10.  The Hydrogen Bomb

1.  Introduction

On July 16, 1945, the United States detonated the world's first, large yield, atomic device at the trinity site near Alamogordo, New Mexico.  With a yield of 20 kilotons of TNT, the bomb changed forever man's view of warfare and opened the possibility of destroying any city or markedly altering any physical feature on the earth's surface with one bomb.  The threat of nuclear proliferation and with it the increased possibility that small bands of terrorists could obtain and use such weapons, threaten all civilized men to this day.  It is therefore with a profound sense of the gravity of the situation that the author gives an explanation of how the bomb works with out using $E=mc^2$.

2.  Potential Energy

An atom of the bomb material attracts a particle of mass $m_p$ at a distance r from the center of mass of the atom with potential energy PE(r).  As the particle goes from $r_o$ to r, $\Delta PE(r) \equiv PE(r) - PE(r_o)$, $r_{pa} \leq r \leq r_o$.  $r_o$ is the radius of the atom, $r_{pa}$ is the radius of the particle where $r_{pa} << r_o$.  PE(r)  is given by 10.1 below, all quantities measured from an inertial frame at rest w.r.t. the center of mass of $m_{At}$

$$10.1 \quad PE(r) = -m_p m_{At} \frac{H}{r_o} \left[ \frac{1}{(p+2)} [p+3 - (\frac{r}{r_o})^{p+2}] \right], r_{pa} \leq r \leq r_o, \ -3 < p \leq 0$$

An atom of the bomb material adsorbs particle $m_p$ (Whose origin is discussed in sec. 4) and the particle is attracted to the center of the atom by the atomic force with $\Delta PE(r) = PE(r) - PE(r_o)$ given by:

$$10.2 \quad \Delta PE(r) = -m_p m_{At} \frac{H}{r_o(p+2)} [1 - (\frac{r}{r_o})^{p+2}], r_{pa} \leq r \leq r_o$$

The translational kinetic energy of the particle increases by $\Delta KE_{pa}$ as given by 10.3

$$10.3 \quad -\Delta PE(r) = m_p m_{At} \frac{H}{r_o(p+2)} [1 - (\frac{r}{r_o})^{p+2}] = \Delta KE_{pa}, \ r_{pa} \leq r \leq r_o$$

10.3 is numerically evaluated for specific values of r and $r_o$ in sec 4.


For future use.
From 3.17, $\frac{1}{2}U_{rms}^2(r) + \Psi(r) = C_1$ and the binding energy of the atom is:

$$BE=T+V=mC_1=\frac{1}{2}mU_{rms}^2(r_0)+m\Psi(r_0), \text{ where } T+V= 2\pi\int_0^{r_0} r^2\rho(r)U_{rms}^2(r)dr + 4\pi\int_0^{r_0} r^2\Psi(r)\rho(r)dr$$

and using 3.8, $\Psi(r)=-\frac{mH(p+3)}{r_0(p+2)}[1-\frac{1}{(p+3)}(\frac{r}{r_0})^{p+2}]$, with $\Psi(r_0)=-\frac{mH}{r_0}$. The binding energy

becomes: $BE=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}$, with density $\rho(r)=\frac{(p+3)m}{4\pi r_0^3}(\frac{r}{r_0})^p$. By direct

computation (See appendix 10A):

$$V=-\frac{m^2H}{h_0}\frac{(p+3)}{(p+2.5)}, \quad T=\frac{m^2H}{h_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(r_0), \quad -2.5^+\le p\le 0, \quad T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$$

$$V=-\infty, \qquad\qquad T=+\infty, \qquad\qquad\qquad -3<p\le-2.5^-, \quad T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$$

A physical process to create bomb material is discussed in the following section.


3. Creation of Bomb Material

As explained below: In order to create bomb material, atoms must be created with $p=-3+10^{-n}$, $n\gtrsim 2$. In order to create atoms with $p=-3+10^{-n}$, $n\gtrsim 2$, consider the following scenario.

A. Using 4.6 and table 4.1, the average pressure two adjacent atoms exert on one another is $P_0=2r_0 nK\frac{T_i}{\overline{R_i}}$ and with Fe as an example at $T_i=293^0K$ and $\overline{R_i}=4.6\cdot 10^{-11}$cm,

$P_0$ becomes: $P_0=1.7\cdot 10^{12}\frac{dy}{cm^2}$.

B. Place the bomb material between the poles of a high voltage source. This will increase the radius of the atoms by $\Delta r_0$ and decrease $\overline{R_i}$ from $\overline{R_i}$ to $\overline{R_i}-2\Delta r_0$. P

becomes $P_{\Delta r_0}=\dfrac{KT_i}{(2r_0+\overline{R_i})^2\overline{R_i}(1-\frac{2\Delta r_0}{\overline{R_i}})}$ and $P_{\Delta r_0}$ becomes: $P_{\Delta r_0}=\dfrac{P_0}{1-\frac{2\Delta r_0}{\overline{R_i}}}$. An arbitrarily

large value of $P_{\Delta r_0}$ occurs as $1-2\frac{\Delta r_0}{\overline{R_i}}->0$ and with $1-2\frac{\Delta r_0}{\overline{R_i}}=0$, $\Delta r_0$ becomes: $\Delta r_0=2.3\cdot 10^{-11}$cm.

C.  The change in binding energy of an atom is $\Delta BE = \frac{1}{2}m\Delta U^2_{rms}(r_o) + \frac{m^2H}{r_o^2}\Delta r_o$ and assuming

$\Delta U^2_{rms}(r_o) = 0$, $\Delta BE$ becomes $\Delta BE = \frac{m^2H}{r_o^2}\Delta r_o$ and evaluating for Fe yields $\Delta BE = 68\Delta r_o$ erg.

Evaluating $\Delta BE$ for $\Delta r_o = 2.3 \cdot 10^{-11}$ cm. yields $\Delta BE = 1.6 \cdot 10^{-9}$ erg $= 1.0 \cdot 10^3$ ev.

D.  The vibrating surface of the atom $A_1$ with average radius $r_o + \Delta r_o$, $\Delta r_o$ positive or

negative, is represented by $\underline{h}_1 = r(\hat{r}) \cdot \hat{r} + \chi_1(r(\hat{r}),t) \cdot \hat{r}$ with $\chi_1(r(\hat{r}),0) = 0$.  Using 3.8 and

3.15 with $[U(\underline{h}(\hat{r}_1,t)_1)]_{rms} = [U(\underline{h}(\hat{r}_2,t)_1)]_{rms}$ averaged over $\hat{r}$ or t.

10.3a     $\frac{1}{2}[U(\underline{h}(\hat{r},t)_1)]^2_{rms} - \frac{mH}{(\underline{h}(\hat{r},t)_1)_{rms}} = C_2 < 0,$

where $(\underline{h}(\hat{r},t)_1)_{rms} \equiv r_o + \Delta r_o$ independent of $\hat{r}$.

The average maximum value of $\underline{h}_1$ is evaluated at the instant for which $\overline{U(\underline{h}(\hat{r},t)_1)} = 0$

and assuming $\overline{\underline{h}(\hat{r},t)_1}$ is given by $\overline{\underline{h}(\hat{r},t)}_{1,M} = -\frac{mH}{C_2}$, the averaged amplitude of $h_o$ becomes:

$A_{mp} = \overline{\underline{h}(\hat{r},t)}_{1,M} - (\underline{h}(\hat{r},t)_1)_{rms} = -\frac{mH}{C_2} - r_o(1 + \frac{\Delta r_o}{r_o}) > 0$.  Set $A_{mp} = a_1\overline{\underline{h}(\hat{r},t)}_{1,M}$ with

$0 < a_1 < 1$ and solve for $C_2$ yielding: $C_2 = -\frac{(1-a_1)}{r_o(1+\frac{\Delta r_o}{r_o})}mH$ and $\overline{\underline{h}(\hat{r},t)}_{1,M} = \frac{r_o(1+\frac{\Delta r_o}{r_o})}{(1-a_1)}$ and $A_{mp} = a_1\frac{r_o(1+\frac{\Delta r_o}{r_o})}{(1-a_1)}$

For future use using $|\frac{\Delta r_o}{r_o}| << 1$ and $TE_{int} = BE_{atom} = (T+V)_{atom} = mC_2$:

10.3b    $TE_{int} = BE_{atom} = (T+V)_{atom} = mC_2 \doteq -\frac{(1-a_1)}{r_o}(1 - \frac{\Delta r_o}{r_o})m^2H$   with $0 < a_1 < 1$ and

$\overline{\underline{h}(\hat{r},t)}_{1,M} \doteq \frac{r_o}{(1-a_1)}$,     $A_{mp} = a_1\overline{\underline{h}(\hat{r},t)}_{1,M} \doteq \frac{a_1}{(1-a_1)} \cdot r_o$     $|\frac{\Delta r_o}{r_o}| << 1$

E.  Note that the total internal energy $TE_{int}$ of a given vibrating atom $A_o$ with average radius $r_o$

is, $TE_{int} = mC_1 = -\frac{m^2H}{r_o} + \frac{1}{2}mU^2(r_o) = -\frac{(1-a_o)}{r_o}m^2H$, where $TE_{int}$ is independent of density $\rho(r)$ and p

for a given $r_o$ where $\rho(r) = \frac{(p+3)m}{4\pi r_o^3}(\frac{r}{r_o})^p$ and $-3 < p \leq 0$.  Due to pressure $P_{\Delta r_o}$, (See B.)

i.  $TE_{int}$ becomes: $mC_2 = -\frac{m^2H}{r_o}(1 - \frac{\Delta r_o}{r_o}) + \frac{1}{2}mU^2(r_o(1+\frac{\Delta r_o}{r_o})) = -(1-a_1)\frac{m^2H}{r_o}(1 - \frac{\Delta r_o}{r_o})$

ii.  The interior mass of the atom is compressed maintaining the atom's average radius

$r_o(1+\frac{\Delta r_o}{r_o})=r_f$ and decreasing p from $p_i$ to $p_f$ where $-3<p_f<p_i\leq0$ and $r_f\equiv r_o+\Delta r_o$. This takes energy $E_{\Delta p}$ where $\Delta p\equiv p_f-p_i$.

iii.  The difference in $TE_{int}$ is $\Delta TE_{int}= m(C_2- C_1)= -(1-a_1)\frac{m^2H}{r_f}[1-\frac{(1-a_0)r_f}{(1-a_1)r_o}]$.

Let $A_1$ represent the atom with radius $r_o$ and let $A_2$ represent the atom with radius $r_f\equiv r_o+\Delta r_o$.

Using 10.3b, the oscillation amplitude of $A_o$ is $A_{mp}=\frac{a_0}{(1-a_0)}\cdot r_o$ with $0<a_0<1$ and the

oscillation amplitude of $A_1$ is $A_{mp}=\frac{a_1}{(1-a_1)}\cdot r_f$ with $0<a_1<1$.

iv.  The total electrical energy $E_{El}$ to change the radius of an atom by $\Delta r_o=r_f-r_o$ and to change p by $\Delta p\equiv p_f-p_i$ is:

10.3c     $E_{El}=E_{\Delta p}+\Delta TE_{int}=E_{\Delta p}-(1-a_1)\frac{m^2H}{r_f}[1-\frac{(1-a_0)r_f}{(1-a_1)r_o}] >0, 0<a_0<1, 0<a_1<1$

10.3c is evaluated in section 5 below.


F.  The change in internal energy $\Delta E_{int}$ of an atom as $A_{mp}->0$ is: $\Delta E_{int}=E_{int,f}- E_{int,i}=$

$-\frac{m^2H}{r_f}+(1-a_1)\frac{m^2H}{r_f} =-a_1\frac{m^2H}{r_f}$  and the work done on the external world $E_{ex}$ is $E_{ex}=|\Delta E_{int}|=a_1\frac{m^2H}{r_f}$.
It is hypothesized that during an unwanted meltdown of nuclear fuel such as at the Russian Chernobyl 1986 nuclear accident, due to contact with a dissimilar substance, the $A_{mp}$ of the atoms of the nuclear fuel decreases resulting in $|\Delta E_{int}|>0$ and an increase in translational kinetic energy and concomitant temperature and melt of the nuclear fuel and containment vessel.

G. The atomic mass within a sphere of radius $r\leq r_f$ is $M(r) \equiv 4\pi\int_0^r r^2\rho(r)dr=m(\frac{r}{r_f})^{p+3}$

and for  $\frac{M(r)}{m_{Fe}}=0.90$:  $\frac{r}{r_f}(p)=(\frac{M(r)}{m_{Fe}})^{\frac{1}{p+3}}=(0.90)^{\frac{1}{p+3}}$ and evaluating $\frac{r}{r_f}(p)$ for given p yields:

$\frac{r}{r_f}(0)=0.97, \frac{r}{r_f}(-1)=0.95, \frac{r}{r_f}(-2)=0.90, \frac{r}{r_f}(-2.9)=0.35, \frac{r}{r_f}(-2.99)=2.7\cdot10^{-5}$

With $p=-3^+$ and consequent $\frac{r}{r_f}(-3^+)<<1$, a particle with radius $r_{pa}<<r_f$ plunges through the less dense outer shell of the atom increasing in energy and momentum and strikes the hard nucleus transforming particle kinetic energy into nuclear and particle internal energy and with small enough r, where r is the distance between the center of the atom and the center of the particle, both the atom and the particle explode with energy $E_{out}$ where $E_{out}$ is measured at R as $R->\infty$.

For some $p=-3^+$ and with $\frac{r}{r_f}(-3^+)<<1$ using 10.3:

10.4a $\quad -\Delta PE(r) = \dfrac{m_p m_{At} H}{(p+2)r_f}\left[1-(\tfrac{r}{r_f})^{p+2}\right] = |BE(r_f)_{At}| + |BE_p| + E_{out}$ .

And with $E_{el} = -\Delta PE(r)$

10.4b $\quad E_{El} = E_{\Delta p} - (1-a_1)\dfrac{m_{At}^2 H}{r_f}[1-\dfrac{(1-a_0)r_f}{(1-a_1)r_0}] = \dfrac{m_p m_{At} H}{(p+2)r_f}\left[1-(\tfrac{r}{r_f})^{p+2}\right] = |BE(r_f)_{At}| + |BE_p| + E_{out}$

10.4b is evaluated in section 5.

## 4. Atom Compression Revisited

Consider an atom undergoing compression with initial radius $r_0$, final radius $r_f$.
From section E above, the difference in internal energy $\Delta TE_{int}$ of an atom undergoing compression with initial radius $r_0$ and final radius $r_f$ is,

$\Delta TE_{int} = m(C_2 - C_1) = -(1-a_1)\dfrac{m^2 H}{r_f}[1-\dfrac{(1-a_0)r_f}{(1-a_1)r_0}] = \tfrac{1}{2}m[U^2(r_f)-U^2(r_0)]-\dfrac{m^2 H}{r_0}(\dfrac{r_0-r_f}{r_f})$ where $a_0 = \dfrac{\frac{1}{2}U^2(r_0)}{\frac{mH}{r_0}}$

and $a_1 = \dfrac{\frac{1}{2}U^2(r_f)}{\frac{mH}{r_f}}$. In general therefore with $r_f < r_0$ and given $0 \le a_0 \le 1$ and $0 \le a_1 \le 1$, derive $0 \le a_0 < a_1 \le 1$.

The compressive force increases the total internal energy $TE_{int}$ of the atom i.e. $\Delta TE_{int} > 0$,

and therefore $[1-\dfrac{(1-a_0)r_f}{(1-a_1)r_0}] < 0$ with resultant $\dfrac{(1-a_0)r_f}{(1-a_1)r_0} > 1$ and using $\dfrac{(1-a_0)r_f}{(1-a_1)r_0} > 1$, it follows

that, $A(r_f)_{mp} = \dfrac{a_1 r_f}{(1-a_1)} > \dfrac{a_1 r_0}{(1-a_0)} > \dfrac{a_0 r_0}{(1-a_0)} = A(r_0)_{mp}$ i.e. $A(r_f)_{mp} > A(r_0)_{mp}$.

Note that if $a_1 = 1-\varepsilon$ with $0 < \varepsilon << 1$ so that $\Delta TE_{int} = -\dfrac{m^2 H}{r_f}[\varepsilon - (1-a_0)\tfrac{r_f}{r_0}]$ and if $(1-a_0)\tfrac{r_f}{r_0} >> \varepsilon$, then

$\Delta TE_{int} \doteq \dfrac{m^2 H}{r_0}$ and $\Delta TE_{int} = \tfrac{1}{2}m[U^2(r_f)-U^2(r_0)]-\dfrac{m^2 H}{r_0}(\dfrac{r_0-r_f}{r_f}) \doteq \dfrac{m^2 H}{r_0}$ and $\tfrac{1}{2}m[U^2(r_f)-U^2(r_0)]-\dfrac{m^2 H}{r_f}=0$.

The resultant compressed atom is quasi stable, $\tfrac{1}{2}U^2(r_f)-\dfrac{mH}{r_f}(1-\varepsilon)=0$, and will become

stable if by direct contact with stable atoms the vibrational energy $\tfrac{1}{2}mU(r_f)^2$ is transferred to

neighboring atoms resulting in $\tfrac{1}{2}U(r)^2 + \Psi(r) = \tfrac{1}{2}U(r_{ff})^2 - \dfrac{mH}{r_f} = const$ with $\tfrac{1}{2}U(r_{ff})^2 << \dfrac{mH}{r_f}$.

Note that from $(1-a_0)\tfrac{r_f}{r_0} >> \varepsilon = (1-a_1)$ derive $\dfrac{\frac{mH}{r_0}-\frac{1}{2}U^2(r_0)}{\frac{mH}{r_f}} >> \dfrac{\frac{mH}{r_f}-\frac{1}{2}U^2(r_f)}{\frac{mH}{r_f}}$ and $\tfrac{1}{2}U^2(r_f)-\tfrac{1}{2}U^2(r_0) >> \dfrac{mH}{r_f}-\dfrac{mH}{r_0} > 0$

## 5. Creation of an Explosion

Continuing the scenario begun above, the energetics of a thermonuclear bomb are developed within the context of a shrunken Fe atom. We consider the bomb material to be 0.5Kg of Fe containing $5.3 \cdot 10^{24}$ atoms. With a yield of 30 megatons of TNT, and if each atom contributes to the total yield, $E_{out}$ of each atom is: $E_{out} = 0.23$ erg$= 140$Bev.
After the initial p value is decreased from $p_i$ to $p_f$, it is sputtered into a diatomic thin film. Fig. 10.1

As will be shown with $-3 < p \lesssim -2.90$, most of the mass of the atom is contained within

a shell of radius $r_{nu} << r_f \doteq 10^{-8}$ cm with $m_{At} - m_{nu} << m_{At}$ .

The binding energy of this atom is:

$$BE_f = \frac{1}{2} m U_{rms}^2 (r_f) - \frac{m^2 H}{r_f} = - (1-a_1) \frac{m^2 H}{r_f} \text{ , where } A_{mp} = \frac{a_1}{(1-a_1)} \cdot r_f \text{ with } 0 < a_1 < 1. \text{ If } a_1 = \frac{1}{3}, \text{ the}$$

atoms oscillate with amplitude $\frac{1}{2} r_f$ : The average least distance $R_{ld}$ between

the center of mass of two adjacent atoms is $R_{ld} = r_f$, the average distance $R_{av}$ is

$R_{av} = 2r_f$, and the average greatest distance $R_{gd}$ is $R_{gd} = 3r_f$ .

FIGURE 10.1
DIATOMIC THIN FILM
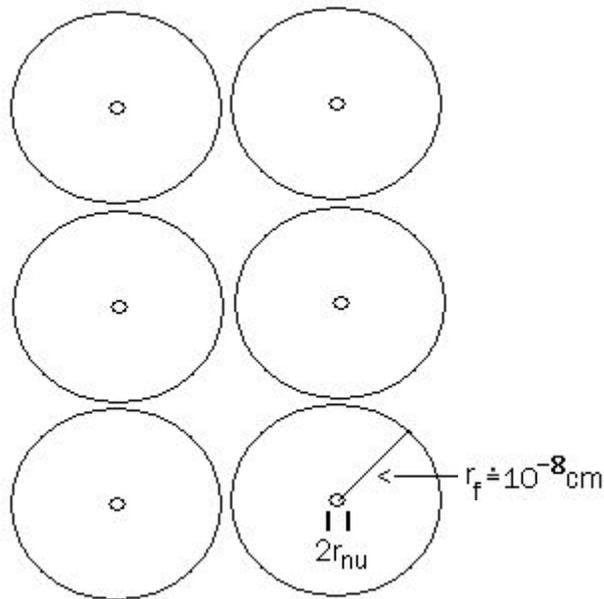


The thin film is submerged in liquid He causing the mass $m_{At} - m_{nu}$ to shrink so that

the average least distance $R_{ld}$ between two adjacent atoms becomes $R_{ld} < r_f$.  The two

$m_{nu}$ are drawn together on a collision course by the strong nuclear force.  Figure 10.2.

FIGURE 10.2
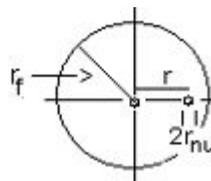


With $m_{nu} \doteq m_{At}$, 10.4b becomes:

10.4c $\quad E_{el}= \frac{m_{At}^2 H}{r_f}(\frac{1}{p+2})\left[1-(\frac{r}{r_f})^{p+2}\right]=2(1+a_1)\frac{m_{At}^2 H}{r_f}+E_{out}$

and solving for $\frac{r}{r_f}$ yields:

10.5 $\quad \frac{r}{r_f}= [-(2p+3) -(p+2)\{(\frac{r_f}{m_{At}^2 H})E_{out}-2a_1\}]^{\frac{1}{(p+2)}} \qquad p \neq -2$

$\quad \frac{r}{r_f}=exp-[2(1+a_1)+(\frac{r_f}{m_{At}^2 H})E_{out}] \qquad\qquad p=-2$

With $E_{out}=280Bev=0.46erg$ evaluate 10.5 for Fe using $r_f \doteq r_o=1.13 \cdot 10^{-8}cm$

10.6 $\quad \frac{r}{r_f}= [-(2p+3) - \{6.0 \cdot 10^5-2a_1\}(p+2)]^{\frac{1}{(p+2)}} \qquad p \neq -2$

$\quad \frac{r}{r_f}=exp-[2(1+a_1)+6.0 \cdot 10^5] \qquad\qquad p=-2$

In order for our model for the hydrogen bomb to represent physical reality, the
surface density of the atom $\rho(r_f)=\frac{(p+3)m_{Fe}}{4\pi r_{f,Fe}^3} =(1+\frac{p}{3})\bar{\rho}$ must be much smaller that the
average density of the atom. i.e. $0<(1+\frac{p}{3})\bar{\rho}<<\bar{\rho}$ and therefore $-3<p \lesssim -2.90$

With $-1 \leq a_1 \leq 0$ and $-3<p \lesssim -2.90$, 10.6 becomes:

10.7 $\quad \frac{r}{r_f} \doteq [- (6.0 \cdot 10^5)(p+2)]^{\frac{1}{(p+2)}}$

From part G above, $\frac{M(r)}{m_{Fe}}=(\frac{r}{r_f})^{(p+3)}$. Computed values of r, $\frac{M(r)}{m_{Fe}}$, and $\rho(r_f)$ as a
function of p are listed in Table 1 and 2 for the special case $a_1=\frac{1}{3}$.

Notice that given $E_{out}$, $a_1$, p and the starting atomic species, the theory as derived
above does not enable us to know which row of table 10.1 or 10.2 represents
physical reality.
The input energy to create the explosion per atom pair is: $E_{in}=2|BE(r_f)|+E_{out}$ and the

theoretic yield ratio $\Phi_y$ is, $0<\Phi_y=\frac{E_{out}}{E_{in}} <1$, where $\Phi_y= \frac{E_{out}}{2|BE(r_f)|+E_{out}} \doteq 1- \frac{2|BE(r_f)|}{E_{out}}$.

Evaluating $\Phi_y$ for Fe with $E_{out}=0.46erg$ yields: $\Phi_y=1- 2.2 \cdot 10^{-6}$

0.5Kgm of naturally occurring Fe contains $5.3 \cdot 10^{24}$ atoms and has dimensions
3.7cm on a side with volume $51cm^3$. To be manageable as a thin film, the bomb

material for example might consist of $10^3$ strips each of dimension $(10)\text{X}(10)\text{X}(5.1\cdot10^{-4})$cm and $\dfrac{5.1\cdot10^{-4}}{2r_0}$ =$2.3\cdot10^4$ atoms thick.

TABLE 10.1

| p | r (cm) | $\dfrac{M(r)}{m_{Fe}}$ | $\rho(r_f)$ |
|---|---|---|---|
| -2.90 | $4.7\cdot10^{-15}$ | 0.23 | $3.3\cdot10^{-2}\bar{\rho}$ |
| -2.91 | $5.5\cdot10^{-15}$ | 0.27 | $3.0\cdot10^{-2}\bar{\rho}$ |
| -2.92 | $6.3\cdot10^{-15}$ | 0.32 | $2.7\cdot10^{-2}\bar{\rho}$ |
| -2.93 | $7.3\cdot10^{-15}$ | 0.37 | $2.3\cdot10^{-2}\bar{\rho}$ |
| -2.94 | $8.4\cdot10^{-15}$ | 0.43 | $2.0\cdot10^{-2}\bar{\rho}$ |
| -2.95 | $9.6\cdot10^{-15}$ | 0.50 | $1.7\cdot10^{-2}\bar{\rho}$ |
| -2.96 | $1.1\cdot10^{-14}$ | 0.58 | $1.3\cdot10^{-2}\bar{\rho}$ |
| -2.97 | $1.3\cdot10^{-14}$ | 0.66 | $1.0\cdot10^{-2}\bar{\rho}$ |
| -2.98 | $1.4\cdot10^{-14}$ | 0.76 | $6.7\cdot10^{-3}\bar{\rho}$ |
| -2.99 | $1.6\cdot10^{-14}$ | 0.87 | $3.3\cdot10^{-3}\bar{\rho}$ |
| -3 | $1.8\cdot10^{-14}$ | 1.0 | 0 |

TABLE 10.2

| p | r (cm) | $\dfrac{M(r)}{m_{Fe}}$ | $\rho(r_{f,Fe})$ |
|---|---|---|---|
| -2.991 | $1.64\cdot10^{-14}$ | 0.89 | $3.0\cdot10^{-3}\bar{\rho}$ |
| -2.992 | $1.66\cdot10^{-14}$ | 0.90 | $2.7\cdot10^{-3}\bar{\rho}$ |
| -2.993 | $1.68\cdot10^{-14}$ | 0.91 | $2.3\cdot10^{-3}\bar{\rho}$ |
| -2.994 | $1.70\cdot10^{-14}$ | 0.92 | $2.0\cdot10^{-3}\bar{\rho}$ |
| -2.995 | $1.72\cdot10^{-14}$ | 0.94 | $1.7\cdot10^{-3}\bar{\rho}$ |
| -2.996 | $1.74\cdot10^{-14}$ | 0.95 | $1.3\cdot10^{-3}\bar{\rho}$ |
| -2.997 | $1.77\cdot10^{-14}$ | 0.96 | $1.0\cdot10^{-3}\bar{\rho}$ |
| -2.998 | $1.79\cdot10^{-14}$ | 0.97 | $6.7\cdot10^{-4}\bar{\rho}$ |
| -2.999 | $1.81\cdot10^{-14}$ | 0.99 | $3.3\cdot10^{-4}\bar{\rho}$ |
| -3 | $1.83\cdot10^{-14}$ | 1.0 | 0 |

To repeat:
The binding energy of the atom is:

$BE_f = -(1-a_1)\frac{m^2H}{r_f}$ , where $A_{mp} = \frac{a_1}{(1-a_1)} \cdot r_f$ with $0 < a_1 < 1$. If $a_1 = \frac{1}{3}$, the atoms oscillate

with amplitude $\frac{1}{2}r_f$ : The average least distance $R_{ld}$ between the center of mass of two adjacent atoms is $R_{ld} = r_f$, the average distance $R_{av}$ is $R_{av} = 2r_f$, and the average greatest distance $R_{gd}$ is $R_{gd} = 3r_f$.

To detonate the bomb, the thin film is submerged in liquid He causing the mass $m_{At} - m_{nu}$ to shrink so that the average least distance $r_f$ between two adjacent atoms becomes less than $r_f$ and one atom captures the $m_{nu}$ of the other. Figure 10.3. The captured $m_{nu}$ plunges through the low density (See table 10.1 and 10.2) outer shell of the remaining atom increasing in energy and momentum. The $2m_{nu}$ strike one another transforming $m_{nu}$ translational kinetic energy into $m_{nu}$ internal energy and both $m_{nu}$ explode with energy $E_{out} = 0.46$ erg.

FIGURE 10.3
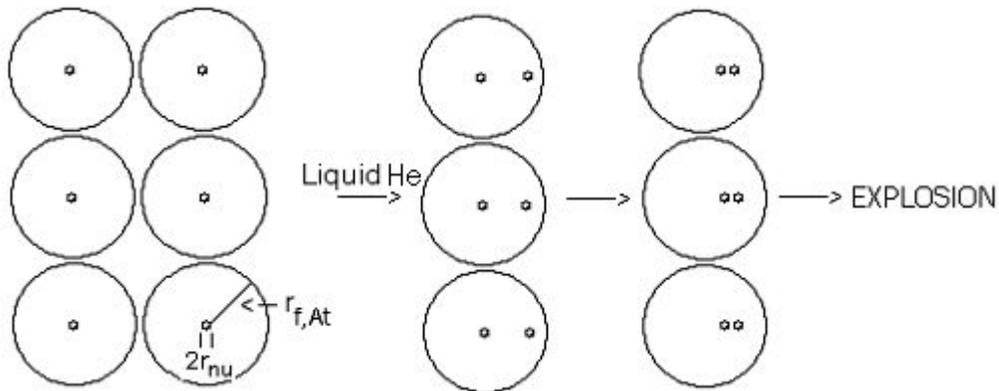


An alternative bomb detonation scenario results if the atom pair do not have enough energy to cause the atoms to explode. The atoms of the thin films coalesce into an incandescent super dense material.


6.    Appendix 10A

Given T+V, $\Psi(r)$ and $\rho(r)$ where $T+V = 2\pi \int_0^{r_0} r^2 \rho(r) U_{rms}^2(r) dr + 4\pi \int_0^{r_0} r^2 \Psi(r)\rho(r) dr = mC_1 = \frac{1}{2}mU_{rms}^2(r_0) - \frac{m^2H}{r_0}$,

$\Psi(r) = -\frac{mH(p+3)}{r_0}\frac{1}{(p+2)}[1 - \frac{1}{(p+3)}(\frac{r}{r_0})^{p+2}]$ and $\rho(r) = \frac{(p+3)m}{4\pi r_0^3}(\frac{r}{r_0})^p$. Show that:

$$V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2.5)},\quad T=\frac{m^2H}{r_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(r_0),\quad -2.5^+\le p\le 0,\quad T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$$

$$V=-\infty,\qquad\qquad T=+\infty,\qquad\qquad\qquad -3<p\le -2.5^-,\quad T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$$

Formally, V becomes:

$$V=-\frac{m^2H}{r_0}\cdot\frac{(p+3)^2}{(p+2)}\int_0^{r_0}[\,1-\frac{1}{(p+3)}(\frac{r}{r_0})^{(p+2)}\,]\frac{r^{(p+2)}}{r_0^{(p+3)}}\cdot dr=-\frac{m^2H}{r_0}\cdot\frac{(p+3)^2}{(p+2)}[\,\frac{1}{(p+3)}(\frac{r}{r_0})^{(p+3)}-\frac{1}{(p+3)(2p+5)}(\frac{r}{r_0})^{(2p+5)}\,]\Big|_0^{r_0}$$

For $-2.5^+\le p\le 0,$  $V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2.5)},$ and $T=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}-V=\frac{m^2H}{r_0}\frac{(0.5)}{(p+2.5)}+\frac{1}{2}mU_{rms}^2(r_0),$

For $-3<p\le -2.5^-,$  $V=-\frac{m^2H}{r_0}\frac{(p+3)}{(p+2)}[1-\frac{1}{2(p+2.5)}+\frac{1}{2(p+2.5)}\lim_{r\to 0}(\frac{r_0}{r})^{2|p+2.5|}]=-\infty$ and

$$T=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}-V==+\infty$$

For $-3<p\le 0,$   $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$

QED

It should be noted that one does not measure T or V independent of one another, one always measures the binding energy T+V where $T+V=\frac{1}{2}mU_{rms}^2(r_0)-\frac{m^2H}{r_0}<0$ for $-3<p\le 0.$ See Chapter 6, Section 5 for an application of T+V.