. This means that x_j must be chosen so that the expression for t is both always, and always not, in agreement with the reading of clock K_{x_i} as determined by O_{x_i}.

This inconsistency may also be expressed as follows. 1.39(i) requires that a T be in 1.39(i) second column. However $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$, in the second column, contains x_j not found in $[P_{x_i}(t_g)]$ in the first column. Thus with fixed x_i,

t_g, α, U; To satisfy the derived requirement that there be a T in 1.39(i) second column, x_j must be chosen so that the expression for t, $t = (\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)$, see figure 1.5, is always in agreement , for $t \geq t_g$, with the reading of clock K_{x_i} as determined by O_{x_i}.
1.39(ii) requires that a F be in 1.39(ii) second column. However $[P_{x_i}(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)]$, in the second column, contains x_j not found in $[P_{x_i}(t_g)]$ in the first column. Thus with fixed x_i, t_g, α, U; To satisfy the derived requirement that there be a F in 1.39(ii) second column, x_j must be chosen so that the expression for t, $t = (\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)$, see figure 1.5, is always not in agreement , for $t \geq t_g$, with the reading of clock K_{x_i} as determined by O_{x_i}. This means that x_j must be chosen so that the expression for Ŧ, $Ŧ = t + \frac{(t_f - t_g)}{\alpha^2}$, Ŧ as defined in figure 1.6 and t as defined in figure 1.5, is always in agreement with the reading of clock K_{x_i} as determined by O_{x_i}. The x_j occurring in Ŧ is by definition the same x_j as occurs in t.
Thus x_j must be chosen so that both t and Ŧ are always in agreement with the reading of clock K_{x_i} as determined by O_{x_i}, and therefore t=Ŧ. However, $Ŧ = t + \frac{(t_f - t_g)}{\alpha^2}$, and therefore $t \neq Ŧ$ for $t_f \neq t_g$. The consequence that t=Ŧ and $t \neq Ŧ$ for $t_f \neq t_g$ means that truth table 1.39 with $\alpha > 0$, $\alpha \neq 1$ is inconsistent with itself and does not represent physical reality. It is the occurrence of x_j in event E and its nonoccurrence in event A that allows this conclusion to be made.
Truth table 1.39 is a consequence of 1.1. Therefore:

1.40 If 1.1 and its consequences are physically true, then $\alpha \neq 1$ is physically false.

With $\alpha > 0$ physically true, and $\alpha \neq 1$ physically false, the only remaining possibility is that $\alpha = 1$. With $t = (\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g)$, and $t\big|_{\alpha=1}$, yields $t = t_g$. Events A,B,C,D,E, are all true at the same instant $t = t_g = t'$. Event A is simultaneous with event C and event C is simultaneous with event A. Event A is simultaneous with event C, event C is simultaneous with event E and event A is simultaneous with event E. Instant is therefore symmetric and transitive. It has been proved that:

1.41 If 1.1 and its consequences are physically true, then $\alpha = 1$ and instant is both symmetric and transitive.

Case II. $\alpha = 1$

If $\alpha = 1$ then $P_{x_i}\left(\frac{(1-\alpha^2)}{\alpha^2 U}(x_j - x_i) + t_g\right) = P_{x_i}(t_g)$ and 1.39 becomes:

1.42(a)

	$[P_{x_i}(t_g)]$ Φ	$[P_{x_i}(t_g)]$
(i)	T	T
(ii)	F	F

Table 1.42(a) is consistent with itself.

It has been proved:

1.42(b) If 1.1 and its consequences are physically true, then:

(i) $\alpha = 1$ is physically true.

(ii) $\alpha \neq 1$ is physically false.

8. The Galilean Transform

It immediately follows from 1.42(b) and 1.6 and 1.7:

1.43 $x' = x - Ut$, $y' = y$

1.44 $t' = t$, $z' = z$

This is the Galilean Transform. It has been proved:

1.45 If postulational system 1.1 and its consequences are physically true, then:

(i) The Galilean and no other transform is a direct consequence of postulational system 1.1

(ii) The Galilean Transform is the physically correct transform relating $\underset{\sim}{x}$,t in inertial frame S to $\underset{\sim}{x}$',t' in inertial frame S'

9. Vector Additivity of the Velocity of Light.

Consider a light source stationary in S' emitting a ray of light in a specified direction. Let $\underset{\sim}{c}'_o \equiv (c'_{o_{x'}}, c'_{o_{y'}}, c'_{o_{z'}})$ represent the vector velocity of the light beam as measured by observers stationary in S'. Let $\underset{\sim}{c} \equiv (c_x, c_y, c_z)$ represent the vector velocity of the light beam, whose source is stationary in S', as measured by observers stationary in S. Differentiating 1.43 and 1.44 with respect to t' yields:

1.46 $$\frac{dx'}{dt'} = \frac{dx}{dt} - U \qquad \frac{dy'}{dt'} = \frac{dy}{dt}, \quad \frac{dz'}{dt'} = \frac{dz}{dt}$$

Let $\underset{\sim}{x}$' = (x',y',z') in S', travel with the light beam and let $\underset{\sim}{x}$ = (x,y,z) in S, travel with the light beam.
From 1.46 therefore:

1.47 $$c'_{o_{x'}} = c_x - U, \qquad c'_{o_{y'}} = c_y, \qquad c'_{o_{z'}} = c_z$$

Rewriting 1.47 in vector form yields:

1.48 $$\underset{\sim}{c} = \underset{\sim}{c}'_o + U\hat{x}$$ That is, the velocity of light is vector ally additive.

Take absolute values of both sides of 1.48 .

1.49 $$|\underset{\sim}{c}| = |\underset{\sim}{c}'_o + U\hat{x}|$$

Let $c'_o \equiv |\underset{\sim}{c}'_o|$, where $|\underset{\sim}{c}'_o|$ is the speed of light as measured in S' from a source at rest in S'. Experimentally $c'_o = 2.99 \cdot 10^{10}$ cm/sec. using as a light source a mercury vapor lamp.
It is a consequence of 1.49:

1.50 $\quad |\underset{\sim}{c}'_o + U\hat{x}| \neq |\underset{\sim}{c}'_o| \equiv c'_o$ for $U \neq 0$ and $U \neq -2|\underset{\sim}{c}'_o|\cos\theta'$

where θ' is the angle between $\underset{\sim}{U}$ and $\underset{\sim}{c}'_o$. From 1.49 and 1.50:

1.51 $\quad |\underset{\sim}{c}| \neq c'_o$ for $U \neq 0$ and $U \neq -2|\underset{\sim}{c}'_o|\cos\theta'$.

From 1.51 and 1.48 it has been proved:

1.52 If postulational system 1.1 and its consequences are physically true:
(i) The II postulate is physically false.
(ii) The velocity of light is vector ally additive,
$\underset{\sim}{c} = \underset{\sim}{c}'_o + \underset{\sim}{U}$.

In the above it is assumed that the L.S. is moving with constant velocity $\underset{\sim}{U} = U\hat{x}$ with respect to S. It is straight forward to generalize the above proof for arbitrary constant velocity $\underset{\sim}{U}$.
This completes the proof of theorem 1.0.

10. Critique of the Derivation of the Lorentz Transform

The Lorentz Transform was derived by Albert Einstein in 1905 from postulational system 1.2. Although the derivation is algebraically correct, Einstein assumed that the II postulate was physically true. It has been proved in sections 1 through 9, from a proper subset of postulational system 1.2, namely postulational system 1.1: "If postulational system 1.1 and its consequences are physically true, then the II postulate is physically false". In the derivation of the Lorentz Transform one assumes that the II postulate is physically correct, however using 1.52(i), the II postulate is physically false and consequently the Lorentz Transform is physically false.
If the Lorentz Transform is physically false, then $E=mc_o^2$ and all formulae derived from the Lorentz Transform are physically false. The $\underset{\sim}{E}$ and $\underset{\sim}{B}$ fields of Maxwell's Equations are postulated to travel in vacuum at the absolute constant speed c_o. Maxwell's Equations must therefore be rederived so as to incorporate the vector additivity of the velocity of light. The results of this section are:

1.53 If postulational system 1.1 and its consequences are physically true, then:
(i) The Lorentz Transform is physically false.
(ii) $E=mc_o^2$ - and all formulae derived from the

Lorentz Transform are physically false.
(iii) Maxwell's Equations must be rederived so as to incorporate the vector additivity of light.

11. The Consistency of the Postulates of Special Relativity

It is an immediate consequence of truth table 1.39:

1.54
(i) Postulational system 1.1 is inconsistent with itself for $\alpha \neq 1$ and $\alpha > 0$.
(ii) If postulational system 1.1 is consistent with itself, then $\alpha = 1$.

Note the following weaker statements.

1.55 Postulational system 1.1 is a proper subset of postulational system 1.2, so that if 1.2 and its consequences are physically true, then 1.1 and its consequences are physically true.

Also:

1.56 The II postulate is a proper subset of postulational system 1.2, so that if 1.2 and its consequences are physically true, then the II postulate is physically true.

It is a direct consequence of 1.54 and 1.55 and 1.52(i):

1.57 If postulational system 1.2 and its consequences are physically true:
(i) The II postulate is physically true.
(ii) The II postulate is physically false.

1.57(i) and (ii) are inconsistent with one another and therefore it is false that postulational system 1.2 and all of its consequences are physically true. Formally:

1.58 If postulational system 1.1 and its consequences are physically true, then:
(i) It is false that postulational system 1.2 and all of its consequences are physically true.

Also:

1.59 If postulational system 1.1 and its consequences are physically false, then:

(i) It is false that postulational system 1.2 and all of its consequences are physically true.

12. Statement of Results

It has been proved that if postulational system 1.1 and its consequences are physically true, then:
Listed in the order proved.

1. The negative results of the Michelson-Morley Experiment do not prove that the speed of light is an absolute constant.
2. The principle of relativity of simultaneity is physically false.
3. The Galilean Transform is the physically correct transform relating $\underset{\sim}{x}$,t in inertial frame S to $\underset{\sim}{x}$',t' in inertial frame S'.
4. The velocity of light is vector ally additive, $\underset{\sim}{c} = \underset{\sim}{c}'_{o} + \underset{\sim}{U}$. In general the proof holds for all electromagnetic radiation.
5. The II postulate is physically false.
6. The Lorentz Transform, $E=mc_0^2$ and all formulae derived from the Lorentz Transform are false and do not represent physical reality.
7. Maxwell's Equations must be rederived so as to incorporate the vector. additivity of light.
8. The two postulates of Special Relativity Theory are inconsistent with one another.

In the following chapters, the implications of the above conclusions for the Atom, the Electromagnetic Field and Quantum Mechanics will be examined.

13. Mathematical Conjectures

During the years 1900 to 1910, Alfred North Whitehead and Bertrand Russell wrote Principia Mathematica. One of their mathematical research aims was to derive the properties of the natural numbers from the axioms of Peano. The properties of the natural numbers were deduced from a finite subset, and the aim was to derive those properties from the axioms of Peano for all the natural numbers. They found in order to do so, it was necessary to make additional assumptions ad infinitum.
In the above chapter, it was found possible to derive consequences of postulational system 1.1 here to fore thought impossible to derive: Namely, the associativity and transitivity of instant and the vectoral additivity of the velocity of light. The mathematical tool that enables the derivations to be made is truth table 1.39 and

the property of truth table 1.39 that enables the derivations to be made is the existence of a variable in the definiens of the second column of 1.39 not found in the definiens of the first column of 1.39.

If within the context of mathematical logic and the axioms of Peano one could construct truth tables Π_i similar to 1.39, it is hypothesized that using Π_i one could derive the properties of the natural numbers that Whitehead and Russell could not derive from Peano's axioms. Of course the operational definitions used to define the connectors T_1 through T_4 in 1.39 cannot be used, and one must use the traditional connectors of mathematical logic, if-then, and-or etc.

It is hypothesized that the desired properties of the natural numbers can be derived by requiring the Π_i to be self consistent.

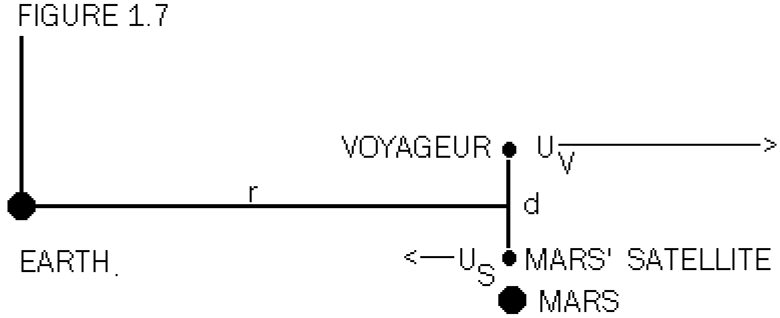
Further, assuming the existence of Π_i , it is hypothesized, that because of the variable in the definiens of the second column not found in the definiens of the first column, that each Π_i does not have a unique Godel number. If so, each Π_i would be a consequence of the axioms of Peano without a unique Godel number and thus a counterexample to the proposition that all consequences of the axioms of Peano have a unique Godel number.

14. Suggested Experiments

Perhaps the most easily tested implication of the II Postulate is that all wavelengths of electromagnetic radiation have the same speed i.e. $c_o \cong (2.99)10^{10}$ cm/sec . Thus radar, infrared, visible, ultraviolet and gamma radiation etc. should all have the same speed in vacuum. The experimentally determined speed of radar waves reflected from the moon should equal the experimentally determined speed of light from a mercury vapor source or a tungsten source or an incandescent $SrCl_2$ source etc. It is of most interest that these experiments be performed and the experimental results published.

The derived equation $\underset{\sim}{c} = \underset{\sim}{c}'_o + \underset{\sim}{U}$ can be tested using a satellite in orbit about Mars and an outward bound Voyageur Explorer.

FIGURE 1.7



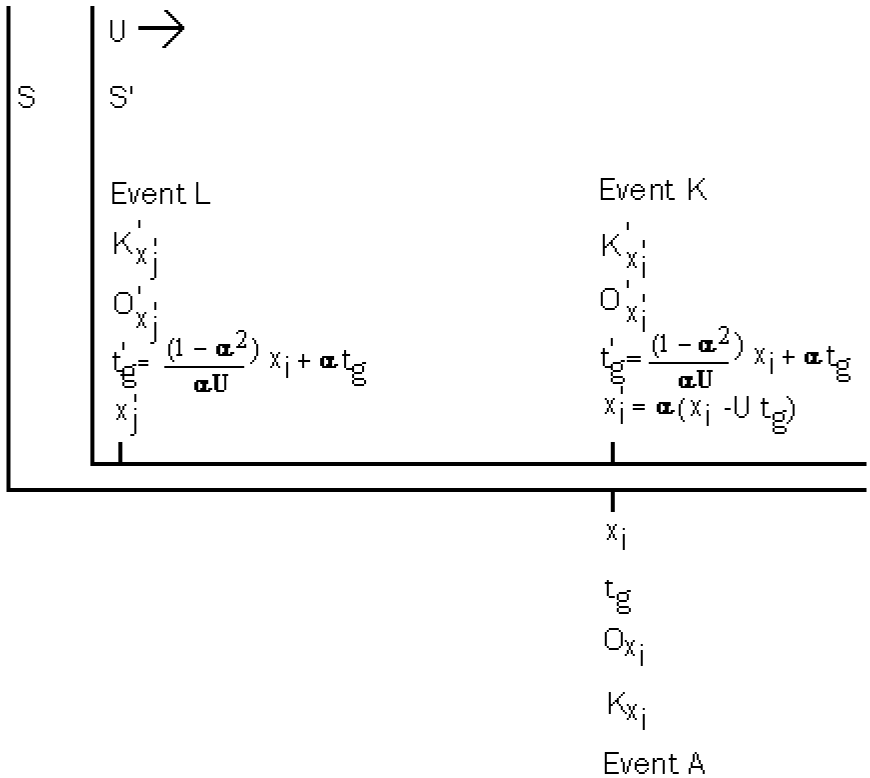
See Figure 1.7 .
d is the closest approach distance between the Voyageur and the Mars' Satellite and r is theperpendicular bisector of d extending to a radio receiver on the earth's surface. $d << r$. U_V is the speed of the outward bound Voyageur, $U_V \cong 40$Km/sec, and U_S is the speed of the Mars' Satellite, $U_S \cong 10$Km/sec . U_V and U_S are measured with respect to earth. At time t_o the Voyageur and the Mars' Satellite are in the positions shown in Figure 1.7, and at time t_o each satellite sends a radio signal to earth. Let c'_R be the rest speed of the radio waves of given frequency, measured with transmitter and receiver at rest on earth. Let Δt be the arrival time difference between the two radio signals emitted by Voyageur and the Mars Satellite as determined by a radio receiver on the earth's surface. Using $\underset{\sim}{c} = \underset{\sim}{c}'_R + \underset{\sim}{U}$, Δt is,

$\Delta t = [\frac{U_V + U_S}{c'_R}]\frac{r}{c'_R}$. At conjunction of Earth and Mars, $r \cong (8)10^7$Km. If $c'_R = (3.0)10^5$ Km/sec , then $\Delta t = (4.4)10^{-2}$sec. If $c'_R = (1.0)10^3$ Km/sec , then $\Delta t = (4.0)10^3$ sec. If $\Delta t = 0$ sec , then the speed of light is an absolute constant. It is of extreme interest that this experiment be performed and the results published.

15. Appendix 1A- Transitivity of Instant

There is one other way that can be used to define the simultaneity of events like event A and event C of truth table 1.35. Let $O'_{x'_i}$ be the observer stationary at x'_i in S' coincident with O_{x_i} at time t_g as determined by O_{x_i} reading clock K_{x_i} stationary at x_i. See figure 1A. x'_i and t'_g are related to x_i and t_g of figure 1A, by equations 1.6 and 1.7.

FIGURE 1A



Event A and event K occur at the same instant as determined by O_{x_i} and $O'_{x'_i}$.

Event A is the same event as in figures 1.5 and 1.6.

Let $O'_{x'_j}$ be the observer stationary at x'_j in S' where x'_j is the same x'_j as in figures 1.5 and 1.6. Event K and event L occur at the same instant t'_g as determined by $O'_{x'_i}$ and $O'_{x'_j}$.

Event A and event B of figure 1.5 occur at the same instant as determined by O_{x_i}. Event B and event C occur at the same instant as determined by $O'_{x'_i}$. By convention , event A and event C occur at the same instant as determined by O_{x_i}. Call this convention, convention$_1$. This is however just a convention and one could just as well use the events defined in figure 1A and conclude: Event A and event K occur at the same instant as determined by O_{x_i}. Event K and event L occur at the same instant as determined by $O'_{x'_i}$. By convention$_2$, event A and event L occur at the same instant as determined by O_{x_i}. This defines convention$_2$. If $\alpha=1$, then event C and event L are the same instant and convention$_1$ = convention$_2$. If $\alpha \neq 1$, then event C and event L are not the same instant and convention$_1 \neq$ convention$_2$.

It is tempting to state that convention_1 must equal convention_2 and that therefore $\alpha=1$. This however is in violation of the assumption that t'=g(x,t). See section 2, corollary I. To conclude that $\text{convention}_1 = \text{convention}_2$ it must be proved that t'=g(t): That is done in the proof of 1.52 without assuming that $\text{convention}_1 = \text{convention}_2$.

References

1.1 A. Einstein, AnnIn. Physics, 1f, 891 (1905a).

1.2 J.M. Kingsley III, On the Consistency of the Postulates of Special Relativity, Found. Physics, 5, 295-300 (1975).

1.3 R.E. Chatham, Consistency in Relativity, Found. Physics, 6, 681-685 (1976).

1.4 O. Gron and M. Nicloa, The Consistency of the Postulates of Special Relativity, Found. Physics, 6, 677-680 (1976).

1.5 J.M. Kingsley III, On the Absolute Constancy of the Speed of Light, Copyright Registration # TXu 173-386 , (September 24, 1984).

1.6 A.P. French, Special Relativity, p79, (W.W. Norton & Company Inc. New York 1968)