Chapter 8. Particle Accelerators and $\underset{\sim}{V} \times \underset{\sim}{B}$ Effects.

1. The Cyclotron

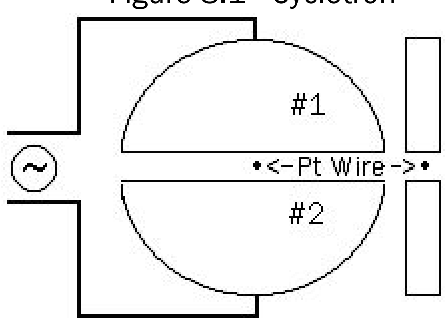
The cyclotron was developed in the years 1929-31 by E.O. Lawrence and M.S. Livingston. Reference 8.1. H_2 gas at a pressure of ~10^{-5} mm of Hg in the interior of two Cu Dees is irradiated by photons emitted by an electrically heated Pt wire. Figure 8.1. The irradiated H atoms are repeatedly accelerated in the y direction, figure 8.2, by an alternating potential applied to 2 electrodes. The atomic paths are bent into a spiral path with increasing radius by a magnetic field in the $-\hat{z}$ direction. With an alternating potential difference across the Dees of 2000V, after ~40 revolutions (Quoted number) about the center of the cyclotron, the H atoms have a translational kinetic energy of $1.6\cdot10^5$ev. in Lawrence and Livingston's first cyclotron. See Sec. 6. Appendix A. The Pt wire is not essential to the functioning of the cyclotron and thus non-irradiated H_2 molecules are also accelerated by the cyclotron.

It is of interest to note that for background gas pressures below a critical minimum, the beam power goes to zero watts. Thus the background gas is essential to the well functioning of the cyclotron. Ref. 1. A verbal description of the cause of the forces acting on the molecules in the interior of the Dees with in the context of the solid mass atom and the solid mass photon is given, followed by a mathematical analysis.

Verbal Description

The heated wire with $1000 \leq T \leq 2000^{0}$K, heats the hydrogen gas so that the

Figure 8.1 - Cyclotron



atoms that strike the wire have a $5.0\cdot10^5 \leq V_{rms} \leq 7.0\cdot10^5 \frac{cm}{sec}$, while those that don't, have a $V_{rms}=2.7\cdot10^5 \frac{cm}{sec}$: (Evaluated at $T=300^{0}$K). A 2000 volt potential is alternatively put on one then the other Dee. This causes the Cu atoms on the excited side of the Dee to emit photons into the gap.

The H atoms in the gap are struck by the photons and experience a force in the $\pm\hat{y}$ direction. Figure 8.2.

An iron core electromagnet with $\underset{\sim}{B}=-B\hat{z}$ emits photons created by the Fe core atoms which pass through the Cu Dees and into the interior of the Dees. The alternating voltage applied to the Dees cause the sides of the Dees to vibrate which causes circular pressure waves to form in the interior of the Dees centered on the center of the cyclotron. The pressure compresses the background gas into thin waves with density $\rho=\rho(\underset{\sim}{x},t)$. The B field photons passing through the atoms in the annular, circular pressure waves excite those atoms into radial oscillation. The radial oscillation causes the two H atoms of the H molecule to come apart with speed v_r. With radius r_s of the circular pressure waves greater than the radius r_H of the trajectory of the H beam, $r_s>r_H$, the explosion products striking the H atom beam cause the atoms of the beam to experience a force toward the center of the cyclotron. For a given B_o, the frequency of the voltage applied to the electrodes is adjusted so that the photons emitted by the electrodes strike the H beam in the direction of motion thus increasing the speed of the H atoms in the beam.

Mathematical Description

The power input to the cyclotron is $P_{in}=150W=9.4\cdot 10^{20}\frac{ev}{sec}$ with one or the other electrodes emitting photons into the gap: Figure 8.1. At $1.6\cdot 10^5$ev, the H atoms of the beam are moving at $U_F=5.8\cdot 10^8\frac{cm}{sec}$ with frequency $f=\frac{U_F}{\pi D}=5.8\cdot 10^8(9\pi)^{-1}=2.1\cdot 10^7$Hertz and time around the circumference $t_c=4.8\cdot 10^{-8}$sec. As shown below, with proper choice of B(x,y,t) where $\underset{\sim}{B}=-B\hat{z}$, f and t_c remain constant as a H atom accelerates around the center of the cyclotron with increasing radius and speed. This means that each beam H atom spends $\frac{1}{2}t_c=2.4\cdot 10^{-8}$ sec traversing half the circumference, independent of beam radius.

The particle frequency f is equal to the voltage frequency f_V: $f=f_V=2.1\cdot 10^7$Hertz.

The minimum time $\Delta\tau_{G_m}$ that a H beam atom spends in the gap is $\Delta\tau_{G_m}=\frac{d_G}{U_F}$ and the maximum time $\Delta\tau_{G_M}$ is $\Delta\tau_{G_M}=\frac{d_G}{V_{rms}}$ where d_G is the distance across the gap. With $d_G=0.2$cm, $U_F=5.8\cdot 10^8\frac{cm}{sec}$, and $U_{min}=2.7\cdot 10^5\frac{cm}{sec}$, (Evaluated at $T=300^oK$): $\Delta\tau_{G_m}=3.4\cdot 10^{-10}$sec. and $\Delta\tau_{G_M}=7.4\cdot 10^{-7}$sec.

The leading edge of the voltage pulse starts at $(0,-4.5,\pm\frac{1}{2})$ at t=0, fig. 8.2 and 8.3, with speed U_{LE} and reaches the gap at time $t_{LE}=\frac{4.4}{U_{LE}}=\frac{1}{2}t_c=2.4\cdot 10^{-8}$ sec. Solving for U_{LE}: $U_{LE}=1.8\cdot 10^8\frac{cm}{sec}$.

As the voltage pulse moves across the Dee, the leading edge of the voltage pulse pumps energy into the Cu atoms on the leading edge of the voltage pulse, increasing the Cu atoms internal energy and average radius, decreasing $\overline{R}$ and increasing the speed of the trailing edge of the voltage pulse. The trailing edge advances on the leading edge forming a narrow voltage pulse. The leading and trailing voltage edges are straight lines parallel to the x-axis.

The trailing edge of the voltage pulse starts at $(0,-4.5,\pm\frac{1}{2})$ at $\frac{1}{2}t_C=2.4\cdot10^{-8}$ sec with speed U_{TE} and reaches the gap at t_{TE} where $t_{TE}-t_{LE}=t_{TE}-2.4\cdot10^{-8}=\frac{4.4}{U_{TE}}$ and $U_{TE}=\frac{4.4}{t_{TE}-t_{LE}}\frac{cm}{sec}$. Apriori $t_{TE}-t_{LE}$ is not known, however in order that the voltage pulse in the #2 Dee does not retard the beam atoms traveling in the $-\hat{y}$ direction in the gap coming from the #1 Dee, it is required that $t_{TE}-t_{LE}<2.4\cdot10^{-8}$ sec. and therefore $U_{TE}>1.8\cdot10^{8}\frac{cm}{sec}$.

FIGURE 8.2

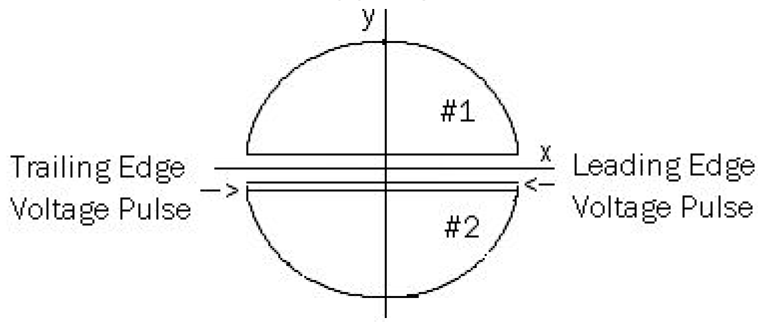


Figure 8.2 illustrates a voltage leading edge and trailing edge for $t_{TE}-t_{LE}<2.4\cdot10^{-8}$ sec.

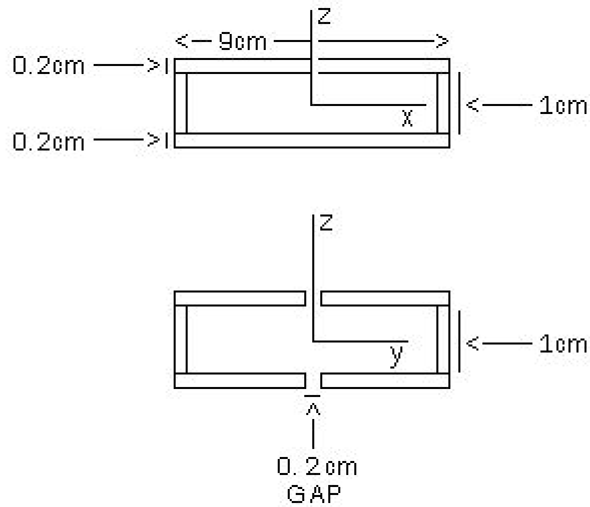
Let r represent the distance from (0,0,0) to (x,y,z) , where r_H is the distance of a H atom from the center of the cyclotron and $-4.5\le x\le4.5$, $-4.5\le y\le4.5$, $-\frac{1}{2}\le z\le\frac{1}{2}$.
At t=0 the cyclotron has been running long enough so that a beam H atom is at $(-x_1,-0.1,0)$, $x_1 \gtrsim 0.1$cm and is about to enter the #2 Dee, figure 8.2 and 8.3, with velocity

$\underset{\sim}{U}_H=U\hat{x}+V\hat{y}=U_hr_H(\sin\theta\,\hat{x}+\cos\theta\,\hat{y})$ where $|\underset{\sim}{U}_H|=U_H=\frac{r_H}{4.5}U_F=1.3\cdot10^{8}r_H=1.3\cdot10^{8}\{(x_1)^2+(0.1)^2\}^{\frac{1}{2}}\frac{cm}{sec}$, and $\underset{\sim}{r}_H=r_H(\cos\theta\,\hat{x}+\sin\theta\,\hat{y})=x_1\hat{x}-0.1\hat{y}$. U_H and r_H are constant for $\tan^{-1}\frac{0.1}{x_1}\le\theta\le\pi-\tan^{-1}\frac{0.1}{x_1}$ and $\pi+\tan^{-1}\frac{0.1}{x_1}\le\theta\le2\pi-\tan^{-1}\frac{0.1}{x_1}$ and are increasing for $0\le\theta<\tan^{-1}\frac{0.1}{x_1}$ and $2\pi-\tan^{-1}\frac{0.1}{x_1}<\theta\le2\pi$ and $\pi-\tan^{-1}\frac{0.1}{x_1}<\theta<\pi+\tan^{-1}\frac{0.1}{x_1}$ i.e. when the atom is in the gap.

The linearized model for the average speed of a H atom during one traverse of the gap is $\bar{U}_G$ where $\bar{U}_G=\frac{1}{2}(U_H+U_f)$ and $U_f=\{3.8\cdot10^{15}+U_H^2\}^{\frac{1}{2}}$. U_f is the speed at which the H atom leaves the gap and $\bar{U}_G$ is derived using the assumption that a H atom gains $2\cdot10^{3}$ev on each traverse of the gap.

Let t_{EG} represent the time that a H atom is about to enter the gap at $(x_1,-0.1,0)$, where $t_{EG}=2.4\cdot10^{-8}(2n+1)$ sec., n=0,1,2,...

FIGURE 8.3



CROSS SECTION CYCLOTRON

Let t_{LG} represent the time that a H atom leaves the gap at $(x_1', 0.1, 0)$, where

$t_{LG} \doteq 2.4\cdot 10^{-8}(2n+1)+\frac{[(\Delta x_H)^2+(0.2)^2]^{\frac{1}{2}}}{\bar{U}_G}$ with $t_{LG}-t_{EG}=\frac{[(\Delta x_H)^2+(0.2)^2]^{\frac{1}{2}}}{\bar{U}_G}$.

$\bar{U}_G \doteq \frac{1}{2}\{1.3\cdot 10^8 r_H+[3.8\cdot 10^{15}+1.3^2\cdot 10^{16} r_H^2]^{\frac{1}{2}}\}=\frac{1}{2}(1.3\cdot 10^8 r_H)\{1+[1+0.23 r_H^{-2}]^{\frac{1}{2}}\}$.

$\Delta x_H \equiv x_1' - x_1$ where $x_1'=\{(r_H')^2-(0.1)^2\}^{\frac{1}{2}}$ and $x_1=\{(r_H)^2-(0.1)^2\}^{\frac{1}{2}}$.

$\frac{r_H'}{4.5} U_F=U_f=\{3.8\cdot 10^{15}+(\frac{r_H}{4.5}U_F)^2\}^{\frac{1}{2}}$ and finally $r_H'=r_H\{1+3.8\cdot 10^{15}(\frac{r_H}{4.5}U_F)^2\}^{\frac{1}{2}}=r_H\{1+0.23(r_H)^{-2}\}^{\frac{1}{2}}$.

x_1' becomes: $x_1'=\{r_H^2\cdot\{1+0.23(r_H)^{-2}\}-(0.1)^2\}^{\frac{1}{2}}=r_H\{1+0.22(r_H)^{-2}\}^{\frac{1}{2}}$ and

$\Delta x_H=r_H[\{1+0.22(r_H)^{-2}\}^{\frac{1}{2}}-\{1+0.01(r_H)^{-2}\}^{\frac{1}{2}}]$. $\Delta r_H\equiv r_H'-r_H=r_H[\{1+0.23(r_H)^{-2}\}^{\frac{1}{2}}-1]$.

Table 8.1 lists computed values of $\Delta r_H\equiv r_H'-r_H$, Δx_H, $\bar{U}_G$ and $\Delta t_H\equiv t_{LG}-t_{EG}$ as a function of r_H.

TABLE 8.1

r_H(cm)	Δr_H(cm)	Δx_H(cm)	$\bar{U}_G(\frac{cm}{sec})$	Δt_H(sec)
0.10	0.38	0.34	$3.8\cdot 10^7$	$1.0\cdot 10^{-8}$
1.0	0.11	0.099	$1.4\cdot 10^8$	$1.6\cdot 10^{-9}$
2.0	0.057	0.052	$2.6\cdot 10^8$	$8.0\cdot 10^{-10}$
3.0	0.038	0.035	$3.9\cdot 10^8$	$5.1\cdot 10^{-10}$
4.0	0.029	0.026	$5.2\cdot 10^8$	$3.8\cdot 10^{-10}$
4.47	0.026	0.023	$5.8\cdot 10^8$	$3.4\cdot 10^{-10}$

With photon binding energy BE_{ph}, and translational kinetic energy KE_{ph}, the total energy to produce a photon emitted by the Dee's is $TE_{ph}=-BE_{ph}+KE_{ph}$.

The area of 1 Cu atom is $(2r_o)^2=4.8\cdot10^{-16}cm^2$ and with 2 emitting surfaces on each Dee, there are $N_o=7.5\cdot10^{15}$ Cu atoms on the gap edge of each Dee. If each of the N_o atoms receives total energy $TE_{Cu}\frac{ev}{atom}$ during the time interval $\Delta\tau$, $3.4\cdot10^{-10}\leq\Delta\tau\leq1.0\cdot10^{-8}$sec, during which all H beam atoms are in the gap then:

$TE_{Cu}=9.3\cdot10^{20}(7.5\cdot10^{15})^{-1}(2.4)10^{-8}=2.9\cdot10^{-3}\frac{ev}{atom}$. It is therefore surmised that due to sputtering, the profile of the gap becomes scalloped and the photons are emitted on the points of the scallops so that N_{en} atoms are energized and emit photons with $N_{en} < N_o$ and $TE_{Cu}=2.2\cdot10^{13}(N_{en})^{-1}\frac{ev}{atom}$.

If each of the N_{en} atoms emits N_{ph} photons during the $3.4\cdot10^{-10}$sec. at which the beam atoms are in the gap then the total energy TE_{ph} available to create 1 photon is:

8.1 $$TE_{ph}=2.2\cdot10^{13}(N_{ph}\cdot N_{en})^{-1}\frac{ev}{photon}.$$

And the photons are emitted in the $\hat{y}$ direction by one Dee at time t and in the $-\hat{y}$ direction by the other Dee at time $t+2.4\cdot10^{-8}$sec.

Let Δz represent the additive sum of the thickness of the two emitting edges of one D of the gap, The energized atoms are in a strip with area $A_1=4.8\cdot10^{-16}N_{en}=9.0\cdot\Delta z\ cm^2$ with $\Delta z=5.3\cdot10^{-17}N_{en}$ (cm). If $-BE_{ph}=KE_{ph}$, then $-BE_{ph}=KE_{ph}=1.1\cdot10^{13}(N_{ph}\cdot N_{en})^{-1}$(ev).

A derivation of the number of collisions N_H between the photons emitted by the electrodes and a H atom in order to yield a H atom with kinetic energy $KE_H=1.6\cdot10^5ev=2.6\cdot10^{-7}erg$ is given below.

We assume the photons produced by the electrodes are adsorbed by the H atoms in the beam and we assume the velocity of the photons $\underset{\sim}{U}_{ph}$ is $\pm U_{ph}\hat{y}$ and that the velocity of the H atoms $\underset{\sim}{V}_H$ in the beam is $\underset{\sim}{V}_H=\pm V_H\hat{y}$. The conservation of energy and momentum equations become:

8.2 a. $$m_{ph}U_{ph}^2+m_H(1+i\varepsilon)V_{H_i}^2= m_H(1+(1+i)\varepsilon)V_{H_{i+1}}^2,\ \varepsilon\equiv\frac{m_{ph}}{m_H},\quad i=0,1,2,3,...$$

b. $$\pm m_{ph}U_{ph}\hat{y}\pm m_H(1+i\varepsilon)V_{H_i}\hat{y}=\pm m_H(1+(1+i)\varepsilon)V_{H_{i+1}}\hat{y}$$

Where U_{ph} is the speed of the photon and V_{H_i} is the speed of the H atom with mass $m_H(1+i\varepsilon)$ after i collisions and $V_{H_{i+1}}$ is the final speed of the H atom with mass $m_H(1+(1+i)\varepsilon)$ after i+1 collisions where $2.7\cdot10^5\leq V_{H_i}\leq 5.8\cdot10^8\frac{cm}{sec}$.

Solving 8.2a for $V_{H_{i+1}}$ yields 8.3a and solving 8.2b for $V_{H_{i+1}}$ yields 8.3b.

8.3 a. $V_{H_{i+1}}=[(\varepsilon U_{ph}^2+(1+i\varepsilon)V_{H_i}^2)(1+(1+i)\varepsilon)^{-1}]^{\frac{1}{2}}$

b. $V_{H_{i+1}}=(\varepsilon U_{ph}+(1+i\varepsilon)V_{H_i})(1+(1+i)\varepsilon)^{-1}$

8.3 a and b are inconsistent with one another for $\varepsilon\neq0$ and consistent with one another for the uninteresting case $\varepsilon=0$. The inconsistency is a consequence of the fact that m_{ph} and m_H are not point particles. Taking into account the internal energy of the photon and H atom, 8.2a becomes,

8.3 c. $\varepsilon U_{ph}^2+2\varepsilon C_{1,ph}+(1+i\varepsilon)V_{H_i}^2+2C_{1,H_i}=(1+(1+i)\varepsilon)V_{H_{i+1}}^2+2C_{1,H_i+1}$

where $\varepsilon C_{1,ph}<0$, $C_{1,H_i}<0$ and $C_{1,H_i+1}<0$. See chap. 3 sec. 4.

For the solid mass atom and photon, translational KE of the masses apart in the amount Δe_{i+1} is transformed into internal kinetic + potential energy of the i^{th} collision product $\Delta e_{i+1}=-(\varepsilon C_{1,ph}+C_{1,H_i})+C_{1,H_i+1}>0$. Define $\Delta e_{i+1}\equiv\frac{1}{2}m_{ph}\phi_{i+1}^2$ in terms of the fictitious speed ϕ_{i+1}. Taking into account Δe_{i+1}, 8.3c and b become:

8.4 a. $\varepsilon U_{ph}^2+(1+i\varepsilon)V_{H_i}^2=(1+(1+i)\varepsilon)V_{H_{i+1}}^2+\varepsilon\phi_{i+1}^2$, $\varepsilon\equiv\frac{m_{ph}}{m_H}$, i=01,2,3,...

b. $\varepsilon U_{ph}+(1+i\varepsilon)V_{H_i}=(1+(1+i)\varepsilon)V_{H_{i+1}}$

When the two particles are apart; The average momentum of m_{ph} about the center of mass of m_{ph} is 0 and the average momentum of m_H about the center of mass of m_H is 0. And when m_{ph} is in the interior of m_H; The average momentum of m_{ph} about the center of mass of m_{ph} is 0 and the average momentum of m_H about the center of mass of m_H is 0. Consequently there is no $\varepsilon\phi_{i+1}$ term on the right hand side of 8.4b. Solve 8.4a and b for $V_{H_{i+1}}$ and set the 2 resultants equal to one another to yield the quadratic, $V_{H_i}^2-2U_{ph}\cdot V_{H_i}+U_{ph}^2-(1+(1+i)\varepsilon)(1+i\varepsilon)^{-1}\cdot\phi_{i+1}^2=0$ with solution:

8.4 c. $V_{H_i}=U_{ph}\pm(1+(1+i)\varepsilon)^{\frac{1}{2}}(1+i\varepsilon)^{-\frac{1}{2}}\cdot\phi_{i+1}$

Only the minus sign is of physical interest, see 8.5a. For large enough i: $i\varepsilon>>1$ and $i>>1$ and using 8.4c: $V_{H_i}\doteq U_{ph}-\phi_{i+1}$. On physical grounds, $\lim_{i\to\infty}V_{H_i}=U_{ph}$ and therefore $\lim_{i\to\infty}\phi_i=0$.

Solving for V_{H_i} with an $\varepsilon\cdot\phi_{i+1}$ on the right hand side of 8.4b results in a non-physical imaginary V_{H_i} and is not investigated further.

8.5a follows from 8.4c and using 8.4b, one can derive the recursive expression 8.5b with $\lim_{i\to\infty} V_{H_i}=U_{ph}$ in agreement with $\lim_{i\to\infty} V_{H_i}=U_{ph}$ derived using 8.5a.

8.5 a. $V_{H_i}=U_{ph}-(1+(1+i)\varepsilon)^{\frac{1}{2}}(1+i\varepsilon)^{-\frac{1}{2}}\cdot\phi_{i+1}$

b. $V_{H_i}=\left[i\varepsilon U_{ph}+V_{H_o}\right](1+i\varepsilon)^{-1}$

Directly solve for ϕ_{i+1} using 8.5a and b yielding:

8.6 a. $\phi_{i+1}=(U_{ph}-V_{H_o})\left[(1+i\varepsilon)(1+(1+i)\varepsilon)\right]^{-\frac{1}{2}}$

b. $\lim_{i\to\infty}\phi_{i+1}=0$

Using 8.5b, the translational kinetic energy and linear momentum of the H atom becomes:

8.7 a. $\frac{1}{2}m_H(1+i\varepsilon)^2 V_{H_i}^2=\frac{1}{2}m_H\left[i\varepsilon U_{ph}+V_{H_o}\right]^2$

b. $m_H(1+i\varepsilon)V_{H_i}=m_H\left[i\varepsilon U_{ph}+V_{H_o}\right]$

Finally, solving 8.5b for i yields:

8.8 $i=(V_{H_i}-V_{H_o})\left[(U_{ph}-V_{H_i})\varepsilon\right]^{-1}$

Set $i=N_H$ where N_H is the number of collisions between the photons emitted by the Dees and a H atom in order to yield a H atom with kinetic energy $KE_H=1.6\cdot10^5$ev$=2.6\cdot10^{-7}$erg. With $V_{H_{N_H}}=5.8\cdot10^8\frac{cm}{sec}$ and $V_{H_o}=10^6\frac{cm}{sec}$ and $V_{H_{N_H}}>>V_{H_o}$, N_H becomes:

$N_H\doteq5.8\cdot10^8\left[(U_{ph}-5.8\cdot10^8)\varepsilon\right]^{-1}$. Assuming that $\frac{1}{2}m_{ph}U_{ph}^2=2\cdot10^3ev=3.2\cdot10^{-9}$erg, (Note that in general $\Delta KE\neq q\Delta V$, see appendix A) ε becomes: $\varepsilon=m_{ph}(1.7\cdot10^{-24})^{-1}=3.8\cdot10^{15}[U_{ph}^2]^{-1}$ and:

8.9 a. $N_H\doteq\left[1.5\cdot10^{-7}U_{ph}\right]\left[1-5.8\cdot10^8(U_{ph})^{-1}\right]^{-1}$, $U_{ph}>5.8\cdot10^8\frac{cm}{sec}$

Table 8.2 lists N_H, ε, m_{ph} and $(1+N_H\varepsilon)$ as a function of U_{ph}. $m_{ph}(1+N_H\varepsilon)$ is the final mass of the H atom in gm, and $(1+N_H\varepsilon)$ is the final mass of the H atom in amu.

TABLE 8.2

$U_{ph}(\frac{cm}{sec})$	N_H(ph)	ε	m_{ph}(amu)	$(1+N_H\varepsilon)$(amu)
10^9	$3.6\cdot10^2$	$3.8\cdot10^{-3}$	$3.8\cdot10^{-3}$	2.4
10^{10}	$1.6\cdot10^3$	$3.8\cdot10^{-5}$	$3.8\cdot10^{-5}$	1.06
10^{11}	$1.5\cdot10^4$	$3.8\cdot10^{-7}$	$3.8\cdot10^{-7}$	1.006

The entry for $U_{ph}=10^9\frac{cm}{sec}$ is suspect as the mass of the H atom is growing appreciably as it is accelerated to its final speed. At its final speed, the mass of the H atom is 2.4amu and technically a constant frequency cyclotron only functions for particles of constant mass.

2. Beam Current

From reference 8.1, with a B field of $4.8\cdot10^3$gauss, a background gas pressure of $1\cdot10^{-5}$mmHg and 2000V between an electrode and ground, the measured beam current is $1.2\cdot10^{-10}$Amps.
With a KE_H for each hydrogen atom of $1.6\cdot10^5$ev=$2.6\cdot10^{-7}$ergs, the beam power P_b is $P_b=n_H\cdot KE_H=2.6\cdot10^{-7}n_H\frac{ergs}{sec}=2.6\cdot10^{-14}n_H$ (W) where n_H is the number of H atoms per second entering the detector. Although the measured electric power, P_{el}=IE, is not given: $P_{el}=1.2\cdot10^{-10}\cdot E$ (W) and with $E=8\cdot10^4$V, P_{el} becomes: $P_{el}=9.6\cdot10^{-6}$(W) and with $P_{el}=P_b$, n_H becomes, $n_H=3.7\cdot10^8$(Hydrogen Atoms per sec)
The total time that 1 H atom is in the beam is $40t_c=(40)4.8\cdot10^{-8}=1.9\cdot10^{-6}$sec. and at any instant of time there are $40t_cn_H=7.0\cdot10^2$ H atoms in a tight bunch and there are $\frac{n_H}{40t_cn_H}=5.3\cdot10^5\,\frac{bunches}{sec}$ entering the detector.

8.10 $\quad n_H=3.7\cdot10^8(\text{H atoms})(\text{sec})^{-1}=7.0\cdot10^2(\text{H atoms})(40t_c)^{-1}$

$P_{el}=P_b=9.6\cdot10^{-6}$(W)

Let $TE_{p,ph}$ represent the total energy needed to produce a photon with kinetic energy KE_{ph}. Using 8.1 and the assumption that $-BE_{ph}=KE_{ph}$ and that $KE_{ph}=2\cdot10^3$ev, yields: $-BE_{ph}=KE_{ph}=2\cdot10^3$ ev and $TE_{p,ph}=4\cdot10^3=2.2\cdot10^{13}(N_{ph}\cdot N_{en})^{-1}\frac{ev}{photon}$ where $N_{ph}\cdot N_{en}=5.5\cdot10^9$=the total number of photons emitted by 1Dee during the time interval $\Delta t_G=3.4\cdot10^{-10}$sec at which time all the $7.0\cdot10^2$ beam H atoms are in the gap.

Assuming $U_{ph}=10^{10}\frac{cm}{sec}$ and using table 8.2, each beam H atom requires an average of $(1.6\cdot10^{3})(80)^{-1}=20\,\frac{photons}{atom}$ collisions on every pass through the gap in order that each H atom attain $KE=1.6\cdot10^{5}$ev in 80 passes through the gap.

The number of energized Cu atoms N_{en} emitting photons into the gap is: $N_{en}=5.5\cdot10^{9}(N_{ph})^{-1}$. With $N_{ph}=20$, N_{en} becomes $N_{en}=2.7\cdot10^{8}<<7.5\cdot10^{15}$=# of Cu atoms on the gap of 1 Dee. From the paragraph below 8.1, $\Delta z=5.3\cdot10^{-17}N_{en}=1.4\cdot10^{-8}$cm i.e. All of the Cu atoms emitting 20 photons during the time interval $\Delta\tau$, $3.4\cdot10^{10}\leq\Delta\tau\leq1.0\cdot10^{-8}$sec, during which all of the 700 beam atoms are in the gap, lie in 2 strips, 9cm by $\frac{\Delta z}{2}=0.7\cdot10^{-8}$cm.

3. $\underset{\sim}{V}\times\underset{\sim}{B}$ Effects.

In order for the B field photons to interact with the background gas in the cyclotron, the mean free path λ_{ph} of the photons in the gas must be $\lambda_{ph}<1$cm. However with a background pressure of $P=10^{-5}$mmHg$=1.3\cdot10^{-2}\frac{dy}{cm^2}$ and with $T=300^{o}K$, $n_{H_2}=\frac{P}{KT}=3.1\cdot10^{11}(\frac{H_2}{cm^3})$ and $\lambda_{ph}>\frac{0.71}{\pi r_{H_2}^2 n_{H_2}}$ =73m where $r_{H_2}=1.0\cdot10^{-8}$cm is the radius of the Hydrogen molecule.

Solving for n_{H_2}, $n_{H_2}=\frac{0.71}{\pi r_{H_2}^2\lambda_{ph}}$ and in general $n_{H_2}=\frac{2.3\cdot10^{15}}{\lambda_{ph}}(\frac{H_2}{cm^3})$. In order that $\lambda_{ph}<1$cm, it is necessary that $n_{H_2}>2.3\cdot10^{15}(\frac{H_2}{cm^3})$.

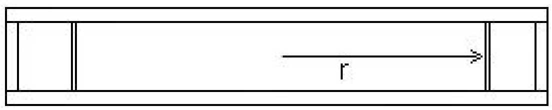
The volume of the cyclotron is $V_{cyl}=\pi\cdot4.5^{2}\cdot1=63cm^{3}$ and when operating contains $n_{H_2}\cdot V_{cyl}=2.0\cdot10^{13}$H molecules.

It is hypothesized that the electric current flowing through the coils of the electromagnet has the same frequency $f=2.1\cdot10^{7}$Hertz as that of the H beam and causes the Dees to vibrate with frequency $f=2.1\cdot10^{7}$Hertz. Further, the Dee vibration is a direct cause of a train of standing wave circular pressure pulses centered on the center of the cyclotron. Fig 8.4 illustrates one such wave.

With $T=400^{o}K$ and $\gamma\doteq1.7$: $c_S=1.6\cdot10^{5}\frac{cm}{sec}$ for molecular H. The time Δt_S for a circular compressive wave to traverse the diameter D of the cyclotron is: $\Delta t_S=\frac{D}{c_S}=5.6\cdot10^{-5}$sec with frequency $f_S=\frac{1}{\Delta t_S}=1.8\cdot10^{4}$ hertz. With $\Delta t_S=nt_c=(4.8\cdot10^{-8})n$ sec, solve for integer n: $n=1.2\cdot10^{3}$ standing waves. Solve for wavelength λ: $\lambda=\frac{9}{1.2\cdot10^{3}}=7.5\cdot10^{-3}$cm.

In order that the mean free path λ_{ph} of the magnetic field photons in the gas is $\lambda_{ph}=\frac{1}{2}$ cm, the number density of H molecules must be $n_{H_2}=6.4\cdot10^{15}(\frac{H_2}{cm^3})$. The volume of 1 standing wave is $V_{sw}=2\pi rh\Delta r_{sw}$ with $\Delta r_{sw}=\Delta r(r)_{sw}$ however assuming Δr_{sw} is a constant $\Delta r_{c,sw}$, the maximum value of V_{sw} is $V_{M,sw}=2\pi(4.5)(1)\Delta r_{c,sw}=28\Delta r_{sw}(cm)^3$ with average value $\overline{V}_{sw}=\frac{1}{2}V_{M,sw}=14\Delta r_{c,sw}$. The total number of H molecules N_{H_2} in $1.2\cdot10^3$ standing waves is $N_{H_2}=1.2\cdot10^3 n_{H_2}\cdot\overline{V}_{sw}=7.7\cdot10^{19}\cdot\Delta r_{c,sw}$ H molecules. Assuming the total number of H molecules in the standing waves equals the number of background H molecules $N_{H_2}=2.0\cdot10^{13}=7.7\cdot10^{19}\cdot\Delta r_{c,sw}$, $\Delta r_{c,sw}$ becomes $\Delta r_{c,sw}=2.6\cdot10^{-7}$cm.

FIGURE 8.4



Cyclotron With Compressive Wave at r

The magnetic field is composed of photons traveling in the $-\hat{z}$ direction with incoming number density $\sigma_B\frac{photons}{sec cm^2}$ where $\sigma_B=\sigma_B(r,t)=\sigma_B(r)\cdot\{\delta[h(t)+\varepsilon(r)-r]\}$, $r\neq r(t)$, $\delta[h(t)+\varepsilon(r)-r]=1$ for $h(t)+\varepsilon(r)=r$ and $\delta[h(t)+\varepsilon(r)-r]=0$ for $h(t)+\varepsilon(r)\neq r$ and $\frac{\varepsilon(r)}{h(t)}<<1$. $0\leq t\leq t_o=4.8\cdot10^{-8}$sec. with $h(0)=0$ and $h(t_o)=4.5$cm. h(t) equals $h(t)=\int_0^t V(\tau)_{r,H}d\tau$, where $V(\tau)_{r,H}$ is the radial speed of the bunched H atoms of the cyclotron beam. Fig. 8.5. For $h(t+t_o)$,

$h(t+t_o)=4.5-\int_0^t V(\tau)_{r,H}d\tau$ where as above, $0\leq t\leq t_o=4.8\cdot10^{-8}$sec. For $t+t_o=2t_o$, the cycle with period $2t_o$ repeats itself and the clock is set back to 0 at $t=2t_o$.

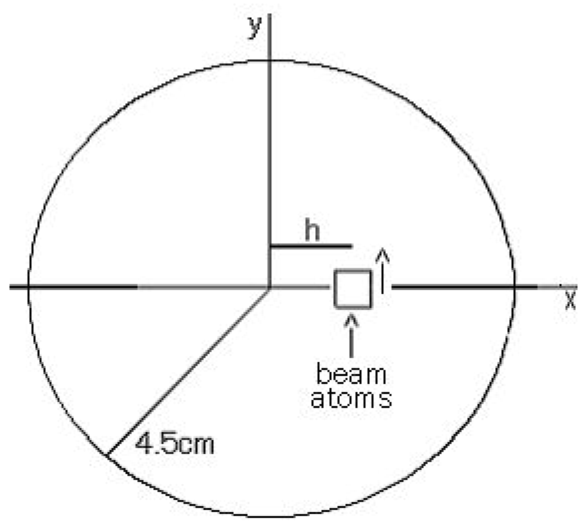
For circular particle paths with constant beam particle frequency f_V, the speed of a H atom in the beam is $V_H=2\pi h(t)f_V$, h(t) as above. The magnitude of the force acting on an individual H atom is:

8.11 $$F(h)_H=\frac{m_H V_H^2}{h}=m_H(2\pi f_V)^2 h=2.9\cdot10^{-8}\cdot h(dy)$$

where F_H is caused by the penetration of magnetic field photons and subsequent excitation and increase in C_1 of the background H atoms of the H_2 molecules in the standing waves. For large enough C_1, H_2 —>2H and the explosion products striking the beam H atoms are the direct cause of $F(h)_H$ where the beam atoms are at h and the H_2 molecules in the standing waves are at $h+\varepsilon(r)$.

Classically the Lorentz Force Equation applied to the cyclotron is: $Bev=\frac{m_H V_H^2}{h}$ with particle period $t_c=\frac{2\pi r}{v}=\frac{2\pi m}{Be}$. For both the small mass magnetic field photon model and the electromagnetic Lorentz Force model, $Be=const\equiv a_o$ and $a_o=\frac{m_H V_H}{h}$. Evaluating at h=4.5cm, $a_o=2.2\cdot 10^{-16}\frac{gm}{sec}$. Formally however, Bev is due to collisions between the beam atoms and the explosion H products of the H_2 molecules in the standing waves and not to an electromagnetic wave.

FIGURE 8.5 Cyclotron Cross Section for z=0



An order of magnitude plausibility computation follows to determine $F(h)_H$ given that the electric power input to the magnetic field coils is $10^3(W)=6.24\cdot 10^{14}(\frac{ev}{sec})$. With total energy to create a photon $KE_{ph}+|BE_{ph}|=E_{ph}(ev)$, the magnetic coils are generating $\frac{6.24\cdot 10^{14}}{E_{ph}}(\frac{photons}{sec})$. The photons are injected into the cyclotron in an area $\Delta A_{ph}\doteq 2\pi r\Delta r$ for $0<\frac{\Delta r}{r}<<1$ and $\sigma_B\frac{photons}{seccm^2}$ becomes: $\sigma_B=\frac{6.2\cdot 10^{14}}{2\pi r\Delta r\, E_{ph}}\cdot\{\delta[h(t)+\varepsilon(r)-r]\}\doteq 1.0\cdot\frac{10^{14}}{r\Delta r\, E_{ph}}\cdot\{\delta[h(t)+\varepsilon(r)-r]\}$ where $\frac{\varepsilon(r)}{h(t)}<<1$,. Δr is of size sufficient to allow the

requisite number of $\frac{photons}{sec}$ to excite and cause the H molecules in the standing waves to decompose i.e. H_2–>2H with sufficient kinetic energy to generate $F(h)_H$ by direct collision with the H atoms of the cyclotron beam as computed below.
The total number impinging on a peaked standing wave is:

$2\pi r\Delta r_{sw}\sigma_B=\frac{6.2\cdot 10^{14}}{E_{ph}}(\frac{\Delta r_{sw}}{\Delta r})\cdot\{\delta[h(t)+\varepsilon(r)-r]\}=\frac{1.6\cdot 10^8}{\Delta r\, E_{ph}}\cdot\{\delta[h(t)+\varepsilon(r)-r]\}\leq\frac{6.2\cdot 10^{14}}{E_{ph}}(\frac{photons}{sec})$,

valid for $\frac{\Delta r}{r}$<<1 with $\frac{\Delta r_{sw}}{\Delta r}\leq 1$. For $\frac{\Delta r}{r}$<<1 with $\frac{\Delta r_{sw}}{\Delta r}>1$: $2\pi r\Delta r_{sw}\sigma_B=\frac{6.2\cdot 10^{14}}{E_{ph}}(\frac{photons}{sec})$.

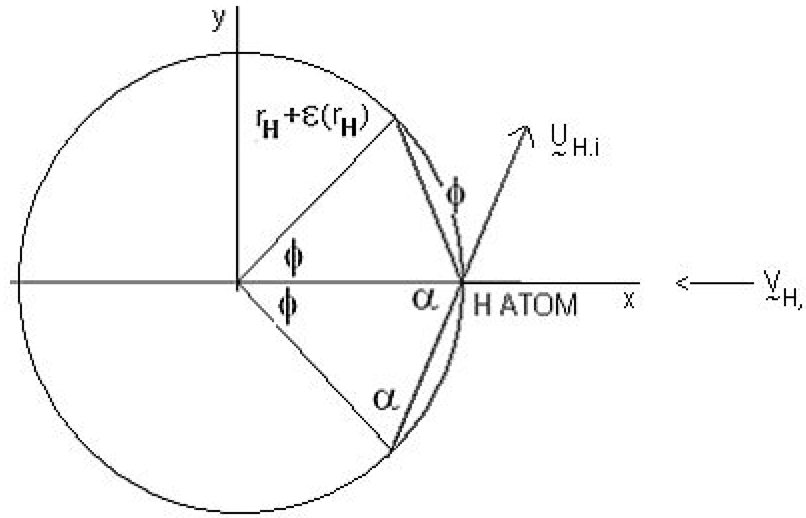
The published molecular binding energy of H_2 is equal to 5ev with equivalent temperature 39,000(^oK). 5ev is based on the false assumption that electromagnetic radiation consists of waves. As a trial, we will use 0.1ev as the molecular binding energy of H_2 with disassociation temperature 800(^oK).

Assuming the magnetic field photon looses $10^{-n_L}KE_{ph}$(ev) on passage through 1 H atom and that 1 photon strikes 2 H atoms on passage through the cyclotron, then the photon looses $10^{-n_L}KE_{ph}$ on passage through the 1st H atom and $(1-10^{-n_L})10^{-n_L}KE_{ph}$ on passage through the 2nd H atom. To disassociate 1 H_2 molecule requires $\frac{0.1}{10^{-n_L}KE_{ph}}=\frac{10^{(n_L-1)}}{KE_{ph}}$ (photons) and to disassociate a 2nd H_2 molecule with the same photon requires $\frac{0.1}{(1-10^{-n_L})10^{-n_L}KE_{ph}}=(1-10^{-n_L})^{-1}\cdot\frac{10^{(n_L-1)}}{KE_{ph}}$ (photons). 1 photon will disassociate $10^{(1-n_L)}\cdot(2-10^{-n_L})KE_{ph}$ H_2 molecules, and with $E_{ph}=2KE_{ph}$, $2\pi r\Delta r_{sw}\sigma_B(\frac{photons}{sec})$ will disassociate:

$\frac{0.8\cdot 10^8}{\Delta r}\cdot 10^{(1-n_L)}\cdot(2-10^{-n_L})\cdot\{\delta[h(t)+\varepsilon(r)-r]\}\leq 3.1\cdot 10^{(15-n_L)}\cdot(2-10^{-n_L})(\frac{H_2}{second})$.

Consider now as a thought experiment a beam H atom at r_H moving with speed U_H fig. 8.6, in zero magnetic field; Further, a 2 dimensional reflecting wall exists at $r_H+\varepsilon(r_H)$ with dimensions $2\pi(r_H+\varepsilon(r_H))$ by 1cm with $\frac{\varepsilon(r_H)}{r_H}$<<1. Every collision between the beam H atom and the wall results in the beam H atom being knocked into a new trajectory with angular deflection ϕ with respect to the old trajectory. Between collisions, the beam atom travels in straight lines. For some N_H, $N_H\phi=360^o$. If N_H is an integer, the chord lines form a regular polygon and after N_H strikes, the H atom regains its starting point. If N_H is not an integer, then for small enough ϕ, the beam H atom will strike the wall N_H times before once again ~ regaining its starting point.

FIGURE 8.6



If instead of the wall, as a beam H atom approaches $(r_H+\varepsilon(r_H))\hat{x}$ with velocity $\underset{\sim}{U}_{H,i}=U_{x,i}\hat{x}+U_{y,i}\hat{y}$, it is struck by an H atom explosion product moving along the x axis with velocity $-V_{x,i}\hat{x}$, $V_{x,i}>0$, such that the final beam H atom velocity is reflected, i.e. $\underset{\sim}{U}_{H,f}=-U_{x,i}\hat{x}+U_{y,i}\hat{y}$, fig. 8.6, then it can be shown using the conservation of momentum and energy equations that $V_{x,i}\hat{x}$ must be $V_{x,i}\hat{x}=-U_{x,i}\hat{x}$ and that the final explosion product H atom must have $\underset{\sim}{V}_{H,f}=V_{x,f}\hat{x}+V_{y,f}\hat{y}$, with $V_{x,f}\hat{x}=U_{x,i}\hat{x}$ and $V_{y,f}=0$.

If the H beam atom at $(r_H+\varepsilon(r_H))\hat{x}$ with velocity $\underset{\sim}{U}_{H,i}=U_{x,i}\hat{x}+U_{y,i}\hat{y}$, is struck by n_H, H atom explosion products moving along the x axis with average velocity $\underset{\sim}{V}_{H,i}=-V_{x,i}\hat{x}$, $V_{x,i}>0$, such that the final beam H atom velocity is reflected, i.e. $\underset{\sim}{U}_{H,f}=-U_{x,i}\hat{x}+U_{y,i}\hat{y}$, fig. 8.6, then it can be shown using the conservation of momentum and energy equations that $V_{x,i}$ must be $n_HV_{x,i}\hat{x}=-U_{x,i}\hat{x}$ and that the final explosion product H atom must have $\underset{\sim}{V}_{H,f}=V_{x,f}\hat{x}+V_{y,f}\hat{y}$, with $n_HV_{x,f}\hat{x}=U_{x,i}\hat{x}$ and $V_{y,f}=0$.

In the above $U_{y,i}$ may be written, $U_{y,i}=U_{x,i}\tan\alpha$ with $\underset{\sim}{U}_{H,i}=U_{x,i}(\hat{x}+\tan\alpha\hat{y})$, and from above $\phi=\frac{360^{\circ}}{N_H}$ with $\alpha=90-\frac{\phi}{2}=90\frac{(N_H-2)}{N_H}$

At $\phi=180^{\circ}$, U_H increases so we will compute $\frac{1}{2}N_H$ instead of N_H. After striking the heater wire at $T=1000^{\circ}K$, a H atom has $U_{H_{min}}=5\cdot10^{5}(\frac{cm}{sec})$ and with $U_{H_{Max}}=5.8\cdot10^{8}(\frac{cm}{sec})$ and $\underset{\sim}{V}_{H,i}=-10^{5}\hat{x}\,(\frac{cm}{sec})$, $\frac{1}{2}N_H$ and n_H are computed as a function of U_H, r_H and ϕ and listed in Table 8.3.

TABLE 8.3

$U_H(\frac{cm}{sec})$	r_H(cm)	α^o	$\frac{1}{2}N_H$	n_H	$U_H(\frac{cm}{sec})$	r_H(cm)	α^o	$\frac{1}{2}N_H$	n_H
$1.3\cdot10^7$	0.1	85	18	11	$1.3\cdot10^7$	0.1	89.5	180	1.1
$1.3\cdot10^8$	1	85	18	110	$1.3\cdot10^8$	1	89.5	180	11
$2.6\cdot10^8$	2	85	18	220	$2.6\cdot10^8$	2	89.5	180	22
$3.9\cdot10^8$	3	85	18	330	$3.9\cdot10^8$	3	89.5	180	33
$5.2\cdot10^8$	4	85	18	440	$5.2\cdot10^8$	4	89.5	180	44
$5.8\cdot10^8$	4.5	85	18	500	$5.8\cdot10^8$	4.5	89.5	180	50
$U_H(\frac{cm}{sec})$	r_H(cm)	α^o	$\frac{1}{2}N_H$	n_H	$U_H(\frac{cm}{sec})$	r_H(cm)	α^o	$\frac{1}{2}N_H$	n_H
$1.3\cdot10^7$	0.1	89.95	1800	0.11	$1.3\cdot10^7$	0.1	89.995	$1.8\cdot10^4$	0.011
$1.3\cdot10^8$	1	89.95	1800	1.1	$1.3\cdot10^8$	1	89.995	$1.8\cdot10^4$	0.11
$2.6\cdot10^8$	2	89.95	1800	2.2	$2.6\cdot10^8$	2	89.995	$1.8\cdot10^4$	0.22
$3.9\cdot10^8$	3	89.95	1800	3.3	$3.9\cdot10^8$	3	89.995	$1.8\cdot10^4$	0.33
$5.2\cdot10^8$	4	89.95	1800	4.4	$5.2\cdot10^8$	4	89.995	$1.8\cdot10^4$	0.44
$5.8\cdot10^8$	4.5	89.95	1800	5.0	$5.8\cdot10^8$	4.5	89.995	$1.8\cdot10^4$	0.50

Note that the total number of explosive H atoms, n_T, necessary for a beam current H atom to complete a polygonal circuit of 180^o is a constant independent of α^o for a given r_H where $n_T=\frac{1}{2}N_H\cdot n_H$ with maximum number $9\cdot10^3$ for r_H=4.5cm.
The magnet generates ex_H explosion H atoms per sec. in an annular volume of dimensions $2\pi(r_H+\varepsilon(r_H))(\Delta r_{c,sw})$(height) where $\Delta r_{c,sw}=2.6\cdot10^{-7}$cm. and cyclotron height =1cm. From above using $\Delta r=\Delta r_{c,sw}=2.6\cdot10^{-7}$cm. and $h(t)+\varepsilon(r)=r$, ex_H becomes: $ex_H=3.1\cdot10^{(15-n_L)}\cdot(2-10^{-n_L})\,(\frac{H_2}{second})$.
From sec. 1, the time $\frac{1}{2}t_C$ for a beam H atom to go from $\phi=0^o$ to $\phi=180^o$ is $\frac{1}{2}t_C=2.4\cdot10^{-8}$ sec. During $\frac{1}{2}t_C$, half of ex_H have a component in the $-\hat{r}$ direction and half of $(\frac{1}{2})ex_H$ have $0^o\le\phi\le180^o$ so that $(\frac{1}{4})(\frac{1}{2}t_C)ex_H=(\frac{1}{4})3.1\cdot10^{(15-n_L)}\cdot(2-10^{-n_L})(\frac{1}{2}t_C)$ explosion product H atoms have the possibility of striking a beam atom. With $n_L=1$: $(\frac{1}{4})(\frac{1}{2}t_C)ex_H=3.5\cdot10^6$(H). From 8.10, with 700 H atoms in the beam, there are $5\cdot10^3$ explosion product H atoms for every beam H atom as the beam H atom goes from $\phi=0^o$ to $\phi=180^o$. The probability of a collision between explosion product H atoms and beam H atoms is increased by the attractive force between explosion product H atoms and beam H atoms.

It should be noted that the figure $5\cdot10^3$ explosion product H atoms for every beam H atom is derived on the assumption that the power input to the magnet is 10^3 W. If the correct figure is 10^4 W, then there are $5\cdot10^4$ explosion product H atoms for every beam H atom as the beam H atom goes from $\phi=0^o$ to $\phi=180^o$ etc.

4. Appendix A

Consider 2 H atoms entering the gap in a cyclotron at the same time with initial speeds $V_{H_o,2}>V_{H_o,1}$. The 1 and 2 stand for atoms #1 and #2 respectively. #2 is struck by N_2 photons before leaving the gap with speed $V_{H_{N_2},2}$ and #1 is struck by N_1 photons before leaving the gap with speed $V_{H_{N_1},1}$.

A basic electrodynamical assumption is that $\Delta KE=q\Delta V$ where ΔKE is the change in kinetic energy of a “charged” particle across distance s_o, q is the charge and ΔV is the voltage drop across distance s_o. Within the context of the assumption that the electric field is composed of small mass photons it will be shown by constructing a counter example, that in general, $\Delta KE\neq q\Delta V$.

Using 8.7a, $\Delta KE_1=\frac{1}{2}m_H(1+i\varepsilon)^2(V_{H_i,1})^2-\frac{1}{2}m_H(V_{H_o,1})^2=\frac{1}{2}m_H i\varepsilon U_{ph}\left[i\varepsilon U_{ph}+2V_{H_o,1}\right]$ and $\Delta KE_2=\frac{1}{2}m_H(1+j\varepsilon)^2(V_{H_j,2})^2-\frac{1}{2}m_H(V_{H_o,2})^2=\frac{1}{2}m_H j\varepsilon U_{ph}\left[j\varepsilon U_{ph}+2V_{H_o,2}\right]$ where $V_{H_o,2}>V_{H_o,1}$ and $\varepsilon\equiv\frac{m_{ph}}{m_H}<<1$. With $i\varepsilon$ and $j\varepsilon$ large enough and $V_{H_o,1}$, and $V_{H_o,2}$ small enough, $\left[i\varepsilon U_{ph}+2V_{H_o,1}\right]\doteq i\varepsilon U_{ph}$ and $\left[j\varepsilon U_{ph}+2V_{H_o,2}\right]\doteq j\varepsilon U_{ph}$.

Assuming $\Delta KE_1=\Delta KE_2$, yields $i\varepsilon U_{ph}=j\varepsilon U_{ph}$ and i=j.

Given that the two atoms are struck by photons at the rate of n_o photons per second and that #2 crosses the gap of width s_o in time t_2 after being struck by j photons: Therefore $j=n_ot_2$ and $i=j=n_ot_2$. However the position of #1 at t_2 is less than s_o and therefore when #1 is at s_o, $\Delta KE(s_o)_1>\Delta KE(s_o)_2$. QED

Reference

M. S. Livingston, Particle Accelerators, p136, (McGraw-Hill Book Company, 1962)