1. Mathematical Theory of Turbulence and the $(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$ Term

The mathematical theory of fluid turbulence is based on the $(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$ term as derived in the Navier-Stokes Equations. It is proved below that this term does not represent physical reality and therefore a new theory is necessary to explain fluid turbulence.

In the derivation of the Navier-Stokes Equations, the assumption is made that the coordinates, velocity and acceleration of a moving point with respect to an inertial frame may be written as $\underset{\sim}{x}=\underset{\sim}{x}(\underset{\sim}{x}_0,t)$, $\underset{\sim}{x}(\underset{\sim}{x}_0,0)=\underset{\sim}{x}_0$, $\underset{\sim}{U}=\dfrac{d\underset{\sim}{x}}{dt}$ and $\underset{\sim}{a}=\dfrac{d^2\underset{\sim}{x}}{dt^2}$. i.e. The derivation explicitly assumes that x,y,z are functions of t. With $\underset{\sim}{U}=\underset{\sim}{U}[x(x_0,t),y(y_0,t),z(z_0,t),t]=$ $U_x\hat{x}+U_y\hat{y}+U_z\hat{z}$. Take the derivative of $\underset{\sim}{U}$ w.r.t. t:

12.1 $\quad \dfrac{d\underset{\sim}{U}}{dt}=\dfrac{\partial\underset{\sim}{U}}{\partial t}+\dfrac{\partial\underset{\sim}{U}}{\partial x}\dfrac{dx}{dt}+\dfrac{\partial\underset{\sim}{U}}{\partial y}\dfrac{dy}{dt}+\dfrac{\partial\underset{\sim}{U}}{\partial z}\dfrac{dz}{dt}=\dfrac{\partial\underset{\sim}{U}}{\partial t}+(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$

Although this expression is mathematically correct, it is not physically correct. Newton's Laws as exemplified by $\rho(\underset{\sim}{x},t)\cdot\dfrac{d\underset{\sim}{U}}{dt}=-\nabla P$, or $\underset{\sim}{F}=\dfrac{d}{dt}(m\underset{\sim}{U})$ are valid only if position, velocity and acceleration are measured from a frame S which is inertial as determined by on board accelerometers at rest in S. To be physically correct the $\underset{\sim}{x}$ occurring in $\underset{\sim}{U}=\dfrac{d\underset{\sim}{x}}{dt}$ must be stationary in inertial frame S and therefore independent of time t as measured by observers at rest w.r.t. clocks at rest in S. The operational definition of $\underset{\sim}{U}$ is thus inconsistent with the mathematical definition of $\underset{\sim}{U}$ i.e. $\underset{\sim}{U}=\dfrac{d\underset{\sim}{x}}{dt}(\underset{\sim}{x}_0,t)$. The physically correct expression for $\underset{\sim}{U}$ is $\underset{\sim}{U}=\dfrac{d\underset{\sim}{h}}{dt}$ where $\underset{\sim}{h}$ is defined below.

To correct $\underset{\sim}{U}=\dfrac{d\underset{\sim}{x}}{dt}(\underset{\sim}{x}_0,t)$, represent a moving point by any other symbol than x,y,z in $\underset{\sim}{x}=\underset{\sim}{x}(\underset{\sim}{x}_0,t)$, $\underset{\sim}{x}(\underset{\sim}{x}_0,0)=\underset{\sim}{x}_0$. Let the function $\underset{\sim}{h}=(h_x,h_y,h_z)$ represent the position of a moving point in rectangular coordinates where h_x,h_y,h_z are linear lengths in the $\hat{x},\hat{y},\hat{z}$ directions respectively.

12.2 $\quad h_x=x+\chi_x(x,y,z,t)$ with $\chi_x(x,y,z,0)=0$.
12.3 $\quad h_y=y+\chi_y(x,y,z,t)$ with $\chi_y(x,y,z,0)=0$.
12.4 $\quad h_z=z+\chi_z(x,y,z,t)$ with $\chi_z(x,y,z,0)=0$.

x,y,z, are stationary in S and independent of t: $\dfrac{dx}{dt}=\dfrac{dy}{dt}=\dfrac{dz}{dt}=0$.

The velocity of the moving point is $\underset{\sim}{U}=\dfrac{d\underset{\sim}{h}}{dt}=\dfrac{\partial h}{\partial t}_x\hat{x}+\dfrac{\partial h}{\partial t}_y\hat{y}+\dfrac{\partial h}{\partial t}_z\hat{z}$ and the

acceleration of the moving point is $\underset{\sim}{a}=\dfrac{\partial \underset{\sim}{U}}{\partial t}=\dfrac{\partial^2\underset{\sim}{h}}{\partial t^2}=\dfrac{\partial^2 h}{\partial t^2}_x\hat{x}+\dfrac{\partial^2 h}{\partial t^2}_y\hat{y}+\dfrac{\partial^2 h}{\partial t^2}_z\hat{z}$ with:

12.5 $\dfrac{\partial h}{\partial t}_x=\dfrac{\partial \chi}{\partial t}_x(x,y,z,t)$ and $\dfrac{\partial^2 h}{\partial t^2}_x=\dfrac{\partial^2 \chi}{\partial t^2}_x(x,y,z,t)$

12.6 $\dfrac{\partial h}{\partial t}_y=\dfrac{\partial \chi}{\partial t}_y(x,y,z,t)$ and $\dfrac{\partial^2 h}{\partial t^2}_y=\dfrac{\partial^2 \chi}{\partial t^2}_y(x,y,z,t)$

12.7 $\dfrac{\partial h}{\partial t}_z=\dfrac{\partial \chi}{\partial t}_z(x,y,z,t)$ and $\dfrac{\partial^2 h}{\partial t^2}_z=\dfrac{\partial^2 \chi}{\partial t^2}_z(x,y,z,t)$

and $\underset{\sim}{a}\neq\dfrac{\partial \underset{\sim}{U}}{\partial t}+(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$. The modified form of the Navier Stokes Equation in Eulerian Coordinates becomes:

12.8 $\rho(\underset{\sim}{h})\cdot\dfrac{d\underset{\sim}{U}(\underset{\sim}{h})}{dt}=-\nabla P=\sum_i \underset{\sim}{F}_{ct}(\underset{\sim}{h})_i+\sum_j \underset{\sim}{F}_{fi}(\underset{\sim}{h})_j$ where $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$ and $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$.

$\sum_i \underset{\sim}{F}_{ct}(\underset{\sim}{h})_i$ represents the sum of the externally applied contact forces, (viscosity in $\dfrac{dyn}{cm^3}$), acting at point $\underset{\sim}{h}(\underset{\sim}{x},t)$ at time t and $\sum_j \underset{\sim}{F}_{fi}(\underset{\sim}{h})_j$ represents the sum of the field

forces, $(\dfrac{dyn}{cm^3})$, acting at point $\underset{\sim}{h}(\underset{\sim}{x},t)$ at time t where $\underset{\sim}{x}\neq\underset{\sim}{f}(t)$ and $\dfrac{d\underset{\sim}{U}}{dt}=\dfrac{\partial \underset{\sim}{U}}{\partial t}\neq\dfrac{\partial \underset{\sim}{U}}{\partial t}+(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$.
To sum up. In Eulerian Coordinates which yield velocity and acceleration following the flow,

12.9 $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$, $\underset{\sim}{U}=\dot{\underset{\sim}{h}}=\dot{\underset{\sim}{\chi}}(\underset{\sim}{x},t)$, $\dot{\underset{\sim}{U}}=\ddot{\underset{\sim}{h}}=\ddot{\underset{\sim}{\chi}}(\underset{\sim}{x},t)$, $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$, $\underset{\sim}{x}\neq\underset{\sim}{f}(t)$.

Newton's 3rd Law becomes:

12.10 $\underset{\sim}{F}=\dfrac{d}{dt}(m\underset{\sim}{U}(\underset{\sim}{h}))$ where $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$ and $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$.

Lagrangian Coordinates yield velocity and acceleration at a fixed point in inertial frame S. Let $\underset{\sim}{x}_0$ be the fixed point is S. We wish to find the velocity and acceleration at $\underset{\sim}{x}_0$, for a given fixed time t_0. Using Eulerian coordinates: $\underset{\sim}{h}=\underset{\sim}{x}_1+\underset{\sim}{\chi}(\underset{\sim}{x}_1,t)$, $\underset{\sim}{\chi}(\underset{\sim}{x}_1,0)=0$, where $\underset{\sim}{\chi}(\underset{\sim}{x}_1,t)$ is as defined above and $\underset{\sim}{x}_1$ satisfies the following. At time t_0 the fluid parcel that was at $\underset{\sim}{x}_1$ at t=0 is at $\underset{\sim}{x}_0$: $\underset{\sim}{x}_0=\underset{\sim}{x}_1+\underset{\sim}{\chi}(\underset{\sim}{x}_1,t_0)$. Define $\underset{\sim}{f}$ by

$\underset{\sim}{f}(\underset{\sim}{x}_1,\underset{\sim}{x}_0,t_0) \equiv \underset{\sim}{x}_1 + \underset{\sim}{\chi}(\underset{\sim}{x}_1,t_0) - \underset{\sim}{x}_0 = 0$. Solve $\underset{\sim}{f}$ for $\underset{\sim}{x}_1$ yielding $\underset{\sim}{x}_1 = \underset{\sim}{g}(\underset{\sim}{x}_0,t_0)$ where $\underset{\sim}{f}(\underset{\sim}{g}(\underset{\sim}{x}_0,t_0),\underset{\sim}{x}_0,t_0) = 0$.

The Lagrangian coordinates for velocity and acceleration are:

12.11 $\underset{\sim}{U} = \dot{\underset{\sim}{h}} = \dot{\underset{\sim}{\chi}}[\underset{\sim}{g}(\underset{\sim}{x}_0,t_0),t_0], \; \dot{\underset{\sim}{U}} = \ddot{\underset{\sim}{h}} = \ddot{\underset{\sim}{\chi}}[\underset{\sim}{g}(\underset{\sim}{x}_0,t_0),t_0]$.

Eqns. 12.11, represent a point-to-point solution for velocity and acceleration. Specifically 12.11 represent the velocity and acceleration at $\underset{\sim}{x}_0$, for time t_0.

In the following sections, a new theory for the physical cause of fluid turbulence in water and air is developed based on a new model for the structure of liquid water and the breakup of water clathrates in water and water droplets in air.

2. Water Molecule Dimension

It is assumed in the derivation of the Navier-Stokes Equations that solids and liquids are made of continuous matter. It is commonly thought among researchers in fluid flow that no major new physics will emerge if one considers liquids and solids to be made up of atoms instead of continuous matter. In what follows it will be shown that this is not the case, and that major new physics emerges if one considers liquids and solids to be made up of atoms instead of continuous matter.

Consider a closed container of degassed water in pressure equilibrium with dry air at t=0(sec) at T=293^{0}K=20^{0}C and air pressure $P_V = 2.34 \cdot 10^4 \frac{dy}{cm^2}$ at rest on the earth's surface. P_V is the saturation water vapor pressure at T=20^{0}C, however in this example the atmosphere is dry air at t=0(sec). Figure 12.1.

If water molecules did not have an attractive force binding one water molecule to another, then all the liquid water would become a gas. This is not the case and therefore there is an attractive force binding one water molecule to its neighbors. The resultant molecular structure is called a clathrate,
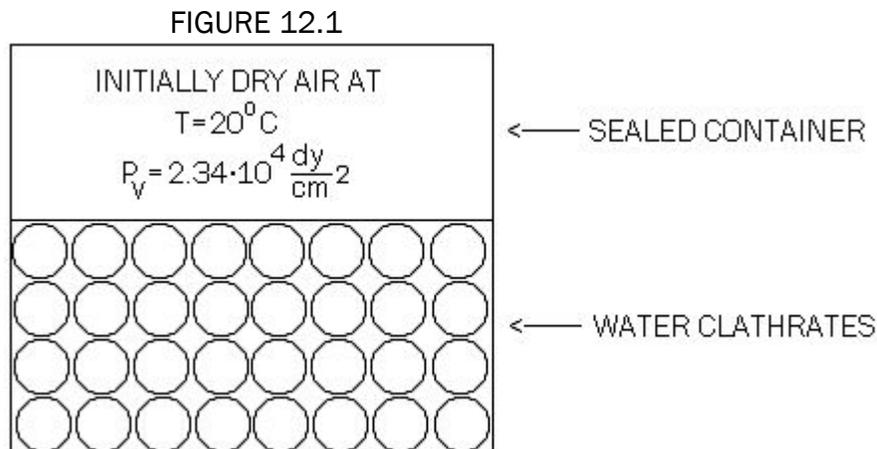
FIGURE 12.1

Considering the liquid water, let r_{CL} be the radius of the, assumed spherical, water clathrate; M_{CL} is its' mass and V_{CL} is its' effective volume where $(2r_{CL}+\overline{R}_{CL})$ is the average distance between 2 clathrates, Figure 12.2.

FIGURE 12.2
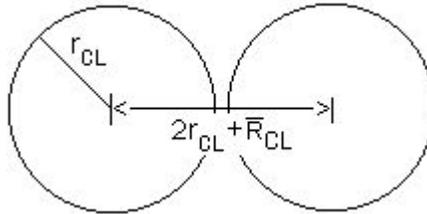
Let N_W represent the number of water molecules in the clathrate. The mass m_W of 1 water molecule is $3.01 \cdot 10^{-23}$gm and $\dfrac{M_{CL}}{V_{CL}} = \dfrac{3.01 \cdot 10^{-23} N_{W}}{(2r_{CL})^3 (1+\frac{\overline{R}_{CL}}{2r_{CL}})^3} = 1\frac{gm}{cm^3}$.

Let L_i be the initial length of a water column where $L_i = N(2r_{CL}+\overline{R}_{CL})$ and N is the number of water clathrates along the length. Let L_f be the final length under compression where $L_f = N(2r_{CL})$ Experimentally it is shown that for compressive pressures up to the bulk modulus, $\dfrac{L_f}{L_i} \doteq 1$ where $B_{\Theta_{Ex}} = 2.1 \cdot 10^{10}\frac{dy}{cm}2$. Consequently

$\dfrac{L_f}{L_i} = \dfrac{1}{(1+\frac{\overline{R}_{CL}}{2r_{CL}})} \doteq 1$ and $\dfrac{\overline{R}_{CL}}{2r_{CL}} <<1$ and therefore $\dfrac{M_{CL}}{V_{CL}} = 1 \doteq \dfrac{3.01 \cdot 10^{-23}(1-\frac{3\overline{R}_{CL}}{2r_{CL}})N_{W}}{(2r_{CL})^3} \doteq \dfrac{3.01 \cdot 10^{-23} N_{W}}{(2r_{CL})^3}\frac{gm}{cm^3}$.

Let r_W be the effective radius of the water molecule where $(2r_W+\overline{R}_W)$ is the average distance between 2 adjacent water molecules within the clathrate, Figure 12.3.

FIGURE 12.3
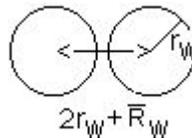
$N_W = \dfrac{\frac{4}{3}\pi r_{CL}^3}{(2r_W)^3(1+\frac{\overline{R}_W}{2r_W})^3}$ and for the same reasons as for the clathrate, $\dfrac{\overline{R}_W}{2r_W} <<1$ and

consequently: $N_W \doteq \dfrac{\frac{4}{3}\pi r_{CL}^3(1-\frac{3\overline{R}_W}{2r_W})}{(2r_W)^3} \doteq \dfrac{\pi r_{CL}^3}{6 r_W^3}$. Also $m_W N_W = M_{CL} \doteq 8(r_{CL})^3$ and

consequently:

12.12 $r_W = (\frac{\pi m_W}{48})^{\frac{1}{3}} = 1.25 \cdot 10^{-8}$cm

3. Surface Tension

Consider the meniscus of water in a glass tube of radius r. The meniscus is due to the attraction of the water molecules on the water surface in contact with the glass graduate, to the glass graduate. Figure 12.4 . The pressure at the bottom of the water column of height h is $P=\rho_w gh$ with force $-F_p\hat{z}=-\pi r^2 P\hat{z}=-\pi r^2\rho_w gh\hat{z}$. The origin of the surface tension force is the attraction between the monomolecular layer of water molecules on the surface of the water in contact with the glass tube, to the glass tube. The total force due to surface tension in the $+\hat{z}$ direction is: $F_T\hat{z}=2\pi r\sigma_T\cos\theta$ where σ_T is the coefficient of surface tension. The forces are in equal and opposite directions so that $F_T\hat{z}=F_p$ and solving for h: $h=2\sigma_T\cos\theta(\rho_w gr)^{-1}$. Experimentally $\sigma_T=72.8\frac{dy}{cm}$ and h becomes; $h=0.149\frac{\cos\theta}{r}$. The maximum value of the force f_M that one surface water molecule exerts on one of its neighbors in the radial direction due to surface tension is: $f_M=2r_W\sigma_T$ and using 12.12, $f_M=1.8\cdot10^{-6}dy$.

FIGURE 12.4
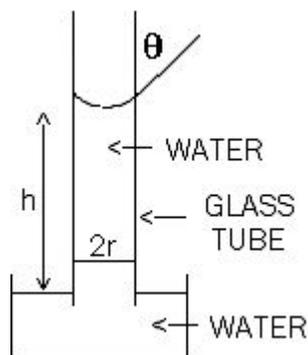
The work W_M to pull apart 2 water molecules held together by surface tension forces at 20^oC is $W_M\approx f_M\cdot\Delta s\approx f_M\cdot10^{-10}=1.8\cdot10^{-16}erg=1.1\cdot10^{-4}ev$ with a ΔT equivalence of, $\Delta T=0.85^o$K. i.e. raising the temperature by 0.85^oK. destroys the surface tension. This is not observed and consequently the actual force f_A holding two water molecules together must be $f_A>f_M$. Note that assuming that $\Delta s\geq10^{-9}$cm yields $|\frac{\Delta\rho}{\rho}|\geq15\%$ as the water climbs up the capillary tube. This is not observed.

4. Dimensions of Water Clathrate

In order to account for the heat of vaporization $0.456\frac{ev}{w}$ at $T=100^o$C and the equilibrium water vapor pressure $P_V(T)$; our model for a clathrate consists of water

molecules tightly bound together within the clathrate but with relatively weak clathrate to clathrate binding. The clathrate to clathrate bonding energy $BE_{CL,CL}$ at $T=20^oC$ must be less than $|BE_{CL,CL}|<\{\frac{3}{2}K(373-293)\}=1.7\cdot10^{-14}erg=.01ev$ in order to account for the explosive boiling of water in a vacuum. Let $f_{CL,CL}$ represent the average force bonding 2 clathrates together at $T=20^oC$ where $f_{CL,CL}\Delta s=1.7\cdot10^{-14}erg$. With $\Delta s=10^{-10}cm$, $f_{CL,CL}$ becomes: $f_{CL,CL}=1.7\cdot10^{-10}dyn$.

The physics of boiling water at $T=100^oC$ is derived in sec. 7.

A 2 dimensional representation of the spherical surface of 2 spherical water clathrates made up of water molecules bound to one another is given in figure 12.5.

FIGURE 12.5
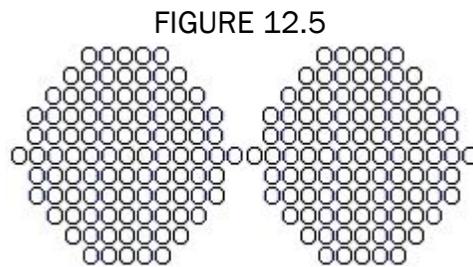
Consider Fig. 12.1 with water in pressure equilibrium with dry air at t=0(sec) at $T=293^oK=20^oC$ and air pressure $P_V=2.34\cdot10^4\frac{dy}{cm}2$.

Using 4.6, the average pressure $P_{CL,CL}$ that 2 clathrates exert on one another is:

$P_{CL,CL}=2(\frac{r_{CL}}{\overline{R}_{CL}})n_{CL}KT$ where $n_{CL}=(2r_{CL})^{-3}$ and $P_{CL,CL}$ becomes: $P_{CL,CL}=\frac{KT}{(2r_{CL})^2\overline{R}_{CL}}$. The

average pressure $P_{A,A}$ that 2 air molecules exert on one another is: $P_{A,A}=n_AKT$ and at

equilibrium $P_{CL,CL}=P_{A,A}$ and $n_A=\frac{1}{(2r_{CL})^2\overline{R}_{CL}}=5.79\cdot10^{17}$. The mass of a clathrate M_{CL}

is: $M_{CL}=\frac{4}{3}\pi(\frac{r_{CL}}{2r_w})^3m_w$ and using $r_{CL}=6.57\cdot10^{-10}(\frac{1}{\overline{R}_{CL}})^{\frac{1}{2}}$, M_{CL} becomes: $M_{CL}=2.4\cdot10^{-27}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}gm$

The volume of a clathrate V_{CL} is: $V_{CL}=(2r_{CL})^3=2.4\cdot10^{-27}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}$ and $M_{CL}=V_{CL}$ (QED).

For future use:

12.13 $r_W=1.25\cdot10^{-8}cm$ and at $T=20^oC$,

$r_{CL}=6.57\cdot10^{-10}(\frac{1}{\overline{R}_{CL}})^{\frac{1}{2}}cm$, $M_{CL}=2.4\cdot10^{-27}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}gm$, $V_{CL}=2.4\cdot10^{-27}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}cm^3$

$N_w=7.9\cdot10^{-5}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}=$number of water molecules per clathrate

Note that by using 12.13 and $n_A = \dfrac{1}{(2r_{CL})^2 \bar{R}_{CL}} = 5.79 \cdot 10^{17}$ one can derive $P_{CL,CL} = P_{A,A} = P_{A,CL}$

where $P_{A,CL}$ is the average pressure that air molecules exert on the clathrate surface.

Evaluating 12.13 for $\bar{R}_{CL} = 2 \cdot 10^{-9}$ cm (See sec. 6,) yields:

12.14 $r_{CL} = 1.4 \cdot 10^{-5}$ cm, $M_{CL} = 2.6 \cdot 10^{-14}$ gm, $V_{CL} = 2.6 \cdot 10^{-14}$ cm$^3 \doteq (2r_{CL})^3$

$N_W = 8.7 \cdot 10^8 =$ number of water molecules per clathrate

As regards figure 12.1 for t>0, N_2 and O_2 molecules are adsorbed by the degassed water and are replaced by water molecules maintaining $P_V = 2.34 \cdot 10^4 \dfrac{dy}{cm}2$. The chemical composition of the dry air therefore changes over time as it becomes "Wet" air. This can be experimentally tested.

5.   Evaporation of Water at S.T.P.

Within the context of our model for a water clathrate fig. 12.1 and 12.5, how is it possible for water to evaporate at S.T.P. The following molecular model for evaporation is designed using the experimentally determined values of specific heat and heat of vaporization.

The specific heat of water is:

$C_P = \dfrac{1cal}{gmK}0$ and $\dfrac{1cal}{gmK}0 = (4.185 \cdot 10^7)(3 \cdot 10^{-23}) \dfrac{erg}{WK}0 = 1.26 \cdot 10^{-15} = 9.13K(\dfrac{erg}{WK}0) = 7.86 \cdot 10^{-4}(\dfrac{ev}{WK}0)$

where as above, W stands for water molecule. A water molecule is made up of 3 atoms so that the specific heat per atom is: $9.13K(\dfrac{erg}{WK}0) = 3.04K(\dfrac{erg}{atomK}0)$. For the transition metals, one atom is bonded to 6 atoms and the specific heats range from 2.9K to 3.4K. It is therefore hypothesized that in a clathrate, every atom in the interior of the clathrate is strongly bonded to 6 neighboring atoms and on the surface of the clathrate, each atom is strongly bonded to its 5 neighboring atoms.

The mass of 1 water molecule is 18amu and the mass of an average air molecule is $m_A = 28.7$ amu. Therefore an unbound water molecule at temperature equilibrium with the atmosphere, will rise in the atmosphere.

The density of air at sea level at T=20°C is $\rho_A = 1.18 \cdot 10^{-3} \dfrac{gm}{cm}3$ with number density $n_A = 2.47 \cdot 10^{19} \dfrac{1}{cm}3$. In general whether or not a water clathrate of mass M_{CL} in thermal equilibrium with a still atmosphere rises from ground level depends on whether M_{CL} is greater than or less than $\rho_A V_{CL}$. Using 12.14, $M_{CL} = 2.6 \cdot 10^{-14}$ gm and $\rho_A V_{CL} = 3.1 \cdot 10^{-17}$ gm and therefore M_{CL} whether bound or unbound to its neighboring clathrates, stays at ground level.   Water vapor is however observed at altitudes over 20 miles high and does not achieve that altitude as a water clathrate. We investigate

the conditions under which single water molecules escape from a water clathrate and then recombine with other single water molecules to form a hollow sphere whose surface is composed of water molecules bound together and whose interior is filled with single water molecules at water vapor pressure concomitant with the pressure and temperature at altitude. The density of the sphere ρ_S is less than the density of air ρ_A i.e. $\rho_S < \rho_A$ and consequently the spheres float upwards.

The heat of vaporization of water at $T=20^0C$ is:

$2.45 \cdot 10^{10} \frac{erg}{gm} = 0.730 \cdot 10^{-12} \frac{erg}{W} = 0.456 \frac{ev}{W} = 5.29 \cdot 10^3 K \frac{erg}{W}$. It therefore takes

$0.730 \cdot 10^{-12} \frac{erg}{W} = 0.456 \frac{ev}{W}$ to remove one water molecule from a clathrate starting with a bound water molecule at $T=20^0C$ and ending with an unbound water molecule at $T=20^0C$. The temperature equivalence of 0.456ev is $T=3.53 \cdot 10^3 (^0K)$.

Assuming the surface temperature of a body of water is greater then that of the air, the source of the energy to vaporize the surface waters during the night is the body of water below the surface driven by a boundary layer temperature distribution with $\frac{\Delta T}{\Delta z} > 0$, z positive downwards.

Let n_W be the number of individual water molecules that escape from the water surface per second per cm^2. The power it takes to liberate n_W is $0.730 \cdot 10^{-12} n_W (\frac{erg}{sec \, cm^2})$ and $0.730 \cdot 10^{-12} n_W = K_W \frac{dT}{dz}$ where $K_W = 5.92 \cdot 10^4 \frac{erg}{sec \, cm \, K^0}$ is the thermal conduction of water. Solving for n_W: $n_W \doteq 8.1 \cdot 10^{16} \frac{\Delta T}{\Delta z} (\frac{W}{sec \, cm^2})$. There are $3.5 \cdot 10^{22} (\frac{W}{cm^3})$ water molecules per cm^3 and $3.5 \cdot 10^{22} z (\frac{W}{z \, cm^2})$ water molecules in volume zx1cmx1cm. Also, $n_W = 3.5 \cdot 10^{22} \frac{dz}{dt} \doteq 3.5 \cdot 10^{22} \frac{\Delta z}{\Delta t}$ and $\frac{\Delta z}{\Delta t} = 2.3 \cdot 10^{-6} \frac{\Delta T}{\Delta z} (\frac{cm}{sec}) = 0.20 \frac{\Delta T}{\Delta z} (\frac{cm}{day})$. A table of values for $\frac{\Delta T}{\Delta z} (\frac{K^0}{m})$, $\frac{\Delta z}{\Delta t} (\frac{cm}{day})$, $n_W (\frac{W}{sec \, cm^2})$ is given in table 12.1. Note the units of $\frac{\Delta T}{\Delta z}$.

TABLE 12.1

$\frac{\Delta T}{\Delta z} (\frac{K^0}{m})$	$\frac{\Delta z}{\Delta t} (\frac{cm}{day})$	$n_W (\frac{W}{sec \, cm^2})$
10	$2.0 \cdot 10^{-2}$	$8.1 \cdot 10^{15}$
1	$2.0 \cdot 10^{-3}$	$8.1 \cdot 10^{14}$
0.1	$2.0 \cdot 10^{-4}$	$8.1 \cdot 10^{13}$
0.01	$2.0 \cdot 10^{-5}$	$8.1 \cdot 10^{12}$

Night Time Production of Water Vapor

The Maxwell-Boltzmann energy distribution is $N_E = 1.13N \int_{\frac{E}{KT}}^{\infty} S^{\frac{1}{2}} e^{-S} dS$ where N is the

number of collisions of a given water molecule with adjacent water molecules and N_E is the number of collisions with energies greater than energy E. With $E = 0.730 \cdot 10^{-12} \frac{erg}{W}$, by direct computation, $\frac{N_E}{N}$ becomes: $\frac{N_E}{N} = 6.80 \cdot 10^{-8}$.

Setting $N_E = 1$ yields $N = 1.47 \cdot 10^7$ which means that for every $1.47 \cdot 10^7$ collisions suffered by a given water molecule, one of those collisions will have an energy greater than or equal to N_E. But with $N_E = 1$ and if the energy of a given molecule can be treated as varying continuously with time, then on average, 1 collision out of $1.47 \cdot 10^7$ will have energy $E = 0.730 \cdot 10^{-12} \frac{erg}{W}$ and no collisions out of $1.47 \cdot 10^7$ will have energies greater then E.

Let $\overline{v}_{x>0}$ represent the average speed in a given direction for a water molecule in the clathrate at $T = 293^\circ K$ and let τ represent the time taken for a water molecule to traverse $\overline{R}_W$ with speed $\overline{v}_{x>0}$ where $\overline{v}_{x>0} \tau = \overline{R}_W$. By direct computation at $T = 293^\circ K$, $\overline{v}_{x>0} = 1.46 \cdot 10^4 \frac{cm}{sec}$, (Ref. 12.1), and using $\overline{R}_W = 2.0 \cdot 10^{-9}$ cm, (See Sec. 6) τ is:

$\tau = \frac{1}{N_\tau} = 1.37 \cdot 10^{-13}$ sec. and $5N_\tau = 3.65 \cdot 10^{13} \frac{hits}{sec}$. Consequently the maximum number

of water molecules n_W that escape the surface of the water per second per $(2r_W)^2$ is:

$n_W = 5 \frac{N_\tau}{N} = \frac{3.65 \cdot 10^{13}}{1.47 \cdot 10^7} = 2.48 \cdot 10^6 \frac{escapes}{sec(2r_W)^2} = 2.48 \cdot 10^6 (1.6 \cdot 10^{15}) = 4.0 \cdot 10^{21} \frac{escapes}{sec(cm)^2}$.

n_W is however severely limited by the thermal conduction of water $K_W = 5.92 \cdot 10^4 \frac{erg}{sec\,cm\,K^0}$.

Using table 12.1, $n_W = 51 \frac{\Delta T}{\Delta z} \frac{escapes}{sec(2r_W)^2}$ and consequently, if water leaves the water

surface as individual water molecules, then $2.48 \cdot 10^6 - 51 \frac{\Delta T}{\Delta z}$ water molecules

recombine with their clathrate and $51 \frac{\Delta T}{\Delta z} \frac{escapes}{sec(2r_W)^2}$ escape the water surface per

second per $(2r_W)^2$. Those water molecules in the interior of the fluid with energies

greater than or equal to $E = 0.456 \frac{ev}{W}$ escape from their clathrate but then are readsorbed by a neighboring clathrate and do not escape to the surface.

On a cloudless day the average value for the energy from the sun at sea level e_S is:

$e_S \doteq 1 \cdot 10^6 (\frac{erg}{sec\,cm^2})$. Let $p \cdot e_S$, $0 < p < 1$, represent the amount of the sun's photon energy per second used to break the chemical bonds holding individual water

molecules to clathrates. $p \cdot e_S$ liberates individual water molecules from clathrates at 20^oC to individual water molecules at 20^oC. Using $p \cdot e_S = 0.73 \cdot 10^{-12} n_W (\frac{erg}{sec\,cm^2})$:

$n_W = 1.4 \cdot 10^{18} p(\frac{w}{sec\,cm^2}) = 3.5 \cdot 10^{22} \frac{\Delta z}{\Delta t}$, with $\frac{\Delta z}{\Delta t} = 4 \cdot 10^{-5} p(\frac{cm}{sec}) = 3.5 p(\frac{cm}{day})$.

A table of values for p, $\frac{\Delta z}{\Delta t}(\frac{cm}{day})$, and $n_W (\frac{w}{sec\,cm^2})$ is given in table 12.2.

TABLE 12.2

P	$\frac{\Delta z}{\Delta t}(\frac{cm}{day})$	$n_W (\frac{w}{sec\,cm^2})$
1	3.5	$1.4 \cdot 10^{18}$
0.1	0.35	$1.4 \cdot 10^{17}$
0.01	0.035	$1.4 \cdot 10^{16}$

Day Time Production of Water Vapor

The tabulated value for $\frac{\Delta z}{\Delta t}$ must be multiplied by $\frac{n}{24}$, where n is the number of daylight hours of sunshine in a 24 hour period in order to derive the measured value of $\frac{\Delta z}{\Delta t}$.

6. Speed of Sound in Water and the Bulk Modulus of Water

The speed of sound in water is given by $C_S = \left[\frac{(C_p + 3K) 2 r_w KT}{C_p} \frac{}{m \bar{R}_w(T)} \right]^{1/2}$. (Reference 12.4.)

Evaluating $\bar{R}_w(293)$ using the values derived above, $C_p = 9K$, $r_w = 1.25 \cdot 10^{-8} cm$, $m = 3.01 \cdot 10^{-23} gm$, And the experimentally determined value, $C_S = 1.5 \cdot 10^5 \frac{cm}{sec}$, yields: $\bar{R}_w(293) = 2.0 \cdot 10^{-9} cm$.

The isothermal bulk modulus B_Θ is defined by $B_\Theta = -V(\frac{dP_e}{dV})_\Theta$ where P_e is the total external pressure applied to a solid, liquid or gas sample where $B_\Theta = \frac{2r_0}{3R_0(T)} (\frac{C_p}{C_p + 3K}) \rho_0 C_S^2$ (Reference 12.5). Applying the formula to 1 water molecule in the clathrate using the values as above with $\rho_0 = \frac{m}{8r_0^3} = 1.93 \frac{gm}{cm^3}$ yields: $B_\Theta = 1.4 \cdot 10^{11} \frac{dy}{cm^2}$ which is 6.7 times the experimental value $B_{\Theta_{Ex}} = 2.1 \cdot 10^{10} \frac{dy}{cm^2}$. However, the experimental value $B_{\Theta_{Ex}}$ has been directly determined by compressing a water sample isothermally in an enclosed pressure cell and measuring the change of length ΔL as a function of applied pressure. The basic measurement $(-\Delta L)_{ex}$, is made by measuring the electrical resistance in a Nirex wire between a point of attachment on the pressure piston and a fixed point on the pressure casing: Once at external pressure $P_{ex} = 0$

dy/cm2 and again at a final external pressure $P_{ex,f}$. Given the difficulty in making consistent electric resistance measurements at high pressure, it is understandable that human error may account for the experimentally determined

value, $B_{\Theta_{Ex}}=2.1\cdot10^{10}\frac{dy}{cm}2$. (Reference 12.2)

The basic measurement $(-\Delta L)_{ex}$, should be made using x-ray negatives, one taken at

external pressure $P_{ex}=0$ dy/cm2 and one taken at final external pressure $P_{ex,f}$.

7. Water Droplet Floating in Air

The model described in sec. 5 postulates that water evaporates as individual water molecules and yet a steaming cup of coffee, water boiling in a tea kettle and a lake steaming on a autumnal day before sunrise, create visible water droplets ~0.1 to ~1mm in diameter. In each case the water molecules are coming into contact with cooler air that causes the water molecules to condense into water droplets.
In order to float and form fogs and clouds, the droplets have a density less than or

equal to the density of earth's atmosphere, $1.19\cdot10^{-3}\frac{gm}{cm}3$ at sea level at 20^{0}C; They

therefore do not consist of water as in a rain droplet with a density of $1\frac{gm}{cm}3$.

The effective volume $\frac{1}{n_0}$ of an atom in the atmosphere at sea level at 293^{0}K is:

$\frac{1}{n_0}=\frac{KT}{P}=4.04\cdot10^{-20}cm3$ with effective radius r_E where $(2r_E)^3=\frac{1}{n_0}$ and $r_E=1.7\cdot10^{-7}$cm.

Hence a droplet with a radius greater than $1.7\cdot10^{-7}$cm must have a density

$\rho_{Dr}\leq1.19\cdot10^{-3}\frac{gm}{cm}3$ in order to float. We are interested in visible droplets with $r_{Dr}>>10^{-7}$cm.

With droplet density $\rho_{Dr}=1.19\cdot10^{-3}\frac{gm}{cm}3$, observed radius $r_{Dr}=1.0\cdot10^{-n}$cm and volume

$V_{Dr}=\frac{4}{3}\pi(r_{Dr})^3=4.2\cdot10^{-3n}cm^3$, the mass of a water droplet M_{Dr} is $M_{Dr}=5.0\cdot10^{-3(n+1)}$gm

and contains $W_{Dr}=1.7\cdot10^{20-3n}$ water molecules. The surface of the droplet contains

$N_S=\frac{A_{Dr}}{A_w}=\frac{4\pi}{4}(\frac{r_{Dr}}{r_w})^2=2.0\cdot10^{2(8-n)}$ water molecules and the droplet contains

$N_L=1.7\cdot10^{20-3n}[2.0\cdot10^{2(8-n)}]^{-1}=0.85\cdot10^{4-n}$ layers of water molecules per droplet.

The hollow droplet consists of a boundary layer of water droplets $d=0.85\cdot10^{4-n}(2.5)\cdot10^{-8}=$ $2.1\cdot10^{-(4+n)}$cm thick.

A table of values for r_{Dr} (Droplet Radius), M_{Dr} (Mass Droplet), W_{Dr} (Number of Water Molecules per Droplet), N_S (Number of Water Molecules on Surface of Droplet), N_L (Number of Layers of Water Molecules in Droplet), and d (Thickness of Boundary Layer of Water Molecules in Droplet) is given in Table 12.3

TABLE 12.3

r_{Dr}(cm)	M_{Dr}(gm)	W_{Dr}	N_S	N_L	d(cm)
10^{-4}	$5.0 \cdot 10^{-15}$	$1.7 \cdot 10^8$	$2.0 \cdot 10^8$	0.85	$2.1 \cdot 10^{-8}$
$1.17 \cdot 10^{-4}$	$7.9 \cdot 10^{-15}$	$2.7 \cdot 10^8$	$2.7 \cdot 10^8$	1.0	$2.5 \cdot 10^{-8}$
10^{-3}	$5.0 \cdot 10^{-12}$	$1.7 \cdot 10^{11}$	$2.0 \cdot 10^{10}$	8.5	$2.1 \cdot 10^{-7}$
10^{-2}	$5.0 \cdot 10^{-9}$	$1.7 \cdot 10^{14}$	$2.0 \cdot 10^{12}$	85	$2.1 \cdot 10^{-6}$
10^{-1}	$5.0 \cdot 10^{-6}$	$1.7 \cdot 10^{17}$	$2.0 \cdot 10^{14}$	850	$2.1 \cdot 10^{-5}$

Notice that our model predicts that the smallest floating water vapor droplet in air at 20°C at sea level has a radius of $r_{min}=10^{-3.93}=1.17 \cdot 10^{-4}$cm. Figure 12.6

FIGURE 12.6

SMALLEST FLOATING WATER DROPLET
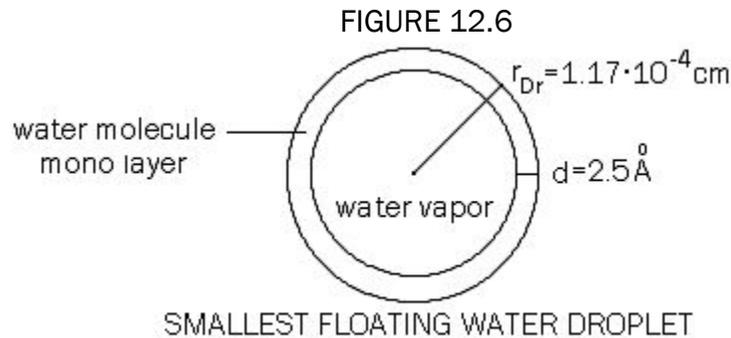
The hollow interior, is hypothesized to consist of water molecules unbonded to one another at saturated vapor pressure $P_V=2.34 \cdot 10^4 \frac{dy}{cm^2}$ containing $N_{in}=2.4 \cdot 10^{-3n+18}$ molecules with total mass $M_{in}=7.1 \cdot 10^{-3n-5}$gm and density $\rho_V=1.7 \cdot 10^{-5} \frac{gm}{cm^3}$.

A table of values for r_{Dr} (Droplet Radius), P_V (Saturated Water Vapor Pressure In the Interior of the Droplet), N_{in} (Number of Water Molecules In the Interior of the Droplet), M_{in} (Mass of Water Molecules In the Interior of the Droplet), and ρ_V (Density of the Water Molecules In the Interior of the Droplet) is given in Table 12.4

TABLE 12.4

r_{Dr}(cm)	$P_V(\frac{dy}{cm^2})$	N_{in}	M_{in}(gm)	$\rho_V(\frac{gm}{cm^3})$
10^{-4}	$2.34 \cdot 10^4$	$2.4 \cdot 10^6$	$7.1 \cdot 10^{-17}$	$1.7 \cdot 10^{-5}$
$1.17 \cdot 10^{-4}$	$2.34 \cdot 10^4$	$3.8 \cdot 10^6$	$1.1 \cdot 10^{-16}$	$1.7 \cdot 10^{-5}$
10^{-3}	$2.34 \cdot 10^4$	$2.4 \cdot 10^9$	$7.1 \cdot 10^{-14}$	$1.7 \cdot 10^{-5}$
10^{-2}	$2.34 \cdot 10^4$	$2.4 \cdot 10^{12}$	$7.1 \cdot 10^{-11}$	$1.7 \cdot 10^{-5}$
10^{-1}	$2.34 \cdot 10^4$	$2.4 \cdot 10^{15}$	$7.1 \cdot 10^{-8}$	$1.7 \cdot 10^{-5}$

The model for liquid water described in sec. 2 and 4 above, is of spherical water clathrates, fig. 12.1 and 12.5, composed of water molecules strongly bonded together ($f \approx 10^{-3}$dy) with each spherical clathrate weakly bonded to its neighbor clathrates ($f \approx 10^{-4}$dy).

If the water droplet volume comprises 5% of the total volume of a given volume of air, and if the water droplets collapse to form raindrops, then assuming the temperature remains constant, the air pressure will drop forming a low-pressure cell. If the starting pressure is P_i, then the pressure will drop to P_f where $P_f = P_i V_i (V_f)^{-1} = P_i (0.95)(1.0)^{-1} = 0.95 P_i$. If the starting pressure is 30" of Hg, then the pressure will drop to final pressure 28.5" of Hg.

If this process occurs over a warm ($T > 80^{\circ}F$) ocean where there is a constant source of water vapor droplets, it is conjectured that a hurricane will form. That is, it is conjectured that the collapse of water droplets, to form raindrops, is the physical cause of a low-pressure cell, and a low-pressure cell occurring over a warm ocean, where there is a constant source of water vapor droplets, is the physical cause of a hurricane.

8. Water to Steam: the Latent Heat of Vaporization

The latent heat of vaporization per water molecule, e_V, for water at $100^{\circ}C$ at 1atms. pressure is: $e_V = 0.730 \cdot 10^{-12} \frac{erg}{W} = 0.456 \frac{ev}{W}$. e_V does not go into increasing the translational kinetic energy of the water molecules. e_V goes into increasing the internal energy $(mC_1)_k = -(\frac{m^2 H}{h})_k$ (eq. 3.28) of each atom in the water molecule where $h(t)_k = <|(\bar{r}_{0_i} \hat{r})_k + \mathcal{X}_k((\bar{r}_{0_i} \hat{r})_k, t)|>$ and $\mathcal{X}_k((\bar{r}_{0_i} \hat{r})_k, 0) = 0$. k=1 is for Oxygen and k=2 is for the first Hydrogen atom and k=3 is for the second Hydrogen atom. The increasing value of $<|\mathcal{X}_k((\bar{r}_{0_i} \hat{r})_k, t)|>$ pushes apart one water molecule from the neighbors to which it is bonded.

At t=0 the water molecules in a given clathrate at the bottom of a saucepan have just reached $T = 100^{\circ}C$. At $t = t_0$ for some t_0, the molecules in the given clathrate break apart from one another with $h(t_0)_k = <|(\bar{r}_{0_i} \hat{r})_k + \mathcal{X}_k((\bar{r}_{0_i} \hat{r})_k, t_0)|> \equiv <|\bar{r}_{0_f}, t_0)|>$ with $\bar{r}_{0_f} > \bar{r}_{0_i}$ and $t_0 > 0$.

The total energy necessary to break the bonds between one water molecule and the 6 water molecules to which it is bonded is:

$e_V = (m_0 \Delta C_{1,0})_1 + 2(m_H \Delta C_{1,H})_2 + 3|B.E._{w,w}| = 0.456 \frac{ev}{w}$, where $(m_0 \Delta C_1)_1 = (\frac{m_0^2 H}{r_0^2} \overline{\Delta r_0})_1$

and $(m_H \Delta C_1)_2 = (\frac{m_H^2 H}{2 r_0^2} \overline{\Delta r_0})_2$. $|B.E._{w,w}|$ is the energy necessary to pull 2 water molecules apart where each water molecule is bonded to 6 water molecules with half the energy going to each of the molecules in the pair; Hence $3|B.E._{w,w}|$ to pull one water molecule from the 6 water molecules to which it is bonded.

If e_V were used solely to pull one water molecule from the 6 water molecules to which it is bonded then $|B.E._{w,w}| = \frac{1}{3}e_V = 0.152 \frac{ev}{w} = 2.43 \cdot 10^{-13} erg$ and with $2.43 \cdot 10^{-13} erg = \overline{f} \cdot s \approx (10^{-10}) \overline{f}$, $\overline{f}$ becomes: $\overline{f} \approx 10^{-3} dyn$.

By direct computation using eq. 3.28, if $\frac{1}{3}e_V = 0.152 \frac{ev}{w}$ were used solely to increase the radius of each atom in a water molecule the increase for O would be,

$< |(\overline{\Delta r_0} \hat{r})_1| > = 1.8 \cdot 10^{-14} cm$ and for H, $< |(\overline{\Delta r_0} \hat{r})_2| > = 2.1 \cdot 10^{-12} cm$.

Consider the following scenario: On rising from the bottom of a sauce pan a bubble of vapor composed of independent water molecules comes in contact with the cooler air above the liquid water. Some of the molecules escape from the surface as individual water molecules and with 18amu, float upwards. The remaining molecules recombine into supra water molecules $(H_2O)_n$ emitting $e_{ir} = 0.456 \frac{ev}{w}$ as infrared photons. Those that form hollow water droplets with radius $10^{-1} \lesssim r_{Dr} \lesssim 10^{-4} cm$ float upwards (See table 12.3) while those that don't, remain in the water.

Those that form hollow water droplets are at first filled with individual water molecules at atmospheric pressure. The individual water molecules over time Δt combine with the molecules on the surface of the hollow sphere until the internal pressure is the vapor pressure of water, $P_V = 2.34 \cdot 10^4 \frac{dy}{cm^2}$ and with a density less than the density of the surrounding air, the droplet floats upward. For an alternative model for turning water to steam, see appendix 12A.

9. Turbulent Flow in Water

We investigate the hypothesis that flowing water is composed of clathrates bound to one another with each clathrate bound to 1 to 12 neighbor clathrates within a bundle composed of N_{CL} clathrates where each clathrate contains N_W water molecules. See 12.13. The bundle of N_{CL} clathrates bound together is here called an S-clathrate or super clathrate. Further, flow separation and turbulent flow of water in the wake of a solid object are hypothesized to be due to the collisions of S-clathrates with the solid object and consequent nonhomogenous breakup and recombination of S-clathrates into clathrates and S-clathrates where n is the number of clathrates plus S-clathrates per unit volume and $n=n(\underset{\sim}{j})$, $\underset{\sim}{j} = \underset{\sim}{x} + \underset{\sim}{\chi}(\underset{\sim}{x},t)$, $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$, $\underset{\sim}{x} \neq \underset{\sim}{x}(t)$ where $\underset{\sim}{x}$ is a stationary point in the water.

Consider a volume of water in a tank with a free surface initially at rest in inertial frame S. Using 4.6, the average pressure that 2 adjacent S-clathrates in still water exert on one another is:

12.15 $\quad P_S \doteq \dfrac{2R_{SCL} n_{SCL} KT}{\overline{R}(h)} = \rho g h, \quad h \geq 2R_{SCL} + \overline{R}(2R_{SCL})$

Where h is the depth of the water at the point where P_S is measured, n_{SCL} is the number of S-clathrates per unit volume in still water, R_{SCL} is the radius of a S-clathrate and $2R_{SCL} + \overline{R}(h)$ is the average distance between two adjacent S-clathrates. Consider a solid cylinder of radius R_0 and length L_0 held stationary in flowing water. Figure 12.7 shows a cross section of the cylinder with the supports not shown. The top, sides and bottom of the tank are far enough away from the cylinder so that surface and wall affects are unimportant. Note that using the ideal gas law in place of 12.15 is incorrect: Assuming $P=nKT=\rho g h$ results in an incorrect sound speed and bulk modulus.

FIGURE 12.7
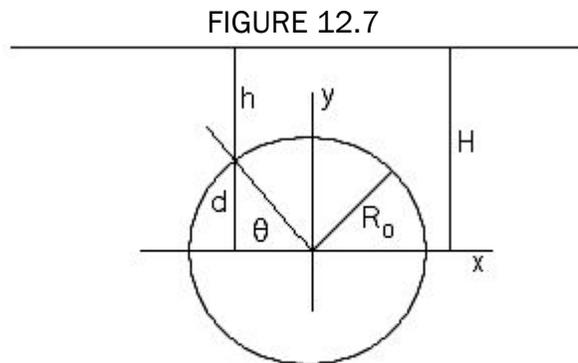
CYLINDER HELD STATIONARY IN FLOWING WATER

The water in the stream is composed of S-clathrates moving from left to right with speed U_X resulting in total pressure P_T normal to the surface of the cylinder on a point on the upstream side of the cylinder where:

$P_T = P_X \cos\theta + P_S$, $-\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$, where $P_X \cdot \hat{x} = |\dfrac{\Delta m U_X}{\Delta t \Delta A}| \cdot \hat{x} = \dfrac{m|\Delta U_X|}{\Delta t \Delta A} \cdot \hat{x}$ and $|\Delta U_X| = U_X + U_X \cos2\theta = U_X(1+\cos2\theta)$ and $\underset{\sim}{\Delta A} = -\Delta A \cdot \hat{x}$. $P_X \cdot \hat{x}$ becomes: $P_X \cdot \hat{x} = \dfrac{m U_X}{\Delta t \Delta A}(1+\cos2\theta) \cdot \hat{x}$ with $m = \rho \Delta V = \rho U_X \Delta t \Delta A = m_{SCL} n_{SCL} U_X \Delta t \Delta A$ and $P_X \cdot \hat{x} = m_{SCL} n_{SCL} U_X^2(1+\cos2\theta) \cdot \hat{x}$. Also with $(1+\cos2\theta) = 2(1-\sin^2\theta) = 2(1-[\dfrac{d}{R_0}]^2)$, $P_X \cdot \hat{x}$ becomes:

12.16 $\quad P_X \cdot \hat{x} = m_{SCL} n_{SCL} U_X^2(1+\cos2\theta) \cdot \hat{x} = 2m_{SCL} n_{SCL} U_X^2(1-[\dfrac{d}{R_0}]^2) \cdot \hat{x}$

For a given θ_0, suppose that U_X is large enough so that for $-\dfrac{\pi}{2} < -\theta_0 \leq \theta \leq \theta_0 < \dfrac{\pi}{2}$, collisions with the cylinder break the S-clathrates into clathrates plus S-clathrates so

that before the collision all clathrates are S-clathrates with n_{SCL} S-clathrates per unit volume and after the collision n_{SCL} becomes $n(\underset{\sim}{j})>n_{SCL}$ for $0<t\leq\tau_0$ for some τ_0 where $\underset{\sim}{j}$ at t=0 represents all points on the cylinder for $-\frac{\pi}{2}<-\theta_0\leq\theta\leq\theta_0<\frac{\pi}{2}$. After the collision $U_X=U(\underset{\sim}{j})_X$, however, $\overline{m}(\underset{\sim}{j})\cdot n(\underset{\sim}{j})=m_{SCL}n_{SCL}$ so that the functional form of P_X before the collision equals the functional form of P_X after the collision i.e.

$$\underset{\sim}{P}_X\cdot\hat{x}=2m_{SCL}n_{SCL}U(\underset{\sim}{j})_X^2(1-[\tfrac{d}{R_0}]^2)\cdot\hat{x}=2\overline{m}(\underset{\sim}{j})\cdot n(\underset{\sim}{j})U(\underset{\sim}{j})_X^2(1-[\tfrac{d}{R_0}]^2)\cdot\hat{x} \text{ for } 0<t\leq\tau_0.$$

P_S however is dependent on $n(\underset{\sim}{j})$. After the breakup, P_S becomes the turbulent pressure P_{tu} where $P_{tu}=\dfrac{2r_0(\underset{\sim}{j})n(\underset{\sim}{j})KT}{\overline{R}(\underset{\sim}{j})}$ and $2r_0(\underset{\sim}{j})+\overline{R}(\underset{\sim}{j})$ with $\overline{R}(\underset{\sim}{j})<<2r_0(\underset{\sim}{j})$ where

$2r_0(\underset{\sim}{j})+\overline{R}(\underset{\sim}{j})=\frac{1}{N_T}\sum d_{ij}$. d_{ij}=the center of mass to center of mass distance between the ith and j^{th} adjacent supra molecules (clathrates and S-clathrates and parts of S-clathrates) within a suitably small volume $\Delta V(\underset{\sim}{j})$ centered at $\underset{\sim}{j}$. The sum is taken over all adjacent pairs in $\Delta V(\underset{\sim}{j})$ and N_T is the total number of adjacent pairs in $\Delta V(\underset{\sim}{j})$.

Assuming that $\dfrac{r_0(\underset{\sim}{j})}{\overline{R}(\underset{\sim}{j})}=\dfrac{R_{SCL}}{\overline{R}(h)}$ for $H-R_0\leq h\leq H+R_0$, (See fig. 12.7), P_{tu} becomes:

12.17 $\qquad P_{tu}=\dfrac{2r_0(\underset{\sim}{j})n(\underset{\sim}{j})KT}{\overline{R}(\underset{\sim}{j})}=\dfrac{2R_{SCL}n(\underset{\sim}{j})KT}{\overline{R}(h)}\dfrac{n(\underset{\sim}{j})}{n_{SCL}}P_S>P_S \text{ for } 0<t\leq\tau_0$

The S-clathrates strike the cylinder with fixed kinetic energy $\frac{1}{2}m_{SCL}U_X^2$.
Using 12.14: $m_{CL}=2.6\cdot10^{-14}$gm and $m_{SCL}=2.6\cdot10^{-14}N_{CL}$ and $r_{CL}=1.4\cdot10^{-5}$cm.
With $-R_0\leq d\leq R_0$, the following scenario is anticipated.
(i) For small enough U_X, $0<U_X\leq U_{Cr}$ for some U_{Cr}, the S-clathrates roll around the cylinder in creep flow. This necessitates a centripetal force $\underset{\sim}{f}_C=-m_{SCL}\dfrac{V_C^2}{R_0}\hat{R}_0$ with $\underset{\sim}{V}_C=V_C\cdot(\hat{R}_0X\hat{z})$

where $\underset{\sim}{f}_C$ is provided by a chemical bonding force $-f_B\hat{R}_0$ between the S-clathrate and the cylinder. In creep flow, $f_B\geq f_C$ where $f_B=m_{SCL}\dfrac{U_{Cr}^2}{R_0}$ for some given fixed N_{CL} where

U_{Cr} is the maximum flow rate at which the flow remains creep flow. Experimentally for R_0=1cm, U_{Cr} is $U_{Cr}\approx0.1\frac{cm}{sec}$ and f_B becomes: $f_B=2.6\cdot10^{-16}N_{CL}$(dy): Also $\frac{1}{2}m_{SCL}U_{Cr}^2=1.3\cdot10^{-14}N_{CL}U_{Cr}^2\approx$
$1.3\cdot10^{-16}$erg.
(ii) For $U_X>U_{Cr}$ and consequent $m_{SCL}\dfrac{U_X^2}{R_0}>m_{SCL}\dfrac{U_{Cr}^2}{R_0}=f_B$ and $\frac{1}{2}m_{SCL}U_X^2>\frac{1}{2}m_{SCL}U_{Cr}^2$, the

flow separates from the cylinder for $\frac{\pi}{2}\leq\theta\leq\frac{3}{2}\pi$.

(iii) The binding energy $Be_{CL,CL}$ holding 2 clathrates together is approximated using $|BE_{CL,CL}|=\frac{3}{2}K(373-T)=\frac{3}{2}K(80)=1.7\cdot10^{-14}$erg=0.01ev. In order to derive the coefficient of viscosity of water, it is assumed that the binding energy $BE_{SCL,SCL}$ holding 2 S-clathrates together is $|BE_{SCL,SCL}|<<|BE_{CL,CL}|$.

With $|BE_{CL,CL}|<<|BE_{w,w}|$, let $|SBE_{SCL}|$ represent the absolute value of the self binding energy of 1 super clathrate. i.e. $|SBE_{SCL}|$ is the energy necessary to pull apart 1 S-clathrate into its N_{CL} constituent clathrates and $|BE_{w,w}|$ represents the energy to pull 2 bound water molecules apart. i.e. $(H_2O)_2 \longrightarrow 2(H_2O)$.

A S-clathrate collides with the cylinder with large enough U_X so that $\frac{1}{2}m_{SCL}U_X^2 = \frac{1}{2}m_{CL}N_{CL}U_X^2 \geq |SBE_{SCL}|$. For large enough N_{CL}, each clathrate is bonded to 6 clathrates in the interior of the clathrate and 5 clathrates on the surface of the clathrate and $|SBE_{SCL}|$ may be approximated by $|SBE_{SCL}|=6|BE_{CL,CL}|N_{CL}$ and $\frac{1}{2}m_{CL}N_{CL}U_X^2 \geq |SBE_{SCL}|$ becomes $\frac{1}{2}m_{CL}U_X^2 \geq 6|BE_{CL,CL}|=1.0\cdot10^{-13}$erg=0.06ev. Solving for U_X yields:

12.18 $U_X \geq 0.29\frac{cm}{sec}$

Note the independence of U_X from N_{CL}.

On striking the cylinder with $U_X \geq 0.29\frac{cm}{sec}$, the S-clathrate breaks into N_{CL} individual clathrates each with mass $m_{CL}=2.4\cdot10^{-12}$gm. The N_{CL} clathrates separate from the cylinder and in those parts of the flow where $\frac{n(j)}{\tilde{n}_{SCL}}>1$ and using 12.17 it follows that $P_{tu}>P_S$. The increased pressure and the random nature of the direction of the vector velocities of the released water clathrates, destroys the laminar flow in the lee of the cylinder and is the physical cause of turbulence.

In the lee of the cylinder, there is a mixture of $N_{CL}=1$ clathrates and N_{SCL} super clathrates. In those regions where $N_{CL}=1$ clathrates surround N_{SCL} super clathrates, there is a pressure gradient inwards towards the super clathrates and a vortex forms that dies away as the $N_{CL}=1$ clathrates reform into N_{SCL} super clathrates. The absence of laminar flow and the creation of vortices, is characteristic of turbulent flow.

In order to break up an individual clathrate into its $N_w=8.7\cdot10^8$ water molecules (See 12.14) requires $6|BE_{w,w}|\cdot N_w =6f_{w,w}\cdot\Delta s\cdot N_w \doteq 6(10^{-3})(10^{-10})N_w =6\cdot10^{-13}N_w(erg)=0.37N_w(ev)$ where $f_{w,w}$ is the average force necessary to pull apart two isolated water molecules and Δs is the distance necessary to pull apart two-isolated water molecules.

The kinetic energy of a clathrate moving with speed U_X is: $\frac{1}{2}m_w\cdot N_w U_X^2 =1.5\cdot10^{-23}\cdot N_w U_X^2$

And setting $1.5 \cdot 10^{-23} \cdot N_W U_X^2 = 6 \cdot 10^{-13} N_W$ yields:

12.19 $U_X = 2 \cdot 10^5 \frac{cm}{sec} = 1.3 \cdot C_S$ C_S is the speed of sound in water

U_X is the minimum speed at which a clathrate must be moving as it strikes a stationary solid object in order that all N_W water molecules of which the clathrate is composed, are freed from one another.

10. Turbulent Flow in Air

Water vapor in air consists of floating water droplets (Sec. 7 and fig. 12.6) and floating individual water molecules. We investigate the hypothesis that the onset of turbulent flow in air is due to the breakup of water droplets into individual water molecules when striking a moving object at speeds<<Mach 1 and due to the breakup of individual water molecules into individual H and O atoms at speeds $\gtrsim$ Mach 10.

In this section we will compute the speed U_X of a solid object (e.g. the wing of an Airplane etc.) necessary to:
1. Break a floating water droplet into its W_{Dr} constituent water molecules.
2. Calculate the resultant increase in pressure ΔP for part 1.

The background air pressure P_0 at sea level at T=293°K, is $P_0 = 1.0 \cdot 10^6 (\frac{dy}{cm}2)$ with $n_0 = 2.5 \cdot 10^{19} (\frac{1}{cm3})$. The physical increase in the number of molecules per unit volume Δn in those volumes where droplets are colliding with the moving solid object, results in an increase in pressure $\Delta P = \Delta n K T$ once temperature equilibrium has been reestablished. It is the physical increase in the number of molecules per unit volume, the resultant increase in the local pressure ΔP, and the random nature of the direction of the vector velocities of the released water molecules that is hypothesized to be the physical cause of turbulence.

Note that with water vapor pressure $P_V = 2.34 \cdot 10^4 \frac{dy}{cm}2$, at T=293°K and assuming that water vapor consists completely of individual water molecules, by direct computation using the ideal gas law, the number of water molecules per unit volume is $n_W = 5.8 \cdot 10^{17} \frac{W}{cm}3$ with saturation density $\rho_W = 1.7 \cdot 10^{-5} (\frac{gm}{cm}3)$.

From table 1, for the smallest floating water droplet at sea level, $r_{Dr} = 1.17 \cdot 10^{-4}$cm;

The maximum number of droplets per unit volume is $n_{M,Dr} = \frac{1}{(2r_{Dr} + \bar{R}_{Dr})}3$ and with

$\bar{R}_{Dr} << 2r_{Dr}$: $n_{M,Dr} = 7.8 \cdot 10^{10} (\frac{Dr}{cm}3)$. Assuming the water vapor consists solely of water vapor droplets, the vapor pressure is $P_V = n_{M,Dr} K T = 3.1 \cdot 10^{-3} << 2.34 \cdot 10^4 (\frac{dy}{cm}2)$.

Consequently the measured water vapor pressure under laboratory conditions is not primarily due to water droplet number density n_{Dr}, but is primarily due to unbonded water molecule number density n_W. The pressure is measured in a glass vessel with closed sides and top and a liquid water bottom. In earth's atmosphere there are no solid sides and top and the bottom consists of ~75% ocean and ~25% earth and through both ocean and earth there is a constant exchange of N_2, O_2, CO_2 and H_2O.

Fogs at ground level at night under a street lamp can be seen to be composed of droplets. We examine the consequences of assuming that in earth's atmosphere (But not in the closed container used to measure water vapor pressure) the water vapor that forms fogs and clouds and the water vapor that forms rain drops (But not the rain drops themselves) is composed of hollow water droplets. See Sec. 7 and appendix 12B.

As regards #1.
The kinetic energy of a floating water droplet colliding with a solid object moving with speed U_X relative to the floating water droplet is: $\frac{1}{2}m_w \cdot W_{Dr} U_X^2 = 1.5 \cdot 10^{-23} \cdot W_{Dr} U_X^2$. For a floating water droplet consisting of a single layer of water molecules, each water molecule is bonded to 4 water molecules and to remove 1 water molecule takes $4|Be_{w,w}| = 4 \cdot \overline{f} \Delta s \approx 4 \cdot 10^{-3}(10^{-10}) = 4 \cdot 10^{-13}$erg. Let N_{rw} represent the number of water molecules removed at 1 collision. Using table 12.3 and setting $1.5 \cdot 10^{-23} \cdot W_{Dr} U_X^2 = 4 \cdot 10^{-13} \cdot N_{rw}$ (erg) and solving for U_X yields: $U_X = 1.6 \cdot 10^5 (\frac{N_{rw}}{W_{Dr}})^{\frac{1}{2}} = 10(N_{rw})^{\frac{1}{2}} \frac{cm}{sec}$.
A table of values of U_X as a function of N_{rw} is given in table 12.5.

TABLE 12.5

N_{rw}	$U_X(\frac{cm}{sec})$	U_X(mph)	N_{rw}	$U_X(\frac{cm}{sec})$	U_X(mph)
1	10	0.22	10^5	$3.2 \cdot 10^3$	70
10	32	0.70	10^6	10^4	220
10^2	10^2	2.2	10^7	$3.2 \cdot 10^4$	700
10^3	320	7.0	10^8	10^5	2200
10^4	10^3	22	$2.7 \cdot 10^8$	$1.6 \cdot 10^5$	3600

In order to calculate the change in pressure ΔP due to the creation of N_{rw}, it is necessary to calculate Δn where $\Delta n = n_{rw} (\frac{1}{cm^3})$ and where n_{rw} is the number of water molecules per cm^3 removed from droplets after 1 collision with a moving object. $\Delta n = n_{rw} = N_{rw} \cdot n_{Dr}$ where n_{Dr} is the number of droplets per cm^3. With $n_{M,Dr} = 7.8 \cdot 10^{10}(\frac{Dr}{cm^3})$, if the total volume of the droplets is 1% of the total volume containing the droplets then $n_{Dr} = 7.8 \cdot 10^8(\frac{Dr}{cm^3})$ and $\Delta n = n_{rw} = 7.8 \cdot 10^8 \cdot N_{rw}$. A table of values is presented below.

<div align="center">TABLE 12.6, T=20° C, Sea Level</div>

N_{rw}	U_X(mph)	$n_{rw}(\frac{1}{cm^3})$	$\Delta P(\frac{dy}{cm^2})$	$\Delta P(\frac{lb}{ft^2})$	$P(\frac{dy}{cm^2})$
1	0.22	$7.8\cdot10^8$	$3.1\cdot10^{-5}$	$6.7\cdot10^{-8}$	$1.2\cdot10^{-1}$
10	0.70	$7.8\cdot10^9$	$3.1\cdot10^{-4}$	$6.7\cdot10^{-7}$	1.2
10^2	2.2	$7.8\cdot10^{10}$	$3.1\cdot10^{-3}$	$6.7\cdot10^{-6}$	$1.2\cdot10^1$
10^3	7.0	$7.8\cdot10^{11}$	$3.1\cdot10^{-2}$	$6.7\cdot10^{-5}$	$1.2\cdot10^2$
10^4	22	$7.8\cdot10^{12}$	$3.1\cdot10^{-1}$	$6.7\cdot10^{-4}$	$1.2\cdot10^3$
10^5	70	$7.8\cdot10^{13}$	3.1	$6.7\cdot10^{-3}$	$1.2\cdot10^4$
10^6	220	$7.8\cdot10^{14}$	31	$6.7\cdot10^{-2}$	$1.2\cdot10^5$
10^7	700	$7.8\cdot10^{15}$	310	$6.7\cdot10^{-1}$	$1.2\cdot10^6$
10^8	2200	$7.8\cdot10^{16}$	3100	6.7	$2.4\cdot10^7$
$2.7\cdot10^8$	3600	$2.1\cdot10^{17}$	8400	18	$3.1\cdot10$

Depending on the geometry of the solid moving object, multiple collisions between a given water droplet and the moving object are possible. If the total number of collisions is N_C, then the N_{rw} , n_{rw} and ΔP columns are multiplied by N_C for $0 \leq N_C \lesssim 2.7\cdot10^6$.

The remaining columns remain as stated. For $2.7\cdot10^6 \leq N_C \leq 2.7\cdot10^8$, one must take into account the mass depletion of the water droplet after every collision. That is not done here.

The P column represents the pressure due to the collision of air molecules with the moving object using $P=\rho U^2$.

The energy to decompose one water molecule, $H_2O \longrightarrow 2H+O$, is $4.2\cdot10^{-12}=2.6$ev.

Setting $\frac{1}{2}m_{H_2O}U_X^2=4.2\cdot10^{-12}$erg yields, $U_X=5.3\cdot10^5\frac{cm}{sec}=12,000$ mph applicable to reentry vehicles and meteorites.

The energy to decompose one Nitrogen molecule, $N_2 \longrightarrow 2N$, is $1.55\cdot10^{-11}$erg=9.7ev.

Setting $\frac{1}{2}m_{N_2}U_X^2=1.55\cdot10^{-11}$erg yields, U_X becomes: $U_X=8.1\cdot10^5\frac{cm}{sec}=18,000$ mph again applicable to reentry vehicles and meteorites.

11. Shock Wave in Earth's Atmosphere

Leading any axis-symmetric solid object traveling faster than or equal to the local speed of sound (Mach 1) is a high pressure, high-density surface of revolution called a shock wave. In Figure 12.8, a solid cube 2 cm on a side is moving from right to left at Mach 2.0. All measurements are made by an observer O in an inertial frame at rest with respect to the ground.

Figure 12.8- Shock Wave
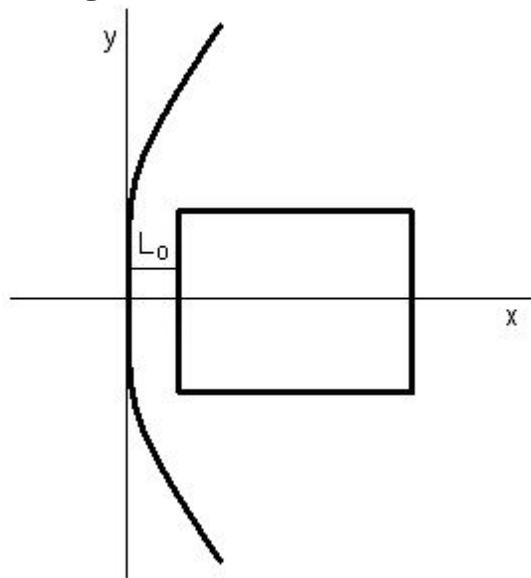
The shock wave acts as a shield in front of the solid object and the majority of incident air molecules reflect off of its surface and a minority penetrate the shock. Figure 12.9 represents the solid moving into still air at arbitrary Mach number M but before the formation of the shock. The air molecules at S.T.P. in A are idealized to be in stationary serried ranks separated by the average distance λ_o between the molecules. Note that assuming that the molecules in serried ranks are separated by the mean free path, yields an air density $\sim 10^3$ times smaller than the measured air density.

The average velocity of the molecules in A is taken to be $0 \frac{cm}{sec}$ except for the molecules directly in contact with the solid. For M>>1 this is a good approximation, but for $M \lesssim 1$, a bad approximation. The struck molecules in A travel the distance λ_o to the left in time t_o with average velocity $-2MC_o\hat{x}$; Strike the molecules in the 2nd rank and exchange velocities. In time t_o, the solid object travels distance $\frac{\lambda_o}{2}$ with velocity $-MC_o\hat{x}$. Considering A through H, it is apparent that $n_1 \doteq 2n_o$ where n_o is the molecule number density in A and n_1 is the molecule number density in H. Also $\rho_1 \doteq 2\rho_o$ where ρ_o is the mass density in A and ρ_1 is the mass density in H. The percussive wave generated by the moving object travels with speed $2MC_o$ as measured by observer O. However the average speed of the molecules is MC_o, i.e.

$$[(2MC_o) \cdot 8 + 0 \cdot 8]16^{-1} = MC_o$$

The adiabatic sound speed in A is $C_o = (\gamma \frac{P_o}{\rho_o})^{\frac{1}{2}} = (\gamma \frac{n_o KT_o}{\rho_o})^{\frac{1}{2}}$ and the adiabatic sound speed in H as measured w.r.t. the moving object is, $C_1 = (\gamma \frac{P_1}{\rho_1})^{\frac{1}{2}} = (\gamma \frac{n_1 KT_1}{\rho_1})^{\frac{1}{2}} = (\gamma \frac{n_o KT_o}{\rho_o} \frac{T_1}{T_o})^{\frac{1}{2}} = C_o (\frac{T_1}{T_o})^{\frac{1}{2}}$.

Figure 12.9 Water Droplets Not Shown
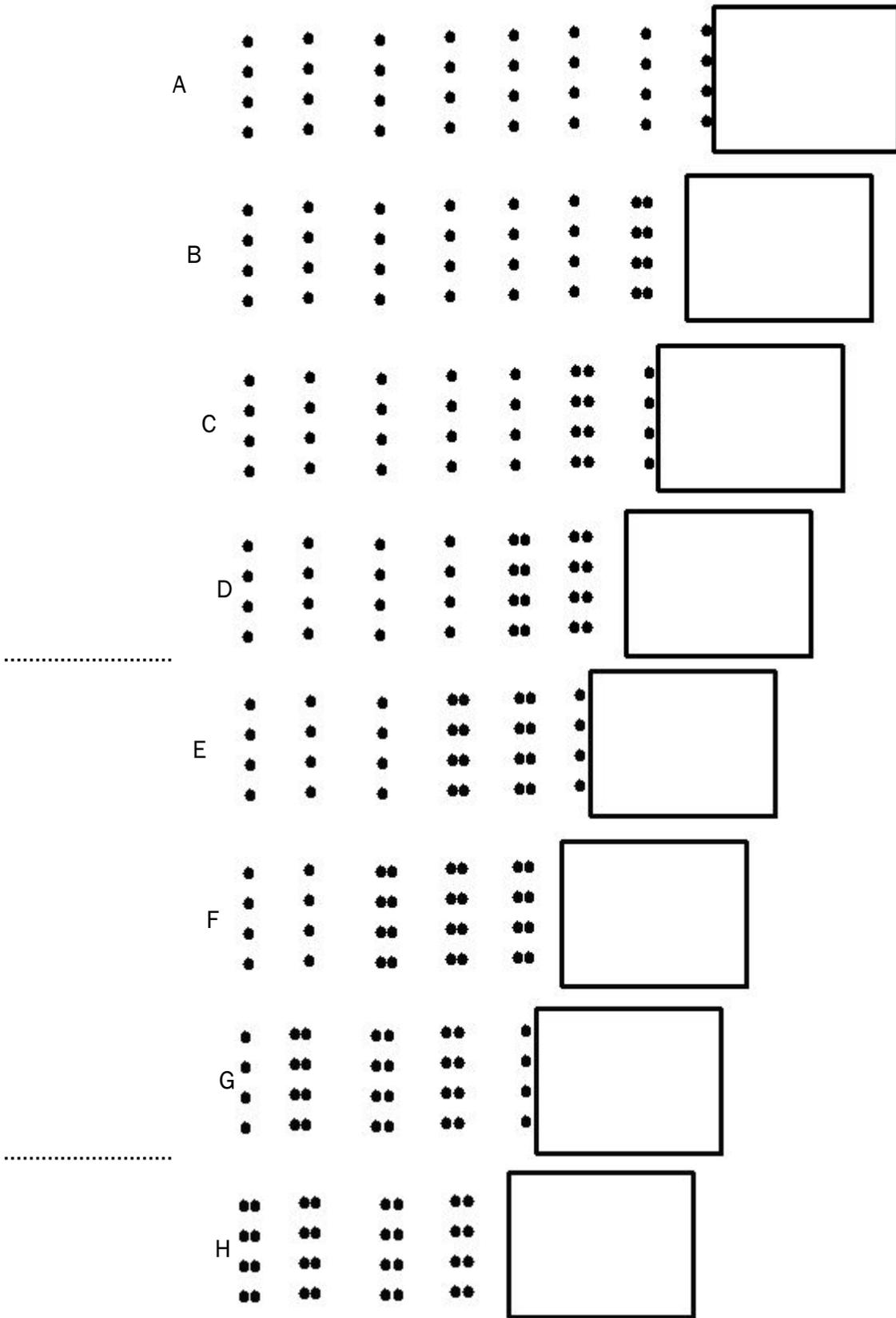
For a shock to form with $C_S = M \cdot C_0 = C_0(\frac{T_1}{T_0})^{\frac{1}{2}}$ yields: $T_1 = M^2 T_0$. This is not observed and consequently a shock wave does not form in air that is in accord with the ideal gas law and $\frac{n_0}{\rho_0} = \frac{n_1}{\rho_1} = \frac{1}{m_0}$.

The density at fig. 12.9 H in front of the moving object ρ_f is $\rho_f = 2\rho_0$ where ρ_0 is the background air density before the passage of the moving object, fig. 12.9 A. The leading edge separating $\rho = \rho_0$ from $\rho = 2\rho_0 = \rho_f$ is traveling with speed $2MC_0$: The moving object is traveling with speed MC_0.

The movement of the solid object causes the breakup of water droplets directly in front of the moving object resulting in an increase in the number of molecules per unit volume from n_0 to $n_1 = n_0 + n_{rw}$ where n_{rw} is the number of newly released water molecules per unit volume (Not Shown). As the water droplets break up, the number density increases from n_0 to n_1 but the mass density ρ_0, at the instant of the creation of n_{rw} directly in front of the moving object, remains the same, fig. 12.9 A (Water droplets not shown).

As the water droplets are pushed away from the moving object, fig. 12.9 H, the number density increases from n_1 to $n_f = 2n_1$ and the mass density increases from ρ_0 to $\rho_f = 2\rho_0$.

The phase speed of sound C_2 at fig. 12.9 H directly in front of the moving object is the speed of sound with respect to the air plus the speed of the air with respect to the ground. With no water droplets, $n_w = 0$, C_2 is:

$C_2 = (\gamma \frac{2n_0 KT_0}{2\rho_0})^{\frac{1}{2}} + MC_0 = (\gamma \frac{n_0 KT_0}{\rho_0})^{\frac{1}{2}} + MC_0 = (1+M)C_0$. With the advent of the breakup of water droplets, the speed of sound C_f at fig. 12.9 H directly in front of the moving object is:

$C_f = (\gamma \frac{n_f KT_0}{\rho_f})^{\frac{1}{2}} + MC_0 = (\gamma \frac{2n_1 KT_0}{2\rho_0})^{\frac{1}{2}} + MC_0 = (\gamma \frac{\overline{m}_f KT_0}{m_0})^{\frac{1}{2}} + MC_0 = (\gamma \frac{n_0(1+\frac{n_{rw}}{n_0})KT_0}{\rho_0})^{\frac{1}{2}} + MC_0 =$

$[(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}} + M]C_0$. For future use:

12.20 $\qquad C_2 = (1+M)C_0$

$\qquad\qquad C_f = [(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}} + M]C_0$

The time interval Δt that it takes the trailing edge of the nascent shock beginning at $x = L_0$ at $t=0$ with wave length λ_0 (Fig. 12.8) to over take the leading edge of the nascent shock (i.e. The leading edge separating $\rho = \rho_0$ from $\rho = 2\rho_0 = \rho_f$, traveling with speed $2MC_0$) is: $\Delta t = (L_0 - \lambda_0)(C_2)^{-1} = L_0(C_f)^{-1} = (L_0 - \lambda_0)[(1+M)C_0]^{-1} = L_0([(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}} + M]C_0)^{-1}$

Solving the last two expressions for $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M$ yields: $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M=(1+M)(1-\frac{\lambda_0}{L_0})^{-1}$.

Δt and $\frac{n_{rw}}{n_0}$ become:

12.21 $\qquad \Delta t=(1-\frac{\lambda_0}{L_0})\frac{L_0}{(M+1)C_0}$

$$\frac{n_{rw}}{n_0}=\left\{\frac{1+\frac{\lambda_0}{L_0}\cdot M}{1-\frac{\lambda_0}{L_0}}\right\}^2 -1$$

Note that using $MC_0=10(N_{rw})^{\frac{1}{2}}\frac{cm}{sec}$ (See sec 10), it follows that: $\lim\limits_{M->0} N_{rw}=0$ and

$\lim\limits_{M->0}\frac{n_{rw}}{n_0}=0$ and using 12.20, $\lim\limits_{M->0} C_f=C_2$. Using $\Delta t=(L_0-\lambda_0)(C_2)^{-1}=L_0(C_f)^{-1}$:

$\lim\limits_{M->0} L_0(C_f)^{-1}=L_0(C_2)^{-1}=(L_0-\lambda_0)(C_2)^{-1}$ and consequently, $\lim\limits_{M->0}\lambda_0=0$. Observationally

$\lim\limits_{M->0} L_0=0$ and using 12.21, $\lim\limits_{M->0}\Delta t=0$. Summing up:

12.22 $\quad \lim\limits_{M->0}\frac{n_{rw}}{n_0}=0, \; \lim\limits_{M->0}\lambda_0=0, \; \lim\limits_{M->0} L_0=0$ and $\lim\limits_{M->0}\Delta t=0$

If the total volume of floating water droplets is 1% of the total volume of air through

which the solid object passes and using $MC_0=10(N_{rw})^{\frac{1}{2}}\frac{cm}{sec}$ (See sec 10), $n_{rw}=7.8\cdot10^8\cdot N_{rw}$

and $\Delta P=n_{rw}KT$ as developed for table 2.5 and 12.6 yields N_{rw} (The number of water

molecules released per floating droplet after 1 collision with the moving object) and n_{rw} (The

number of water molecules released per cm^3 after 1 collision with the moving object)
and ΔP (The change in pressure as measured from an inertial frame at rest with
respect to the center of mass of the moving air. The change in pressure is due to the
release of water molecules from droplets thereby increasing n_f). C_0 as experimentally

determined at sea level for $T=20^0C$ is, $C_0=3.4\cdot10^4(\frac{cm}{sec})=760$mph.

Using $MC_0=2C_0$, a table of values of N_{rw}, n_{rw} and ΔP computed for Mach2 is presented

TABLE 12.7, Sea Level, Mach2

N_{rw}	$2C_0$(mph)	$n_{rw}(\frac{1}{cm^3})$	$\Delta P(\frac{dy}{cm^2})$	$\Delta P(\frac{lb}{ft^2})$
$4.6\cdot10^7$	1520	$3.6\cdot10^{16}$	$1.5\cdot10^3$	0.54
$4.6\cdot10^7N_C$	1520	$3.6\cdot10^{16}N_C$	$1.5\cdot10^3N_C$	$0.54N_C$
$7.8\cdot10^8$	1520	$6.1\cdot10^{17}$	$2.55\cdot10^4$	9.2

in table 12.7. If the total number of collisions is N_C, then the N_{rw}, n_{rw} and ΔP columns are multiplied by N_C where $0 \leq N_C \leq 17$.

The change in pressure ΔP_{M2} due to the collision of air molecules with the moving object at Mach2 is: $\Delta P_{M2} = \rho(Mach2)^2 = (2\rho_0)(6.8 \cdot 10^4)^2 = 1.1 \cdot 10^7 \frac{dy}{cm^2}$.

Using 12.21, Δt and $\frac{n_{rw}}{n_0}$ are evaluated and compiled in table 12.8 for a solid of cross section 2cmX2cm and $L_0 \approx 0.6$cm traveling at Mach 2 as in fig. 12.8. At T=20^{0}C at sea level, $n_0 = 2.5 \cdot 10^{19}$ and for the droplet under consideration, fig 12.6, $0 \leq \frac{n_{rw}}{n_0} \leq 2.4 \cdot 10^{-2}$

Table 12.8, Sea Level, Mach2

$\frac{n_{rw}}{n_0}$	$\frac{\lambda_0}{L_0}$	Δt(sec)
0	0	$5.9 \cdot 10^{-6}$
$1.0 \cdot 10^{-3}$	$1.67 \cdot 10^{-4}$	$5.9 \cdot 10^{-6} (1 - 1.67 \cdot 10^{-4})$
$1.0 \cdot 10^{-2}$	$1.67 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 1.67 \cdot 10^{-3})$
$2.0 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 3.3 \cdot 10^{-3})$
$2.4 \cdot 10^{-2}$	$6.6 \cdot 10^{-3}$	$5.9 \cdot 10^{-6} (1 - 6.6 \cdot 10^{-3})$

As $\bar{t} \rightarrow \Delta t = t_0 - 0$, the trailing edge overtakes the leading edge and the energy density, mass density and pressure between the leading and trailing edge increase forming a shock. Observationally, the shock is traveling with group speed MC_0 at approximately constant distance L_0 in front of the moving object. This means that water droplets in front of the shock are being broken up by collision with the molecules making up the shock and the shock is entering an atmosphere where the speed of the shock is

$C_S = MC_0 = (1 + \frac{n_{w,s}}{n_0})^{\frac{1}{2}} C_0$ and $(M^2 - 1)n_0 = n_{w,s}$ where $n_{w,s}$ is the number of water molecules per cm^3 that are released from water droplets by collision with the shock and reflect in front of the shock.

How is it that the speed of the shock remains constant at MC_0? If $n_{w,s} > (M^2 - 1)n_0$ the shock starts to travel away from the moving object, loses energy and ceases to exist and if $n_{w,s} < (M^2 - 1)n_0$, the moving object overtakes the shock and again the shock ceases to exist. If $n_{w,s} = (M^2 - 1)n_0$ the shock remains at a ~fixed distance in front of the moving object and energy is constantly pumped into it by the moving object.

The shock acts as a shield and depending on the mass density and thickness of the shock, a certain percentage P of the incident air molecules in front of the shock reflect off the shock and 100-P percent go through the shock. Among those that reflect off

of the shock are the water droplets that are broken up and release $n_{w,s}$ water molecules per cm^3 in front of the shock.

Of the $100-P$ percent that go through the shock, $n(100-P)$ percent reflect off the moving body and make up the molecules that are pumped back into the shock and $(1-n)(100-P)$ move around and behind the moving body where $0<n<1$.

12. Appendix 12A. Water to Steam: the Latent Heat of Vaporization

The latent heat of vaporization per water molecule, e_V, for water at 100^oC at 1atms. pressure is: $e_V=0.730\cdot10^{-12}\frac{erg}{W}=0.456\frac{ev}{W}$. e_V does not go into increasing the translational kinetic energy of the water molecules. e_V goes into increasing the internal energy $(mC_1)_k=-(\frac{m^2H}{h_0})_k$ (eq. 3.28) of each atom in the water molecule where $h_{o,k}=<|(\overline{r}_{0_i}\hat{r})_k+\underset{\sim}{\chi}_k((\overline{r}_{0_i}\hat{r})_k,t)|>$ and $\underset{\sim}{\chi}_k((\overline{r}_{0_i}\hat{r})_k,0)=0$. k=1 is for Oxygen and k=2 is for Hydrogen. The increasing value of $<|\underset{\sim}{\chi}_k((\overline{r}_{0_i}\hat{r})_k,t)|>$ pushes apart the H and O atoms of a given water molecule.

The disassociation temperature for a water molecule is $T=3400^oK$ with an energy equivalence of $0.44\frac{ev}{W}$. (There is a wide range of values in the literature ranging from $\sim800^oK$ to $\sim3700^oK$ presumably due to the number of water molecules that a given water molecule is bonded too). This makes it conceivable given $e_V=0.456\frac{ev}{W}$, that the latent heat of vaporization disassociates N_W water molecules into $2N_W\cdot H+N_W\cdot O$ atoms in a bubble of gas in the bottom of a heated saucepan resulting in a bubble of gas that rises through the boiling water.

At t=0 the water molecules in a given clathrate at the bottom of the saucepan have just reached $T=100^oC$. At $t=t_0$ for some t_0, the molecules in the given clathrate break apart into individual atoms with $h_{f,k}=<|(\overline{r}_{0_f}\hat{r})_k +\underset{\sim}{\chi}_{f,k}((\overline{r}_{0_f}\hat{r})_k,t)|>$ and $\underset{\sim}{\chi}_{f,k}((\overline{r}_{0_f}\hat{r})_k,t_0)=0$ with $\overline{r}_{0_f}>\overline{r}_{0_i}$ and $t \geq t_0$. k=1 is for Oxygen and k=2 is for Hydrogen.

The total energy necessary to form 2H and O from H_2O is:

$(m_o\Delta C_1)_1+(m_H\Delta C_1)_2+3B.E._{w,w}+B.E._{OH,H}+B.E._{O,H}$, where $(m_o\Delta C_1)_1=(\frac{m_o^2H}{r_o^2}\overline{\Delta r_0})_1$

and $(m_H\Delta C_1)_2=(\frac{m_H^2H}{r_o^2}\overline{\Delta r_0})_2$. $B.E._{w,w}$ is the energy necessary to pull 2 water molecules apart where each water molecule is bonded to 6 water molecules with half the energy going to each of the molecules in the pair; Hence $3B.E._{w,w}$ to pull one water molecule from 6 water molecules to which it is bonded. $B.E._{OH,H}$ is the energy necessary to pull one H atom from H_2O and $B.E._{O,H}$ is the energy necessary to pull one H atom from OH .

By direct computation using eq. 3.28, if $\frac{1}{3}e_V$=0.152$\frac{ev}{w}$ were used solely to increase the radius of each atom in a water molecule the increase for O would be,

$<|(\overline{\Delta r_0}\hat{r})_1|>$=1.8·10^{-14}cm and for H, $<|(\overline{\Delta r_0}\hat{r})_2|>$=2.1·10^{-12}cm.

Consider the following testable scenario: On rising from the bottom of the sauce pan in the gas bubble through the cooler liquid water, 2H and O atoms recombine into one water molecules H_2O emitting e_{ir} ($\frac{erg}{w}$) as infrared photons. The water molecules bind themselves into a circular, bilayered micelle of radius r_{mi} and surface area $2\pi \cdot r_{mi}^2$. Figure 12.7.
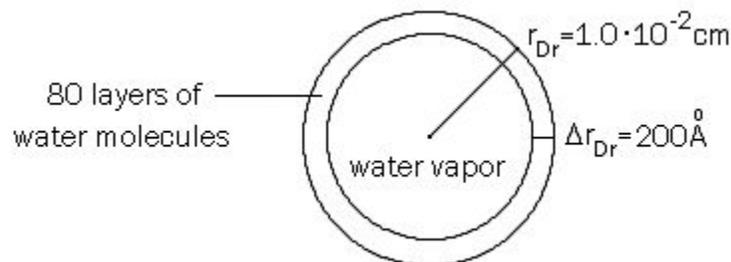
FIGURE 12.7
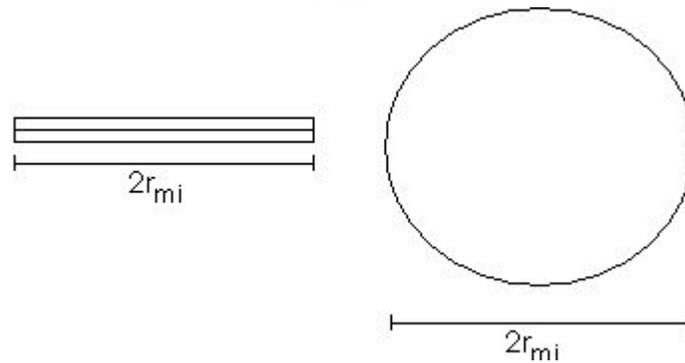
DOUBLE LAYERED MICELLE

The kinetic energy of the N_W water molecules of the micelle cause the micelle to form

FIGURE 12.8

WATER DROPLET

a hollow sphere filled with unbound water molecules at water vapor pressure inside the micelle. Figure 12.8

The radius of the droplets is r_{Dr}=(2)$^{-\frac{1}{2}}r_{mi}$ with surface area $4\pi r_{Dr}^2=2\pi \cdot r_{mi}^2$. The expansion takes $P_{atm}\Delta V$=10^6($\frac{4}{3}\pi \cdot r_{Dr}^3$) (erg). Using table 12.3 for a droplet of arbitrary radius: $P_{atm}\frac{\Delta V}{N_W}$=10^6(2.6·10^{-20})=2.6·10$^{-14}\frac{erg}{w}$=0.016$\frac{ev}{w}$.

Let be_{ir} ($\frac{erg}{W}$), $0<b<1$ represent the fraction of the infrared energy that escapes the boiling water let $2.6\cdot10^{-14}\cdot a$ ($\frac{erg}{W}$) represent the fraction of the $P_{atm}\Delta V$ expansion work that does not do work on the boiling water (e.g. by creating surface waves that move air). Balancing energy terms yields:

$(m_0\Delta C_1)_1+(m_H\Delta C_1)_2+3B.E._{w,w}+B.E._{OH,H}+B.E._{O,H}+be_{ir}+2.6\cdot10^{-14}\cdot a= e_V=0.730\cdot10^{-12}\frac{erg}{W}=$

$0.456\frac{ev}{W}$ where $(m_0\Delta C_1)_1+(m_H\Delta C_1)_2+3B.E._{w,w}+B.E._{OH,H}+B.E._{O,H} >> be_{ir}+2.6\cdot10^{-14}\cdot a$

With a density less than the density of the surrounding air, the droplet floats upward.

13. Appendix 12B Absolute Humidity

The absolute humidity is defined as the mass of water vapor per unit volume $A.H.=n_W m_W(\frac{gm}{cm^3})$, where n_W is the number of water molecules per unit volume and $m_W=3.01\cdot10^{-23}$gm is the mass of 1 water molecule. As measured in an ENCLOSED VESSEL designed to measure water vapor pressure, pure water vapor has no floating water droplets in it with $\rho_{air}=0.0012(\frac{gm}{cm^3})$. However by direct observation in the ATMOSPHERE, fogs at ground level near sea level are composed of floating water droplets. By inference, although there are no floating water droplets in an ENCLOSED VESSEL designed to measure water vapor pressure, there are floating water droplets in clouds and fogs. And although the floating water droplets do not substantially contribute to the water vapor pressure in the atmosphere, they do contribute substantially to the absolute humidity. e.g. If water droplets with $N_L=1$, (Table 12.3) compose 1% by volume of the atmosphere, then by direct computation there are $n_{Dr}=7.8\cdot10^8(\frac{Dr}{cm^3})$ and $n_{Dr}W_{Dr}=7.8\cdot10^8(2.7\cdot10^8)=2.1\cdot10^{17}$ water molecules per cm^3 bound in water droplets. And at saturation at 20^oC there are $1.7\cdot10^{-5}(\frac{gm}{cm^3})$ of water vapor composed of $n_{f.w.s.}=5.8\cdot10^{17}$free water molecules at saturation as measured in an ENCLOSED VESSEL designed to measure water vapor pressure. n_W becomes: $n_W=n_{Dr}W_{Dr}+n_{f.w.s}\cdot R.H.=2.1\cdot10^{17}+5.8\cdot10^{17}\cdot R.H.$ where R.H. is the relative humidity in decimal equivalent. Thus by measuring the dew point in air and using a table to determine the relative and absolute humidity, one would have missed the $2.1\cdot10^{17}$ water molecules bound up in floating water droplets and thus computed the wrong absolute humidity.

<div align="center">Reference</div>

12.1.   Milton Van Dyke, "An Album of Fluid Motion", The Parabolic Press, Stanford California, 1982