1. Mathematical Theory of Turbulence and the $(U \cdot \nabla)U$ Term

The mathematical theory of fluid turbulence is based on the $(U \cdot \nabla)U$ term as derived in the Navier-Stokes Equations. It is proved below that this term does not represent physical reality and therefore a new theory is necessary to explain fluid turbulence.

In the derivation of the Navier-Stokes Equations, the assumption is made that the coordinates, velocity and acceleration of a moving point with respect to an inertial frame may be written as $x = x(x_0, t)$, $x(x_0, 0) = x_0$, $U = \frac{dx}{dt}$ and $a = \frac{d^2x}{dt^2}$. i.e. The derivation explicitly assumes that x,y,z are functions of t.

With $U = U[x(x_0,t), y(y_0,t), z(z_0,t), t] = U_x \hat{x} + U_y \hat{y} + U_z \hat{z}$. Take the derivative of U w.r.t. t:

12.1 $$\frac{dU}{dt} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x}\frac{dx}{dt} + \frac{\partial U}{\partial y}\frac{dy}{dt} + \frac{\partial U}{\partial z}\frac{dz}{dt} = \frac{\partial U}{\partial t} + (U \cdot \nabla)U$$

Although this expression is mathematically correct, it is not physically correct.

Newton's Laws as exemplified by $\rho(x,t) \cdot \frac{dU}{dt} = -\nabla P$, or $F = \frac{d}{dt}(mU)$ are valid only if position, velocity and acceleration are measured from a frame S which is inertial as determined by on board accelerometers at rest in S. To be physically correct the x occurring in $U = \frac{dx}{dt}$ must be stationary in inertial frame S and therefore independent of time t as measured by observers at rest w.r.t. clocks at rest in S. The operational definition of U is thus inconsistent with the mathematical definition of U i.e. $U = \frac{dx}{dt}(x_0, t)$. The physically correct expression for U is $U = \frac{dh}{dt}$ where h is defined below.

To correct $U = \frac{dx}{dt}(x_0, t)$, represent a moving point by any other symbol than x,y,z in $x = x(x_0, t)$, $x(x_0, 0) = x_0$. Let the function $h = (h_x, h_y, h_z)$ represent the position of a moving point in rectangular coordinates where h_x, h_y, h_z are linear lengths in the $\hat{x}, \hat{y}, \hat{z}$ directions respectively.

12.2 $h_x = x + \chi_x(x,y,z,t)$ with $\chi_x(x,y,z,0) = 0$.
12.3 $h_y = y + \chi_y(x,y,z,t)$ with $\chi_y(x,y,z,0) = 0$.
12.4 $h_z = z + \chi_z(x,y,z,t)$ with $\chi_z(x,y,z,0) = 0$.

x,y,z, are stationary in S and independent of t: $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$.

The velocity of the moving point is $\underset{\sim}{U}=\dfrac{d\underset{\sim}{h}}{dt}=\dfrac{\partial h}{\partial t}_x\hat{x}+\dfrac{\partial h}{\partial t}_y\hat{y}+\dfrac{\partial h}{\partial t}_z\hat{z}$ and the acceleration

of the moving point is $\underset{\sim}{a}=\dfrac{\partial\underset{\sim}{U}}{\partial t}=\dfrac{\partial^2\underset{\sim}{h}}{\partial t^2}=\dfrac{\partial^2 h_x}{\partial t^2}\hat{x}+\dfrac{\partial^2 h_y}{\partial t^2}\hat{y}+\dfrac{\partial^2 h_z}{\partial t^2}\hat{z}$ with:

12.5 $\dfrac{\partial h}{\partial t}_x=\dfrac{\partial \chi}{\partial t}_x(x,y,z,t)$ and $\dfrac{\partial^2 h_x}{\partial t^2}=\dfrac{\partial^2 \chi_x(x,y,z,t)}{\partial t^2}$

12.6 $\dfrac{\partial h}{\partial t}_y=\dfrac{\partial \chi}{\partial t}_y(x,y,z,t)$ and $\dfrac{\partial^2 h_y}{\partial t^2}=\dfrac{\partial^2 \chi_y(x,y,z,t)}{\partial t^2}$

12.7 $\dfrac{\partial h}{\partial t}_z=\dfrac{\partial \chi}{\partial t}_z(x,y,z,t)$ and $\dfrac{\partial^2 h_z}{\partial t^2}=\dfrac{\partial^2 \chi_z(x,y,z,t)}{\partial t^2}$

and $\underset{\sim}{a}\neq\dfrac{\partial\underset{\sim}{U}}{\partial t}+(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$. The modified form of the Navier Stokes Equation in Eulerian Coordinates becomes:

12.8 $\rho(\underset{\sim}{h})\cdot\dfrac{d\underset{\sim}{U}(\underset{\sim}{h})}{dt}=-\nabla P=\sum_i \underset{\sim}{F}_{ct}(\underset{\sim}{h})_i+\sum_j \underset{\sim}{F}_{fi}(\underset{\sim}{h})_j$ where $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$ and $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$.

$\sum_i \underset{\sim}{F}_{ct}(\underset{\sim}{h})_i$ represents the sum of the externally applied contact forces, (viscosity in $\dfrac{dyn}{cm^3}$), acting at point $\underset{\sim}{h}(\underset{\sim}{x},t)$ at time t and $\sum_j \underset{\sim}{F}_{fi}(\underset{\sim}{h})_j$ represents the sum of the field forces, ($\dfrac{dyn}{cm^3}$), acting at point $\underset{\sim}{h}(\underset{\sim}{x},t)$ at time t where $\underset{\sim}{x}\neq\underset{\sim}{f}(t)$ and $\dfrac{d\underset{\sim}{U}}{dt}=\dfrac{\partial\underset{\sim}{U}}{\partial t}\neq\dfrac{\partial\underset{\sim}{U}}{\partial t}+(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$.
To sum up. In Eulerian Coordinates which yield velocity and acceleration following the flow,

12.9 $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$, $\underset{\sim}{U}=\dot{\underset{\sim}{h}}=\dot{\underset{\sim}{\chi}}(\underset{\sim}{x},t)$, $\dot{\underset{\sim}{U}}=\ddot{\underset{\sim}{h}}=\ddot{\underset{\sim}{\chi}}(\underset{\sim}{x},t)$, $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$, $\underset{\sim}{x}\neq\underset{\sim}{f}(t)$.

Newton's 3rd Law becomes:

12.10 $\underset{\sim}{F}=\dfrac{d}{dt}(m\underset{\sim}{U}(\underset{\sim}{h}))$ where $\underset{\sim}{h}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$ and $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$.

Lagrangian Coordinates yield velocity and acceleration at a fixed point in inertial frame S. Let $\underset{\sim}{x}_0$ be the fixed point is S. We wish to find the velocity and acceleration at $\underset{\sim}{x}_0$, for a given fixed time t_0. Using Eulerian coordinates: $\underset{\sim}{h}=\underset{\sim}{x}_1+\underset{\sim}{\chi}(\underset{\sim}{x}_1,t)$, $\underset{\sim}{\chi}(\underset{\sim}{x}_1,0)=0$, where $\underset{\sim}{\chi}(\underset{\sim}{x}_1,t)$ is as defined above and $\underset{\sim}{x}_1$ satisfies the following. At time t_0 the fluid parcel that was at $\underset{\sim}{x}_1$ at t=0 is at $\underset{\sim}{x}_0$: $\underset{\sim}{x}_0=\underset{\sim}{x}_1+\underset{\sim}{\chi}(\underset{\sim}{x}_1,t_0)$. Define $\underset{\sim}{f}$ by

$\underset{\sim}{f}(\underset{\sim}{x}_1, \underset{\sim}{x}_o, t_o) \equiv \underset{\sim}{x}_1 + \underset{\sim}{\chi}(\underset{\sim}{x}_1, t_o) - \underset{\sim}{x}_o = 0$. Solve $\underset{\sim}{f}$ for $\underset{\sim}{x}_1$ yielding $\underset{\sim}{x}_1 = \underset{\sim}{g}(\underset{\sim}{x}_o, t_o)$ where $\underset{\sim}{f}(\underset{\sim}{g}(\underset{\sim}{x}_o, t_o), \underset{\sim}{x}_o, t_o) = 0$.

The Lagrangian coordinates for velocity and acceleration are:

12.11 $\qquad \underset{\sim}{U} = \dot{\underset{\sim}{h}} = \dot{\underset{\sim}{\chi}}[\underset{\sim}{g}(\underset{\sim}{x}_o, t_o), t_o], \ \dot{\underset{\sim}{U}} = \ddot{\underset{\sim}{h}} = \ddot{\underset{\sim}{\chi}}[\underset{\sim}{g}(\underset{\sim}{x}_o, t_o), t_o].$

Eqns. 12.11, represent a point-to-point solution for velocity and acceleration. Specifically 12.11 represent the velocity and acceleration at $\underset{\sim}{x}_o$, for time t_o.

In the following sections, a new theory for the physical cause of fluid turbulence in water and air is developed based on a new model for the structure of liquid water and the breakup of water clathrates in water and water droplets in air.

2. Water Molecule Dimension

It is assumed in the derivation of the Navier-Stokes Equations that solids and liquids are made of continuous matter. It is commonly thought among researchers in fluid flow that no major new physics will emerge if one considers liquids and solids to be made up of atoms instead of continuous matter. In what follows it will be shown that this is not the case, and that major new physics emerges if one considers liquids and solids to be made up of atoms instead of continuous matter.

Consider a closed container of liquid water in pressure equilibrium with an atmosphere of air at $T = 293^oK = 20^oC$, $P = 10^6 \frac{dy}{cm}2$ and water vapor pressure $P_V = 2.34 \cdot 10^4 \frac{dy}{cm}2$ at rest on the earth's surface. Figure 12.1.

Rapidly removing the air above water in a closed container causes the water to explosively boil. It is external pressure and not surface tension that cause water to remain in the liquid state. It will be shown that if water molecules did not have an attractive force binding one water molecule to another, then all the liquid water would rapidly boil off. This is not the case and therefore there is an attractive force binding one water molecule to its neighbors. The resultant molecular structure is called a clathrate.

FIGURE 12.1

AIR AT $T = 20^oC$

$P_V = 2.34 \cdot 10^4 \frac{dy}{cm}2$

← SEALED CONTAINER
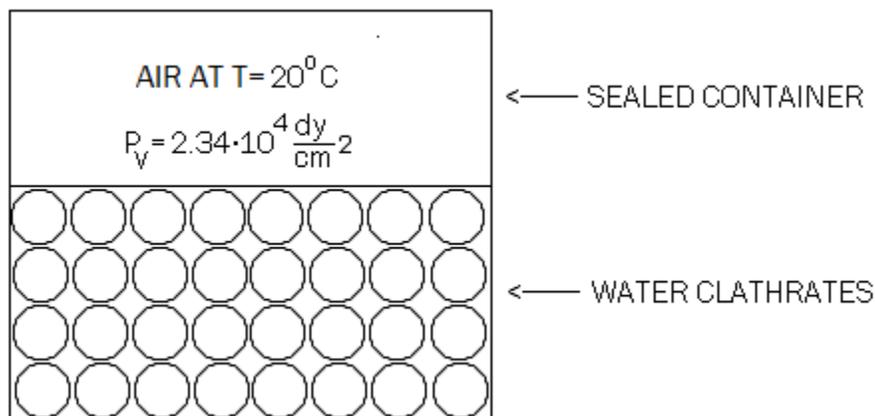
← WATER CLATHRATES

Considering the liquid water, let r_{CL} be the radius of the assumed spherical water clathrate; M_{CL} is its' mass with volume $v_{CL}=\frac{4}{3}\pi r_{CL}^3$ and effective volume $V_{CL}=(2r_{CL}+\bar{R}_{CL})^3$ where $(2r_{CL}+\bar{R}_{CL})$ is the average distance between 2 clathrates, Figure 12.2.

FIGURE 12.2
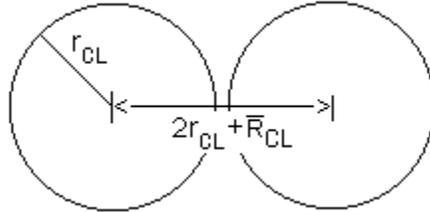
Let N_W represent the number of water molecules in the clathrate. The mass m_W of 1 water molecule is $3.01\cdot10^{-23}$gm and $\frac{M_{CL}}{V_{CL}}=\frac{m_W N_W}{(2r_{CL})^3(1+\frac{\bar{R}_{CL}}{2r_{CL}})^3}=1\frac{gm}{cm^3}$.

Let L_i be the initial length of a vertical water column where $L_i=N(2r_{CL}+\bar{R}_{CL})$ and N is the number of water clathrates along the length. Let L_f be the final length under compression where $L_f=N(2r_{CL})$ Experimentally it is shown that for compressive pressures up to the bulk modulus, $\frac{L_f}{L_i}\doteq1$ where $B_{\Theta_{Ex}}=2.1\cdot10^{10}\frac{dy}{cm^2}$. Consequently

$\frac{L_f}{L_i}=\frac{1}{(1+\frac{\bar{R}_{CL}}{2r_{CL}})}\doteq1$ and $\frac{\bar{R}_{CL}}{2r_{CL}}<<1$ and therefore $\frac{M_{CL}}{V_{CL}}=1\doteq\frac{m_W N_W}{(2r_{CL})^3}\frac{gm}{cm^3}$.

Let r_W be the effective radius of the water molecule where $(2r_W+\bar{R}_W)$ is the average distance between 2 adjacent water molecules within the clathrate, Figure 12.3.

FIGURE 12.3
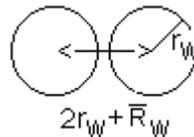
$N_W=\frac{(2r_{CL})^3}{m_W}=2.66\cdot10^{23}\cdot r_{CL}^3=\frac{\frac{4}{3}\pi r_{CL}^3}{(2r_W+\bar{R}_W)^3}$ and for the same reasons as for the

clathrate, $\frac{\bar{R}_W}{2r_W}<<1$. N_W becomes: $N_W=2.66\cdot10^{23}\cdot r_{CL}^3=0.523\cdot\frac{r_{CL}^3}{r_W^3}$ and $r_W=1.25\cdot10^{-8}$cm

12.12 $r_W=1.25\cdot10^{-8}$cm, $N_W\doteq2.66\cdot10^{23}\cdot r_{CL}^3$

3. Dimensions of Water Clathrate

In order to account for the heat of vaporization $0.456\frac{ev}{w}$ at T=100°C and the equilibrium water vapor pressure $P_V(T)$; our model for a clathrate consists of water molecules tightly bound together within the clathrate but with relatively weak clathrate to clathrate binding.

A 2 dimensional representation of the spherical surface of 2 spherical water clathrates made up of water molecules bound to one another is given in figure 12.5.

FIGURE 12.4
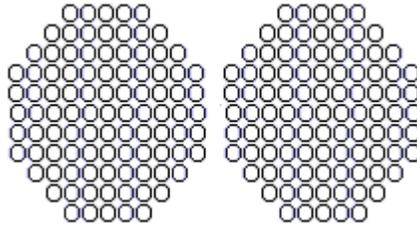
Consider Fig. 12.1 with water in pressure equilibrium with air at T=293°K=20°C, $P_{Atm}=1.0 \cdot 10^6 \frac{dy}{cm}2$ and water vapor pressure $P(293°)_V=2.34 \cdot 10^4 \frac{dy}{cm}2$. It is atmospheric pressure P_{Atm} that keeps water in the liquid state at STP. Once the pressure is reduced to the water vapor pressure $P(T)_V$, the water explosively boils decreasing the temperature of the water that remains.

Using 4.6, the average pressure $P_{CL,CL}$ that 2 clathrates exert on one another is:

$$P_{CL,CL}=2\frac{r_{CL}}{\overline{R}_{CL}} n_{CL}KT=1.0 \cdot 10^6 \frac{dy}{cm}2 . \text{ Using } n_{CL}=(2r_{CL})^{-3}, P_{CL,CL} \text{ becomes: } P_{CL,CL}=\frac{KT}{\overline{R}_{CL}(2r_{CL})^2}2 .$$

Solve for $r_{CL}^2 \overline{R}_{CL}$: $r_{CL}^2 \overline{R}_{CL}=1.0 \cdot 10^{-20} cm^3$. The mass of a clathrate M_{CL} is:

$$M_{CL}=\frac{4}{3}\pi(\frac{r_{CL}}{2r_w})^3 m_w=8.0 \cdot r_{CL}^3 \text{ and using } r_{CL}=1.0 \cdot 10^{-10}(\frac{1}{\overline{R}_{CL}})^{\frac{1}{2}}, M_{CL} \text{ becomes:}$$

$$M_{CL}=8.0 \cdot 10^{-30}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}gm.$$

The effective volume of a clathrate $V_{CL,eff}$ is: $V_{CL,eff}=(2r_{CL})^3=8.0 \cdot 10^{-30}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}$ and

$M_{CL}=V_{CL,eff}$ (QED). For future use: At T=20°C,

12.13 $r_{CL}=1.0 \cdot 10^{-10}(\frac{1}{\overline{R}_{CL}})^{\frac{1}{2}}cm, M_{CL}=8.0 \cdot 10^{-30}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}gm, V_{CL}=8.0 \cdot 10^{-30}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}} cm^3$

$N_w=2.7 \cdot 10^{-7}(\frac{1}{\overline{R}_{CL}})^{\frac{3}{2}}=$ number of water molecules per clathrate

Assuming the turnaround Δt_{Ar} time for the clathrate is $\Delta t_{Ar} << \Delta t_S =$ the time it takes sound in water at $P = 10^6 \frac{dy}{cm}2$ and $T = 293^0 K$ to travel $2r_{CL} + \bar{R}_{CL}$: calculate using sec. 12, $\bar{R}_{CL} = 2 \cdot 10^{-9} cm$ and using $\bar{R}_{CL}$, evaluate 12.13:

12.14 $r_{CL} = 2.2 \cdot 10^{-6} cm$, $M_{CL} = 8.8 \cdot 10^{-17} gm$, $V_{CL} = 8.8 \cdot 10^{-17} cm^3 \doteq (2r_{CL})^3$

       $N_W = 2.9 \cdot 10^6 =$ number of water molecules per clathrate

Let $f(10^6 \frac{dy}{cm}2)_{CL,CL}$ represent the average force between 2 clathrates at $P = 10^6 \frac{dy}{cm}2$ and $T = 20^0 C$. $f(10^6 \frac{dy}{cm}2)_{CL,CL} = PA = 10^6 (2r_{CL})^2 = 1.9 \cdot 10^{-5} dy$.

As atmospheric pressure is reduced to $P(293^0)_V = 2.34 \cdot 10^4 \frac{dy}{cm}2$ the clathrates on the water surface are released into the partial vacuum with zero initial speed. By direct collision with the clathrates still bonded together on the water surface, the initially free clathrates attain average kinetic energy $6.06 \cdot 10^{-14} erg = .0378 ev$ and attain average height 0.70cm above the liquid surface.

At $P(293^0)_V$, $\bar{R}_{CL}$ becomes: $\bar{R}_{CL} = \frac{KT}{P_V(2r_{CL})}2 = 0.89 \cdot 10^{-7} cm$.

Let $f(P_V)_{CL,CL}$ represent the average force between 2 clathrates at $P_V = 2.34 \cdot 10^4 \frac{dy}{cm}2$ and $T = 20^0 C$. $f(P_V)_{CL,CL} = PA = 2.34 \cdot 10^4 (2r_{CL})^2 = 0.45 \cdot 10^{-10} cm$.

4. A New Mathematical, Physical Model for Turbulent Flow

As normally written, the Navier-Stokes Equations are: $\rho(\underset{\sim}{X},t) \cdot \frac{d\underset{\sim}{U}}{dt} = -\nabla P + (\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U} + \Sigma \underset{\sim}{F}_i$. This is wrong on two counts. $\underset{\sim}{X}$ is assumed to be a function of time t: $\underset{\sim}{X} = \underset{\sim}{X}(t)$ and the sum of the external forces term, $\Sigma \underset{\sim}{F}_i$ is on the wrong side of the equation. This is crucial as the derivation of the $(\underset{\sim}{U} \cdot \nabla) \underset{\sim}{U}$ term (Used in the derivation of mathematical fluid turbulence and the derivation of the oceanic tides) is dependent on $\underset{\sim}{X} = \underset{\sim}{X}(t)$. However $\underset{\sim}{X}(t)$ represents a point moving with respect to a coordinate frame S, but to measure $\underset{\sim}{U}$ requires that $\underset{\sim}{X}$ represent points that are stationary in S. This is inconsistent notation and may be resolved by letting $\underset{\sim}{h}$ and $\underset{\sim}{\chi}$ represent the position of a moving point where $\underset{\sim}{h} = \underset{\sim}{h}(\underset{\sim}{X},t) = \underset{\sim}{X} + \underset{\sim}{\chi}(\underset{\sim}{X},t)$ with $\underset{\sim}{\chi}(\underset{\sim}{X},0) = 0$ and $\frac{dx}{dt} = 0$. $\underset{\sim}{h}$ is an Eulerian coordinate and if Newton's Laws are to be used, S must be an inertial frame. This means the Navier Stokes Equations are only valid when experimentally determined from an inertial frame.   As regards the $\Sigma \underset{\sim}{F}_i$ term. The pressure term is a result of external forces and is not an innate force such as gravity. Without external forces there is no pressure term and consequently the $\Sigma \underset{\sim}{F}_i$ term should be written on the other side of the equation. The correct form of the Navier Stokes Equations are therefore:

$$\rho(\underset{\sim}{h})\cdot\frac{d^2\underset{\sim}{h}}{dt^2}=-\nabla P(\underset{\sim}{h})=\sum \underset{\sim}{F}(\underset{\sim}{h})_i \, , \, \underset{\sim}{h}=\underset{\sim}{h}(\underset{\sim}{x},t)=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t) \text{ with } \underset{\sim}{\chi}(\underset{\sim}{x},0)=0, \frac{dx}{dt}\equiv 0 \text{ or if you will:}$$

$$\rho(\underset{\sim}{h})\cdot\frac{d\underset{\sim}{U}(\underset{\sim}{h})}{dt}=-\nabla P(\underset{\sim}{h})=\sum \underset{\sim}{F}(\underset{\sim}{h})_i$$

Notice the absence of the $(\underset{\sim}{U}\cdot\nabla)\underset{\sim}{U}$ term as it is dependent on the physically false assumption that $\underset{\sim}{x}$ is a function of t (Chap. 12, sec 1). Consider atmospheric air striking a motionless airplane wing with partial pressure due to water droplets $P_{wd}(\underset{\sim}{h})$ where $P_{wd}(\underset{\sim}{h})=n_{wd}(\underset{\sim}{h})KT(\underset{\sim}{h})$, $\frac{dx}{dt}\equiv 0$ and $n_{wd}(\underset{\sim}{h})=\#$ of water droplets per cm^3. Dropping the subscript where it is understood, $-\nabla P(\underset{\sim}{h})$ becomes:

$$-\nabla P(\underset{\sim}{h})=-\left\{\frac{\partial P}{\partial x}\hat{x}+\frac{\partial P}{\partial y}\hat{y}+\frac{\partial P}{\partial z}\hat{z}\right\}=-\left\{\left(\frac{\partial n}{\partial x}KT+nK\frac{\partial T}{\partial x}\right)\hat{x}+\left(\frac{\partial n}{\partial y}KT+nK\frac{\partial T}{\partial y}\right)\hat{y}+\left(\frac{\partial n}{\partial z}KT+nK\frac{\partial T}{\partial z}\right)\hat{z}\right\}$$

and

12.15 $\quad \rho(\underset{\sim}{h})\cdot\frac{d\underset{\sim}{U}(\underset{\sim}{h})}{dt}=-\nabla P(\underset{\sim}{h})=-\left\{\left(\frac{1}{n}\frac{\partial n}{\partial x}+\frac{1}{T}\frac{\partial T}{\partial x}\right)\hat{x}+\left(\frac{1}{n}\frac{\partial n}{\partial y}+\frac{1}{T}\frac{\partial T}{\partial y}\right)\hat{y}+\left(\frac{1}{n}\frac{\partial n}{\partial z}+\frac{1}{T}\frac{\partial T}{\partial z}\right)\hat{z}\right\}P(\underset{\sim}{h}), \frac{dx}{dt}\equiv 0$

The $\frac{\partial n}{\partial x},\frac{\partial n}{\partial y},\frac{\partial n}{\partial z}$ terms are caused by the airplane wing striking water vapor droplets with n_o water vapor droplets in still air and each droplet composed of m_o water molecules, hydrogen bonded together in the droplet. If the wing is going fast enough so that the energy imparted to the droplet is sufficient to break the hydrogen bonds holding the droplet together, the droplet breaks up into m_o water molecules. It is the increase in number density $n(\underset{\sim}{x},t)$ that is hypothesized to cause hydrodynamic turbulence.

It is interesting to note that an object such as an asteroid or reentry vehicle would have to be traveling some ~18,000 mph to break apart Nitrogen molecules and thus increase $n(\underset{\sim}{x},t)$.

Consider a fluid under the influence of gravity where $\underset{\sim}{g}(\underset{\sim}{h})$ is the local acceleration of gravity and $-\nabla P=\pm P(\underset{\sim}{h})\underset{\sim}{g}(\underset{\sim}{h})$. The upper sign is used in the case of static equilibrium (no net movement with respect to the center of mass of gas or liquid) or in the case of net acceleration in the $+\hat{r}$ direction (due to heating from below). The lower sign is used in the case of net acceleration in the $-\hat{r}$ direction; e.g. water flowing down hill. Writing the Navier-Stokes Equations for a fluid under the influence of gravity:

FIGURE 12.5A
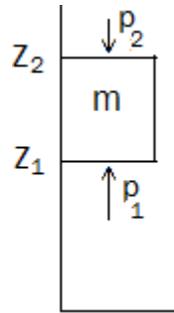
Consider P to be a contact force and with $P_1 > P_2 > 0$,

$$-\nabla P = -\frac{\partial P}{\partial Z} = -\frac{\Delta P}{\Delta Z}\hat{z} = -\frac{P_2 - P_1}{Z_2 - Z_1}\hat{z} = \frac{P_1 - P_2}{Z_2 - Z_1}\hat{z}$$ there is a net force in the $+\hat{z}$ direction. The

physical origin of the net force per unit volume is not $-\nabla P$; $-\nabla P$ is the result of an external physical force per unit volume $\underset{\sim}{f}$. Although P has been specified to be a contact force, the physical origin has not been specified: A hammer? Etc. The physical origin of the force should be specified in order to completely understand the behavior of the medium. Note that P is a scalar while $\underset{\sim}{f}$ is a vector so,

$$\underset{\sim}{f} = \frac{P_2 + P_1}{Z_2 - Z_1} = \frac{-P_2\hat{z} + P_1\hat{z}}{Z_2 - Z_1} = \frac{P_1 - P_2}{Z_2 - Z_1}\hat{z}$$ in agreement with $-\nabla P$ in the $\lim\limits_{\Delta z \to 0}\frac{P_1 - P_2}{Z_2 - Z_1}\hat{z}$

The situation is somewhat different if the force causing P is a field force e.g. gravity near the earth's surface with $\underset{\sim}{f} = -\rho g\hat{z}\frac{dy}{cm^3}$. How is it that an external force acting in the $-\hat{z}$ direction can create a force acting in the $+\hat{z}$ direction to support e.g. the oceans and atmosphere?

Matter is atomic not continuous. Consider an atom vibrating between Z_1 and Z_2 either chemically bonded to the surrounding atoms at Z_1 and Z_2 as in a solid or not strongly bonded to surrounding molecules as in earth's atmosphere. Either way conserving energy and momentum, atom 1 in the diagram rebounds from atom 2 at Z_2 and atom 1 rebounds from atom 3 at Z_1 where $Z_2 - Z_1 = 2r_0 + \overline{R}$,

FIGURE 12.5B
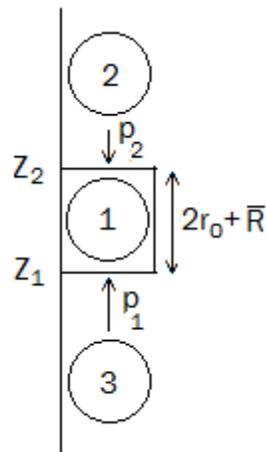
It is the rebound, conserving energy and momentum, that enable a field

force $\underset{\sim}{f}=-\rho g\hat{z}$ in the $-\hat{z}$ direction, to cause a contact force in the $+\hat{z}$ direction. Using

the diagram, r_0 is the radius of the atom, $r_0 \doteq 10^{-8}$ cm and $\overline{R}$ is the average distance

traveled by atom 1 between collisions at Z_1 and Z_2.

In the transition metals $\overline{R} \doteq 10^{-10}$ cm and in air $\overline{R} \doteq 10^{-7}$ cm. It is commonly claimed by

many chemists and physicists that for solids $\overline{R} \doteq 10^{-8}$ cm. Using the $\overline{R} \doteq 10^{-8}$ cm

figure to compute the mass density of solid materials such as the transitions metals

yields a density too small by a factor of $\frac{1}{3}$ while using the figure $\overline{R} \doteq 10^{-10}$ cm yields the

experimentally determined mass density value as computed in chap. 4, sec 2.

Assuming atoms 1,2,3 have the same mass, one can derive the pressure at Z_2 as

12.16 $PV=(\frac{2r_0}{\overline{R}})NKT$ (Chap. 4, sec. 3). As a special case, $2r_0=\overline{R}$, yields the ideal gas

law, valid for earth's atmosphere where r_0 and $\overline{R}$ are averaged over the total number

of N_2, O_2, CO_2, H_2O molecules per cm^3. For the case under consideration, the

pressure at Z_2, $PV(Z_2)=(\frac{2r_0}{\overline{R}})NKT$, $V(Z_2)=(2r_0+\overline{R})^3$ and with $N=1$, yields,

$P(Z_2)=(\frac{2r_0}{\overline{R}})\frac{KT_2}{(2r_0+\overline{R})^3}$. Under the influence of gravity, the time t taken by atom 1 to

fall a distance $\overline{R}$ is: $\overline{R}=U_{z,i}t_d+\frac{1}{2}gt_d^2$ with solution $t_d=\frac{U_{z,i}}{g}\left\{[1+2\frac{\overline{R}g}{U_{z,i}^2}]^{\frac{1}{2}}-1\right\}$ where $U_{z,i}$ is

the speed of atom 1 in the $-\hat{z}$ direction when the atom is at Z_2 directly after rebound

at time 0^+. Let $U_{z,f}$ represent the speed of atom 1 in the $-\hat{z}$ direction when the atom

is at Z_1 directly before the collision with atom 2 at time t_d^- where:

$U_{z,f}=U_{z,i}+gt_d=U_{z,i}+U_{z,i}\left\{[1+2\frac{\overline{R}g}{U_{z,i}^2}]^{\frac{1}{2}}-1\right\}=U_{z,i}[1+2\frac{\overline{R}g}{U_{z,i}^2}]^{\frac{1}{2}}$. However in the derivation of

$PV=(\frac{2r_0}{\overline{R}})NKT$, in order to derive the average pressure at $P(Z)$, the down and back

time is used. The calculated time t_d is just the down time hence rederiving $P(Z_1)$

using t_d one must use $2t_d$. See Chap 4, sec 3. This results in:

$P(Z_1)=(\frac{r_0}{\overline{R}})\frac{KT_f}{(2r_0+\overline{R})^3}$ and $P(Z_2)=(\frac{r_0}{\overline{R}})\frac{KT_i}{(2r_0+\overline{R})^3}$. derive $\frac{T_f}{T_i}=\frac{U_{rms,f}^2}{U_{rms,i}^2}=\frac{3U_{z,f}^2}{3U_{z,i}^2}=[1+2\frac{\overline{R}g}{U_{z,i}^2}]$

and with $U_{z,i}^2=\frac{1}{3}U_{rms,1}^2=\frac{KT_i}{m}$. T_f becomes: $T_f=[1+2\frac{\overline{R}g}{U_{z,i}^2}]T_i=(1+2\frac{mg\overline{R}}{KT_i})T_i$. $-\nabla P$

becomes:

$-\nabla P \doteq -\{\frac{P(Z_2)-P(Z_1)}{Z_2-Z_1}\}\hat{z}=-\{\frac{P(Z_2)-P(Z_1)}{(2r_0+\overline{R})}\}\hat{z}=-(\frac{r_0}{\overline{R}})\frac{K}{(2r_0+\overline{R})^4}(T_i-T_f)\hat{z}$ And:

12.17 $-\nabla P \doteq (\frac{1}{2\bar{R}}) \frac{K}{(2r_0+\bar{R})^3}(T_i-T_f)\hat{z} = \frac{mg}{(2r_0+\bar{R})^3}\hat{z} = +\rho g\hat{z}.$

Note the positive sign in front of $\rho g\hat{z}$. It is the rebound of atom 1 from atom 3, conserving energy and momentum, that enable a field force $\underset{\sim}{f} = -\rho g\hat{z}$ in the $-\hat{z}$ direction, to cause a contact force in the $+\hat{z}$ direction.

In order to further develop a model turbulence it is necessary to develop a model for a water droplet, sec 5, and absolute humidity, sec 6 and the latent heat of vaporization, sec 7, as set down below.

5. Water Droplet Floating in Air

Clouds, fogs, mists, the marine layer, a steaming cup of coffee, and a lake steaming on a autumnal day before sunrise, create visible water droplets ~0.1 to ~1mm in diameter. In order to understand how water droplets are formed and why they float, it is necessary to have a model for the atmosphere. The model created here is based on the following equations.

12.18

a. P=nKT

b. $\rho(z) \cdot \frac{dW}{dt}\hat{z} = -\nabla P = +\rho g\hat{z}$

c. $\frac{1}{2}\bar{m} V_{rms}^2 + \bar{m}gz = \frac{3}{2}KT(z) + \bar{m}gz = \frac{3}{2}KT(0),\quad 0 \le z \lesssim 10^6 cm = 10km$

The 3rd equation hypothesizes that the average kinetic energy plus average potential energy is a constant for every parcel of air at z for all z, $0 \le z \lesssim 10km$. The air at z is kept at constant temperature T(z) from heat from the earth at temperature T(0) and loses heat by emitting infrared radiation into space. It does not include warming from the sun and thus errors are to be expected especially for 12km< z where T(z) increases with z due to warming from the sun. Using T(0)=293°K, and $\bar{m}=4.82 \cdot 10^{-23}$gm, 12.18c becomes:

T(z)=293-22.8$\cdot 10^{-5}$z(cm) where by direct measurement T(z)= 293-7.3$\cdot 10^{-5}$z, $0 \le z \lesssim 10^6$cm. 12.18c is a direct analogue of chap 3, eq. 3.15

Using 12.18a and b, find: $\frac{dp}{dz}=-n\bar{m}g=K(T\frac{dn}{dz}+n\frac{dT}{dz})$ and $-\frac{\bar{m}g}{KT}=(\frac{dlnn}{dz}+\frac{dlnT}{dz})$. With

dz=-1.37$\cdot 10^4$dT, integrate $-\frac{\bar{m}g}{KT}$ to find $ln\{\frac{n(z)}{n(0)}(\frac{T(0)}{T(z)})^{3.7}\}=0$ and n(z)=n(0)[1-2.5$\cdot 10^{-7}$z]$^{3.7}$.

Using 12.18a, P= n(0)KT(0)[1-2.5$\cdot 10^{-7}$z]$^{4.7}$=10^6[1-2.5$\cdot 10^{-7}$z]$^{4.7}$. At 10km P(10)=0.26$\cdot 10^6\frac{dy}{cm^2}$

Where 10$^6\frac{dy}{cm^2}$= 1 atmosphere. Collating results:

12.19 With T(0)=293°K, $0 \le z \lesssim 10^6$cm=10km

a. P=10^6[1-2.5$\cdot 10^{-7}$z]$^{4.7}\frac{dy}{cm^2}$

b. n(z)=2.5$\cdot 10^{19}$[1-2.5$\cdot 10^{-7}$z]$^{3.7}\frac{1}{cm^3}$

c. $\rho(z)=1.2 \cdot 10^{-3}[1-2.5 \cdot 10^{-7}z]^{3.7}\frac{gm}{cm^3}$

d. $T(z)=[293-7.3 \cdot 10^{-5}z]\,°K$,

A table of values of 12.19 is listed in Table 12.1

<div align="center">TABLE 12.1</div>

Z km	$P \frac{dy}{cm^2}$	$n\frac{1}{cm^3}$	$\rho\frac{gm}{cm^3}$	T °K
0	10^6	$2.5 \cdot 10^{19}$	$1.2 \cdot 10^{-3}$	293
1	$0.89 \cdot 10^6$	$2.3 \cdot 10^{19}$	$1.1 \cdot 10^{-3}$	286
2	$0.79 \cdot 10^6$	$2.1 \cdot 10^{19}$	$1.0 \cdot 10^{-3}$	278
5	$0.53 \cdot 10^6$	$1.5 \cdot 10^{19}$	$0.73 \cdot 10^{-3}$	257
10	$0.26 \cdot 10^6$	$0.85 \cdot 10^{19}$	$0.41 \cdot 10^{-3}$	220

Let M_D represent the mass of a spherical water vapor droplet and R_D its radius. Using

12.18b, $-|\nabla P|=\rho_{air}g \doteq \frac{\Delta P}{\Delta z}=\frac{3M_D g}{4\pi R_D^3}$ and $\rho(z)_{air}=1.2 \cdot 10^{-3}[1-2.5 \cdot 10^{-7}z]^{3.7}=\frac{3M_D}{4\pi R_D^3}$.

With m_w the mass of a water molecule, $m_w=3.01 \cdot 10^{-23}$gm, and N_w the number of water molecules in the droplet, $M_D=m_w N_w$ and considering M_D to be a hollow micelle, Fig. 12.6,

$N_w=\frac{A_D}{A_w}N_L=\frac{4\pi R_D^2}{(2r_w)^2}N_L$ where N_L is the number of layers of water molecules. M_D

becomes: $M_D=m_w\frac{4\pi R_D^2}{(2r_w)^2}N_L$ and $\rho(z)_{air}=\frac{3M_D}{4\pi R_D^3}=m_w\frac{4\pi R_D^2}{(2r_w)^2}N_L(\frac{3}{4\pi R_D^3})=1.44 \cdot 10^{-7}(\frac{1}{R_D})N_L$.

With $1.2 \cdot 10^{-3}[1-2.5 \cdot 10^{-7}z]^{3.7}=1.44 \cdot 10^{-7}(\frac{1}{R_D})N_L$, solve for $R(z)_D$ where $R(z)_D$

represents the radius of the micelle that will float and remain at height z and $N(z)_w$ is the number of water molecules in the micelle with radius $R(z)_D$.

12.20 $R(z)_D=1.2 \cdot 10^{-4}N_L[1-2.5 \cdot 10^{-7}z]^{-3.7}cm$

       $N(z)_w=2.9 \cdot 10^8 N_L^3[1-2.5 \cdot 10^{-7}z]^{-7.4}$

       $M(z)_D=8.7 \cdot 10^{-15}N_L^3[1-2.5 \cdot 10^{-7}z]^{-7.4}gm$

Values for R_D, N_w and M_D have been computed and are tabulated in Table 12.2

TABLE 12.2

Z km	R_D cm	N_w	M_D gm	N_L
0	$1.2 \cdot 10^{-4}$	$2.9 \cdot 10^{8}$	$8.7 \cdot 10^{-15}$	1
1	$1.3 \cdot 10^{-4}$	$3.5 \cdot 10^{8}$	$1.1 \cdot 10^{-14}$	1
2	$1.4 \cdot 10^{-4}$	$4.3 \cdot 10^{8}$	$1.3 \cdot 10^{-14}$	1
5	$1.9 \cdot 10^{-4}$	$7.8 \cdot 10^{8}$	$2.4 \cdot 10^{-14}$	1
10	$3.5 \cdot 10^{-4}$	$2.4 \cdot 10^{9}$	$7.2 \cdot 10^{-14}$	1

Z km	R_D cm	N_w	M_D gm	N_L
0	$1.2 \cdot 10^{-3}$	$2.9 \cdot 10^{11}$	$8.7 \cdot 10^{-12}$	10
1	$1.3 \cdot 10^{-3}$	$3.5 \cdot 10^{11}$	$1.1 \cdot 10^{-11}$	10
2	$1.4 \cdot 10^{-3}$	$4.3 \cdot 10^{11}$	$1.3 \cdot 10^{-11}$	10
5	$1.9 \cdot 10^{-3}$	$7.8 \cdot 10^{11}$	$2.4 \cdot 10^{-11}$	10
10	$3.5 \cdot 10^{-3}$	$2.4 \cdot 10^{12}$	$7.2 \cdot 10^{-11}$	10

If $\rho(z)_{air} < \dfrac{3M_D g}{4\pi R_D^3} = 1.44 \cdot 10^{-7}(\dfrac{1}{R_D})N_L$, and $R(z)_D < 1.2 \cdot 10^{-4} N_L[1 - 2.5 \cdot 10^{-7}z]^{-3.7}$ and

$N(z)_w > 2.9 \cdot 10^{8} N_L^3 [1 - 2.5 \cdot 10^{-7}z]^{-7.4}$ and $M(z)_D > 8.7 \cdot 10^{-15} N_L^3 [1 - 2.5 \cdot 10^{-7}z]^{-7.4}$ gm,

then the micelle will descend to a new level until $\rho(z)_{air} = \dfrac{M_D}{2\pi R_D^3}$. If $\rho(z)_{air} > \dfrac{3M_D g}{4\pi R_D^3}$ and

$R(z)_D > 1.2 \cdot 10^{-4} N_L[1 - 2.5 \cdot 10^{-7}z]^{-3.7}$ and $N(z)_w < 2.9 \cdot 10^{8} N_L^3 [1 - 2.5 \cdot 10^{-7}z]^{-7.4}$ and

$M(z)_D < 8.7 \cdot 10^{-15} N_L^3 [1 - 2.5 \cdot 10^{-7}z]^{-7.4}$ gm, then the micelle will ascend until $\rho(z)_{air} = \dfrac{3M_D g}{4\pi R_D^3}$.

The hollow interior of the floating water droplet, is hypothesized to consist of N_{wv} water vapor, wv, molecules unbonded to one another at saturated water vapor pressure containing $N(z)_{wv} = \dfrac{P(z)_{wv} V(z)}{KT(z)}$ water vapor molecules, where $V(z) = \dfrac{4}{3}\pi R_D^3$.

The total mass $M(z)_{wv}$ becomes, $M(z)_{wv} = m_w N(z)_{wv}$ gm with density $\rho(z)_{wv} = \dfrac{M(z)_{wv}}{V(z)} \dfrac{gm}{cm^3}$ for $d << R(z)$, See Fig. 12.6.

A table of values for $R(z)_D$ (Droplet Radius), $P(z)_{wv}$ (Saturated Water Vapor Pressure In the Interior of the Droplet), $N(z)_{wv}$ (Number of Water Molecules unbonded to one another in the Interior of the Droplet), $M(z)_{wv}$ (Total Mass of Water Molecules in the Interior of the Droplet), and $\rho(z)_{wv}$ (Mass Density of the Water Molecules in the

Interior of the Droplet) has been computed using Table 12.1 and 12.2 and published on line data for vapor pressure over water and over ice. The results are tabulated in Table 12.3
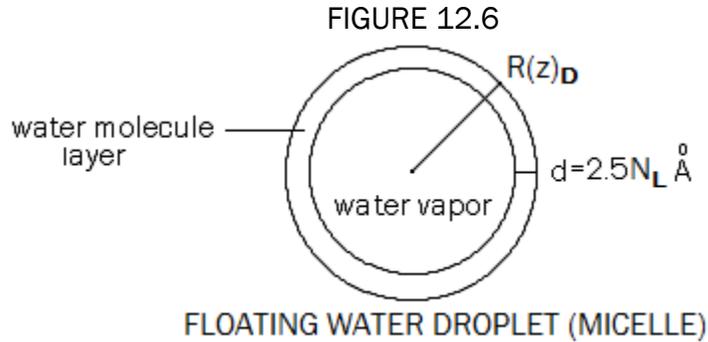
FIGURE 12.6

FLOATING WATER DROPLET (MICELLE)

TABLE 12.3

Z km	T °K	$P(z)_{wv} \frac{dy}{cm^2}$	N_L	R_D cm	$N(z)_{wv}$	$M(z)_{wv}(gm)$	$\rho_{wv} (\frac{gm}{cm^3})$
0	293	$2.3 \cdot 10^4$	1	$1.2 \cdot 10^{-4}$	$4.1 \cdot 10^6$	$1.2 \cdot 10^{-16}$	$1.7 \cdot 10^{-5}$
1	286	$1.5 \cdot 10^4$	1	$1.3 \cdot 10^{-4}$	$3.3 \cdot 10^6$	$9.9 \cdot 10^{-17}$	$1.1 \cdot 10^{-5}$
2	278	$8.6 \cdot 10^3$	1	$1.4 \cdot 10^{-4}$	$2.5 \cdot 10^6$	$7.5 \cdot 10^{-17}$	$6.8 \cdot 10^{-6}$
5	257	$1.5 \cdot 10^3$(ice)	1	$1.9 \cdot 10^{-4}$	$1.2 \cdot 10^6$	$3.6 \cdot 10^{-17}$	$1.3 \cdot 10^{-6}$
10	220	27 (ice)	1	$3.5 \cdot 10^{-4}$	$1.6 \cdot 10^5$	$4.8 \cdot 10^{-18}$	$2.7 \cdot 10^{-8}$

Z km	T °K	$P(z)_{wv} \frac{dy}{cm^2}$	N_L	R_D cm	$N(z)_{wv}$	$M(z)_{wv}(gm)$	$\rho_{wv} (\frac{gm}{cm^3})$
0	293	$2.3 \cdot 10^4$	10	$1.2 \cdot 10^{-3}$	$4.1 \cdot 10^9$	$1.2 \cdot 10^{-13}$	$1.7 \cdot 10^{-5}$
1	286	$1.5 \cdot 10^4$	10	$1.3 \cdot 10^{-3}$	$3.3 \cdot 10^9$	$9.9 \cdot 10^{-14}$	$1.1 \cdot 10^{-5}$
2	278	$8.6 \cdot 10^3$	10	$1.4 \cdot 10^{-3}$	$2.5 \cdot 10^9$	$7.5 \cdot 10^{-14}$	$6.8 \cdot 10^{-6}$
5	257	$1.5 \cdot 10^3$(ice)	10	$1.9 \cdot 10^{-3}$	$1.2 \cdot 10^9$	$3.6 \cdot 10^{-14}$	$1.3 \cdot 10^{-6}$
10	220	27 (ice)	10	$3.5 \cdot 10^{-3}$	$1.6 \cdot 10^8$	$4.8 \cdot 10^{-15}$	$2.7 \cdot 10^{-8}$

6. Absolute Humidity

The absolute humidity is defined as the mass of water vapor in grams, per cubic meter of air. A.H. $= n_W m_W (\frac{gm}{m^3})$, where n_W is the number of water molecules per

cubic meter and $m_W=3.01\cdot10^{-23}$gm is the mass of 1 water molecule. At ground level with no micelles present, water vapor exists as unbonded water molecules and for such air $\rho_{air}=0.0012(\frac{gm}{cm}3)$ and the measured absolute humidity has values $0<A.H.\leq 30$. However by direct observation clouds and fogs are composed of floating water droplets and although the floating water droplets do not substantially contribute to atmospheric pressure, they do contribute substantially to the absolute humidity. As micelles are present in clouds and fogs, $0<A.H.\leq 30$ will be ignored.

a. If water droplets with $N_L=1$ at z=2km (Table 12.2) compose 0.5% by volume of the atmosphere, then by direct computation there are $n_D=4.5\cdot10^{14}(\frac{Dr}{m}3)$ with total mass:

$M_{T,Dr}=4.5\cdot10^{14}(1.3\cdot10^{-14})=5.9=A.H.(\frac{gm}{m}3)$.

b. If water droplets with $N_L=1$ at z=2km (Table 12.2) compose 1% by volume of the atmosphere, then by direct computation there are $n_{Dr}=9.0\cdot10^{14}(\frac{Dr}{m}3)$ with total mass:

$M_{T,Dr}=9.0\cdot10^{16}(1.3\cdot10^{-14})=12(\frac{gm}{m}3) =A.H.=1.2\cdot10^{-5}(\frac{gm}{cm}3)$

If 1% of the volume of a rectangular volume of height 1km and cross section A is filled with the water vapor droplets of part b and the droplets should condense into water drops, the area A would be covered by water to a depth of 1.2cm=0.47inches. The partial pressure P_{wv} due to the water vapor droplets with $N_L=1$ at z=2km is:

$P_{wv}=n_{Dr}KT=9.0\cdot10^8(3.8\cdot10^{-14}) =3.4\cdot10^{-5}\frac{dy}{cm}2<<10^6\frac{dy}{cm}2= 1$ atmosphere, however as the water droplet micelles form, they push 1% of the atmosphere to the side causing a geostrophic flow.

Relative humidity, R.H. is defined as $R.H.\equiv\frac{P_{wv}}{P_V}\cdot10^2$ where P_{wv} is the partial pressure of water molecules striking one another and P_V is the vapor pressure P_V of air striking liquid water at temperature T and R.H. is expressed as a percent. Assuming the formula is correct, at z=0 at $T=293^oK$ and $P_V=2.34\cdot10^4\frac{dy}{cm}2$; if R.H.=100% then

$P_{wv}= P_V=2.34\cdot10^4\frac{dy}{cm}2$ and by direct measurement the air contains water in the

amount $1.8\cdot10^{-5}(\frac{gm}{cm}3)$. If the water molecules are unbonded to one another then

$n_W=\frac{1.8\cdot10^{-5}}{3.0\cdot10^{-23}}=0.6\cdot10^{18}$ and $P_{wv}= n_WKT =0.6\cdot10^{18}(4\cdot10^{-14})=2.4\cdot10^4\frac{dy}{cm}2 =P_V$ O.K.

However if the water exists as micelle water droplets, then the definition of relative humidity is wrong in that as above, from direct measurement at $T=293^oK$, the air contains water in the amount $1.8\cdot10^{-5}(\frac{gm}{cm}3)$ and using table 2.2 the air contains n_{Dr} micelle water droplets per cm^3 where $n_{Dr}=2.1\cdot10^9\frac{Dr}{m}3$. P_{wv} becomes $P_{wv}=n_{Dr}KT=8.4\cdot10^{-5}\frac{dy}{cm}2<<P_V$

and R.H.$=\frac{P_{wv}}{P_V}\cdot10^2=3.6\cdot10^{-7}$% does not represent physical reality.

This raises the question: Is it possible that the sling hygrometer in earth's atmosphere with at a given absolute humidity A.H., break up the micelles into individual water molecules? If so, both the unbonded together water molecules case and the water molecules bonded together as a micelle case would have the same ΔT reading as a result of the breakup of the micelles into individual water molecules.

Lastly, what is the physical cause of micelles to bond together to form a liquid water drop or snowflake at the dew point temperature? To be completed.

If the water droplet volume comprises 5% of the total volume of a given volume of air, and if the water droplets collapse to form raindrops, then assuming the temperature remains constant, the air pressure will drop forming a low-pressure cell. If the starting pressure is P_i, then the pressure will drop to P_f where $P_f=P_iV_i(V_f)^{-1}=P_i(0.95)(1.0)^{-1}=0.95P_i$

If the starting pressure is 30" of Hg, then the pressure will drop to final pressure 28.5" of Hg.

If this process occurs over a warm (T>80°F) ocean where there is a constant source of water vapor droplets, it is conjectured that a hurricane will form. That is, it is conjectured that the collapse of water droplets, to form raindrops, is the physical cause of a low-pressure cell, and a low-pressure cell occurring over a warm ocean, where there is a constant source of water vapor droplets, is the physical cause of a hurricane.

7. Water to Steam: the Latent Heat of Vaporization

The latent heat of vaporization per water molecule, e_V, for water at 100°C at 1atms.

pressure is: $e_V=0.730\cdot10^{-12}\frac{erg}{W}=5.29\cdot10^3K=0.456\frac{ev}{W}$. Note that the specific heat of water between 0$^+$ and 99.9$^+$degrees C is 9.1K.

The total energy necessary to break the bonds between one water molecule on the surface of a complete spherical clathrate at the bottom of the sauce pan and the water molecules to which it is bonded, and to increase the kinetic energy of the just

released water molecule from 0 to $\frac{3}{2}KT$ is: $e_V=0.456\frac{ev}{w}=.0482+M_W(\Delta C_{1,1}+2\Delta C_{1,2})$

where .0482ev is the energy necessary to increase the kinetic energy of the just

released water molecule from 0 to $\frac{3}{2}KT$. With T=373°K, the necessary energy is:

$\frac{3}{2}K\cdot(373)=7.72\cdot10^{-14}$erg=.0482ev.

Let B.E.$_W$ represent the binding energy between one water molecule on the surface of the clathrate and the molecules to which it is attached.

Using 3.15, $[\frac{1}{2}U_{rms}^2(r)+\Psi(r)]=[\frac{1}{2}U_{rms}^2(r_0)+\Psi(r_0)]=C_1$ and to insure a stable atom it is

required that, $\frac{1}{2}U_{rms}^2(r_0)<<|\Psi(r_0)|$. Using 3.8, $\Psi(r_0)\dot=C_1=-\frac{m_wH}{r_0}=-2.4\cdot10^{15}(\frac{cm}{sec})^2$ and

consequently, the total energy T.E. to convert a clathrate from water at 100^oC to steam at

100^oC is, T.E.$=0.730\cdot10^{-12}N_{CL,w}erg=\{7.7\cdot10^{-14}+|B.E._w|\}N_{CL,w}$erg where $N_{CL,w}$
is the total number of water molecules per clathrate.

Assuming all of the energy $0.730\cdot10^{-12}-7.7\cdot10^{-14}=0.65\cdot10^{-12}$erg goes into

increasing $U_{rms}(r_0)_i$ to $U_{rms}(r_0)_f$ while maintaining $\Psi(r_0)$ yields:

12.20A　　a. $[\frac{1}{2}U_{rms}^2(r_0)_i+\Psi(r_0)]=C_1$　　　　c. $[\frac{1}{2}m_wU_{rms}^2(r_0)_i+m_w\Psi(r_0)]=m_wC_1$

　　　　b. $[\frac{1}{2}U_{rms}^2(r_0)_f+\Psi(r_0)]=C_1+\Delta C_1$　　d. $[\frac{1}{2}m_wU_{rms}^2(r_0)_f+m_w\Psi(r_0)]=m_wC_1+m_w\Delta C_1$

d-c$=\frac{1}{2}m_w[U_{rms}^2(r_0)_f-U_{rms}^2(r_0)_i]=m_w\Delta C_1=0.65\cdot10^{-12}$erg and assuming

$U_{rms}^2(r_0)_f>>U_{rms}^2(r_0)_i$ find $U_{rms}^2(r_0)_f=4.3\cdot10^{10}(\frac{cm}{sec})^2$.

For time $0<t\le t_0$ the amplitude of the oscillation A(t) satisfies, energy $E(t)=0.65\cdot10^{-12}\frac{t}{t_0}=k_1A(t)^2$.

with solution $K_1=\frac{0.65\cdot10^{-12}}{A^2(t_0)}\frac{gm}{sec^2}$ and $A(t)=A(t_0)(\frac{t}{t_0})^{\frac{1}{2}}$, $0<t\le t_0$. $A(t_0)$ is the separation
distance at which the bonding force holding two water molecules together goes to
zero where the center to center distance between two water molecules is:

$2r_0+\bar{R}(t)\xrightarrow{t\to t_0}2r_0+A(t_0)$ and $A(t)<\bar{R}(t)$ for $t<t_0$ and $A(t_0)=\bar{R}(t_0)$ and $A(t_0)\sim10^{-9}$cm and

$10^{-10}\le\bar{R}(t)\le10^{-9}$cm
As the water temperature reaches 100^oC, all the water molecules in the clathrate
enter into the resonance condition:

12.20B　　　　$h(r_0,t)=r_0+A(t)\sin\omega t\cos\omega t,$

　　　　　　$\dot{h}(r_0,t)=\dot{A}(t)\sin\omega t\cos\omega t+A(t)\omega(\cos^2\omega t-\sin^2\omega t)$

In reality, ω is a function of t. From above, with $U_{rms}(r_0,t_0)_f=2.1\cdot10^5\frac{cm}{sec}$ and

$\overline{U_{rms}(r_0,t)}_f=1.1\cdot10^5\frac{cm}{sec}$ and $10^{-10}\le\bar{R}(t)\le10^{-9}$cm, find, $\bar{\omega}=1.1\cdot10^{14}cps=2.2\pi\cdot10^{14}\frac{Rad}{sec}$.
12.20B becomes:

12.20C　$h(r_0,t)=1.25\cdot10^{-10}+10^{-9}(\frac{t}{t_0})^{\frac{1}{2}}[\sin(6.9\cdot10^{14}t)\cos(6.9\cdot10^{14}t)]$,　$0<t\le t_0$

　　　　$\dot{h}(r_0,t)=$

$$(\tfrac{1}{2}\tfrac{10^{-9}}{t_o})\cdot(\tfrac{t}{t_o})^{-\tfrac{1}{2}}[\sin(6.9\cdot10^{14}t)\cos(6.9\cdot10^{14}t)]+ 6.9\cdot10^{5}(\tfrac{t}{t_o})^{\tfrac{1}{2}}[\cos^2(6.9\cdot10^{14}t)- \sin^2(6.9\cdot10^{14}t)]$$

Consider the following scenario: On rising from the bottom of a sauce pan a bubble of vapor composed of independent water molecules comes in contact with the cooler air above the liquid water. As the vapor bubble bursts at the water surface, some of the water molecules escape as individual water molecules and with mass 18amu, float upwards. Those molecules that form hollow micelles are filled with water vapor at water vapor pressure $P_V=2.34\cdot10^{4}\frac{dy}{cm}2$ and those with a density less than the density of the surrounding air float upward while those with a density greater than air fall back into the boiling water.

8. Turbulent Flow in Air Under review but still worth a read.

Water vapor in air consists of floating water droplets (Sec. 7 and fig. 12.6) and floating individual water molecules. We investigate the hypothesis that the onset of turbulent flow in air is due to the breakup of water droplets into individual water molecules when striking a moving object at speeds<<Mach 1 and due to the breakup of individual water molecules into individual H and O atoms at speeds $\gtrsim$ Mach 10.

In this section we will compute the speed U_X of a solid object e.g. the wing of an aircraft, the wing of a bird etc. that is necessary to:

1. Break a floating water droplet into its W_{Dr} constituent water molecules.
2. Calculate the resultant increase in pressure ΔP for part 1.

The background air pressure P(2) at z=2km at $T=278^{o}K$, is $P(2)=0.786\cdot10^{6}(\frac{dy}{cm}2)$ with $n_o=2.05\cdot10^{19}(\frac{1}{cm^3})$. The physical increase in the number of water molecules per unit volume Δn due to the breakup of water droplets on striking a solid object, results in an increase in pressure $\Delta P=\Delta nKT$ once temperature equilibrium has been reestablished. The increase in the local pressure ΔP and the random nature of the direction of the vector velocities of the released water molecules is hypothesized to be the cause of turbulence.

With $N_L=1$, table 12.2, the radius of a micelle droplet R_{Dr} at 2 km is: $R_{Dr}=0.96\cdot10^{-4}cm$ and contains $2.0\cdot10^{8}$ water molecules per micelle droplet with mass $5.9\cdot10^{-15}gm$. Saturated air at $278^{o}K$ contains $6.8\frac{gm}{m^3}$ of water and contains $1.1\cdot10^{15}\frac{Dr}{m^3}$. The average center to center distance between two adjacent micelles is $(2R_{Dr}+\bar{r}_{Dr})$ and the maximum number of micelles per unit volume is $n_{M,Dr}=\frac{1}{(2R_{Dr}+\bar{r}_{Dr})}3 =1.1\cdot10^{9}\frac{Dr}{cm^3}$

and solving for $\bar{r}_{Dr}$: $\bar{r}_{Dr}$=0.97·10^{-3}cm. Assuming the water vapor consists solely of water vapor micelle droplets, the maximum pressure that water vapor droplets exert on one another is $P_{M,Dr}$=$n_{M,Dr}$KT=4.2·10^{-5}<<P_V=2.34·10^4($\frac{dy}{cm}$2).

Fogs at ground level at night under a street lamp can be seen to be composed of droplets. We examine the consequences of assuming that in earth's atmosphere (But not in the closed container used to measure water vapor pressure) the water vapor that forms fogs and clouds and the water vapor that forms rain drops (But not the rain drops themselves) are composed of hollow water droplets.

As regards #1.

The kinetic energy of a floating water droplet after an elastic collision with a solid object with mass M_{SO}>>m_{Dr} moving with speed U_X relative to the floating water droplet is: $2m_{Dr}·U_X^2$=1.2·10^{-14}· U_X^2 erg.

For a floating water droplet consisting of a single layer of water molecules, each water molecule is bonded to 4 water molecules but a little more contemplation reveals that with N_W>>10^2 and given that there are N_W=2.0·10^8 water molecules per micelle, that the average number of bonds it takes to deconstruct a micelle into 2.0·10^8 individual water molecules is 2. Given that $|Be_{w,w}|$ is the binding energy of two water molecules in the micelle, $\frac{1}{2}|Be_{w,w}|$ is the energy necessary to pull one of the two water molecules apart and $|Be_{w,w}|$ is the energy necessary to pull one water molecule from two water molecules to which it is bound in the micelle and it requires $N_O|Be_{w,w}|$ to pull N_O molecules from the micelle with an additional $N_O|Be_{w,w}|$ distributed among the remaining bound N_W-N_O water molecules in the micelle.

When $N_{0,f}$= (N_W-$N_{0,f}$)=$\frac{1}{2}N_W$. There is now enough energy in the remaining $\frac{1}{2}N_W$ water molecules in the micelle, so that no further work is required to deconstruct the remaining $\frac{1}{2}N_W$ micelle water molecules apart. The total energy to pull apart all N_W micelle water molecules with N_L=1 is (N_W+N_W)$|Be_{w,w}|$=$2N_W|Be_{w,w}|$ for large enough N_W.

Let N_{rw} represent the number of water molecules removed at 1 collision. Setting 1.2·10^{-14}·U_X^2= $N_{rw}|Be_{w,w}|$ and solving for U_X yields: U_X=0.91·10^7($N_{rw}|Be_{w,w}|$)$^{\frac{1}{2}}$ $\frac{cm}{sec}$ for $N_{rw}\gtrsim 10^2$. For N_{rw}=1 it requires a total of $4|Be_{w,w}|$ergs to remove 1 water molecule and for N_{rw}=2 it requires (4+3) $|Be_{w,w}|$=$7|Be_{w,w}|$ and for N_{rw}=10 it requires (28) $|Be_{w,w}|$.

For the standard atmosphere; eqns. 12.19, at z=2km, T=5.4^{o}C=42 ^{o}F with kinetic

energy equivalent $5.8 \cdot 10^{-14}$erg$=3.6 \cdot 10^{-2}$ev. Before the removal of a single water molecule from the micelle and for every molecule in the micelle it is assumed that $|Be_{w,w}| = |K.E.+P.E.| \doteq |P.E.|$ and assuming the Virial theorem holds;

$2K.E.=1.16 \cdot 10^{-13}erg=|P.E.|$, and $|Be_{w,w}|=1.16 \cdot 10^{-13}erg=7.2 \cdot 10^{-2}$ev.

A table of values of U_X as a function of N_{rw} is given in table 12.4 using $|Be_{w,w}|=1.16 \cdot 10^{-13}$erg

TABLE 12.4

N_{rw}	$U_X(\frac{cm}{sec})$	U_X(mph)	N_{rw}	$U_X(\frac{cm}{sec})$	U_X(mph)
1	6.2	0.14	10^5	980	22
10	16	0.35	10^6	3,100	69
10^2	31	0.69	10^7	9,800	220
10^3	98	2.2	10^8	31,000	690
10^4	310	6.9	$2.7 \cdot 10^8$	44,000	990

In order to calculate the change in pressure ΔP due to the creation of N_{rw}, it is necessary to calculate Δn where $\Delta n = n_{rw} (\frac{1}{cm^3})$ and where n_{rw} is the number of water molecules per cm^3 removed from droplets after 1 collision with a moving object. $\Delta n = n_{rw} = N_{rw} \cdot n_{Dr}$ where n_{Dr} is the number of droplets per cm^3. With $n_{M,Dr}=7.8 \cdot 10^{10}(\frac{Dr}{cm^3})$, if the total volume of the droplets is 1% of the total volume containing the droplets then $n_{Dr}=7.8 \cdot 10^8 (\frac{Dr}{cm^3})$ and $\Delta n = n_{rw} = 7.8 \cdot 10^8 \cdot N_{rw}$. A table of values is presented in table 12.5 below.

Depending on the geometry of the solid moving object, multiple collisions between a given water droplet and the moving object are possible. If the total number of collisions is N_C, then the N_{rw}, n_{rw} and ΔP columns are multiplied by N_C for $0 \leq N_C \lesssim 2.7 \cdot 10^6$. The remaining columns remain as stated. For $2.7 \cdot 10^6 \leq N_C \leq 2.7 \cdot 10^8$, one must take into account the mass depletion of the water droplet after every collision. That is not done here.
The P column represents the pressure due to the collision of air molecules with the moving object using $P=\rho U^2$.

TABLE 12.5, T=20^0 C, Sea Level

N_{rw}	U_X(mph)	$n_{rw}(\frac{1}{cm^3})$	$\Delta P(\frac{dy}{cm^2})$	$\Delta P(\frac{lb}{ft^2})$	$P(\frac{dy}{cm^2})$
1	0.22	$7.8 \cdot 10^8$	$3.1 \cdot 10^{-5}$	$6.7 \cdot 10^{-8}$	$1.2 \cdot 10^{-1}$
10	0.70	$7.8 \cdot 10^9$	$3.1 \cdot 10^{-4}$	$6.7 \cdot 10^{-7}$	1.2
10^2	2.2	$7.8 \cdot 10^{10}$	$3.1 \cdot 10^{-3}$	$6.7 \cdot 10^{-6}$	$1.2 \cdot 10^1$
10^3	7.0	$7.8 \cdot 10^{11}$	$3.1 \cdot 10^{-2}$	$6.7 \cdot 10^{-5}$	$1.2 \cdot 10^2$

10^4	22	$7.8 \cdot 10^{12}$	$3.1 \cdot 10^{-1}$	$6.7 \cdot 10^{-4}$	$1.2 \cdot 10^3$
10^5	70	$7.8 \cdot 10^{13}$	3.1	$6.7 \cdot 10^{-3}$	$1.2 \cdot 10^4$
10^6	220	$7.8 \cdot 10^{14}$	31	$6.7 \cdot 10^{-2}$	$1.2 \cdot 10^5$
10^7	700	$7.8 \cdot 10^{15}$	310	$6.7 \cdot 10^{-1}$	$1.2 \cdot 10^6$
10^8	2200	$7.8 \cdot 10^{16}$	3100	6.7	$2.4 \cdot 10^7$
$2.7 \cdot 10^8$	3600	$2.1 \cdot 10^{17}$	8400	18	$3.1 \cdot 10$

The energy to decompose one water molecule, $H_2O \longrightarrow 2H+O$, is $4.2 \cdot 10^{-12} = 2.6 ev$.

Setting $\frac{1}{2} m_{H_2O} U_X^2 = 4.2 \cdot 10^{-12}$ erg yields, $U_X = 5.3 \cdot 10^5 \frac{cm}{sec} = 12,000$ mph applicable to reentry vehicles and meteorites.

The energy to decompose one Nitrogen molecule, $N_2 \longrightarrow 2N$, is $1.55 \cdot 10^{-11}$ erg $= 9.7 ev$.

Setting $\frac{1}{2} m_{N_2} U_X^2 = 1.55 \cdot 10^{-11}$ erg yields, U_X becomes: $U_X = 8.1 \cdot 10^5 \frac{cm}{sec} = 18,000$ mph again applicable to reentry vehicles and meteorites.

9. The Evolution of a Vortex Over an Airplane Wing

As is well known, the shape of an object causing turbulent flow is a determining factor in the properties of turbulent flow. e.g. The vortices caused by an elliptical winged Spitfire have a rotational axis parallel to the long axis of the fuselage while the vortices caused by a square winged ME 109 have a rotational axis parallel to the long axis of the wing.
The problem with statistical approaches to turbulent flow is that they do not account for the geometric shape of the object causing the turbulence. Consider the square winged aircraft, fig. 12.7A, with cross sectional wing area as shown.
$0 < t \leq t_0$

FIGURE 12.7A
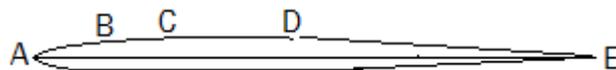
The slope of the upper wing surface decreases continuously from ∞ at A^+ to 0 at C

and remains 0 from C to D^- and becomes negative from D^+ to E^-. Between A and C,

the H_2O micelle strikes a wing with a vertical component and if the wing is traveling fast enough, one or more H_2O molecules will break away from the micelle.
Figure 12.7B

FIGURE 12.7B,

x axis perpendicular to the yz plane
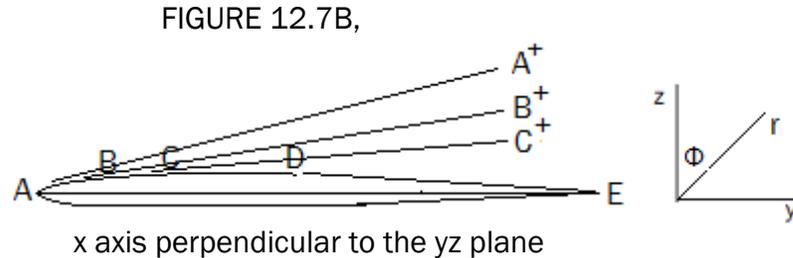
The H_2O molecules that break away from the clathrate close to A travel on the line A- A^+, those that break away close to B travel on the line B-B^+ and those that break away close to C travel on the line C-C^+. The air pressure above A^+ is greater than the air pressure below C^+ and above the wing and the pressure difference bends the particle paths into the beginnings of an arc.

The vertical component of the slope near A is greater than the vertical component of the slope near B that is greater than the vertical component of the slope near C and consequently $n_{w,A^+} > n_{w,B^+} > n_{w,C^+}$ where n_{w,A^+} is the number of water molecules per cm^3 at A^+ with similar definitions for n_{w,B^+} and n_{w,C^+}. It is hypothesized that the resultant $\frac{dn(h)}{dr}_w > 0$ is the physical cause of the centrifugal force holding a downstream vortex together where the Eulerian coordinates $\underset{\sim}{h}(\underset{\sim}{x},t)$ of a water molecule in a vortex are as follows with Φ as in Fig. 12.5B. The xyz coordinate frame with the origin at the center of the vortex is assumed to be inertial.

12.21 $\underset{\sim}{h}(\underset{\sim}{x},t)=h(\underset{\sim}{x},t)_x\hat{x}+h(\underset{\sim}{x},t)_y\hat{y}+h(\underset{\sim}{x},t)_z\hat{z}=(x+\int_0^t \frac{dh}{dt}_x dt)\hat{x}+r\sin(\Phi+\omega t)\hat{y}+r\cos(\Phi+\omega t)\hat{z}, \frac{dx}{dt}=0,$

$\underset{\sim}{h}(\underset{\sim}{x},0)=x\hat{x}+r\sin\theta\hat{y}+r\cos\theta\hat{z}, 0\leq x \leq x_o, 0\leq y \leq r_o, 0\leq z \leq r_o, 0\leq r\leq r_o, 0\leq \Phi \leq 2\pi$

If xyz is not inertial, then an inertial frame S must be found such that a vector $\underset{\sim}{R}(t)$ from the origin of S to the origin of the now noninertial frame xyz is drawn and the position of a water molecule becomes $\underset{\sim}{R}(t)+\underset{\sim}{h}(\underset{\sim}{x},t)$.

The integral term $\int_0^t \frac{dh}{dt}_x dt$, represents vortex stretching. In more advanced models $\frac{dn(\underset{\sim}{h})}{dt}$ is considered and constant r and ω become: r=r(t) and $\omega=\omega(t)$.

The velocity and acceleration terms of a water molecule $\underset{\sim}{U}$ and $\dot{\underset{\sim}{U}}$ become:

12.22A $\underset{\sim}{U}=\frac{dh}{dt}_x\hat{x}+r\omega\cos(\Phi+\omega t)\hat{y}-r\omega\sin(\Phi+\omega t)\hat{z}$

$$\dot{\underset{\sim}{U}}=\frac{dU}{dt}x\hat{x}-r\omega^2\sin(\phi+\omega t)\hat{y}-r\omega^2\cos(\phi+\omega t)\hat{z}$$

Using 12.15 and 12.22A find:

12.22B $\quad \rho(\underset{\sim}{h})\cdot\frac{dh}{dt}x=-(\frac{1}{n}\frac{\partial n}{\partial x}+\frac{1}{T}\frac{\partial T}{\partial x})P(\underset{\sim}{h})$,

$$\rho(\underset{\sim}{h})\cdot r\omega^2\sin(\phi+\omega t)=(\frac{1}{n}\frac{\partial n}{\partial y}+\frac{1}{T}\frac{\partial T}{\partial y})P(\underset{\sim}{h})$$

$$\rho(\underset{\sim}{h})\cdot r\omega^2\cos(\phi+\omega t)=(\frac{1}{n}\frac{\partial n}{\partial z}+\frac{1}{T}\frac{\partial T}{\partial z})P(\underset{\sim}{h})$$

and with $\underset{\sim}{r}=r\sin(\phi+\omega t)\hat{y}+r\cos(\phi+\omega t)\hat{z}$, $\hat{r}=\sin(\phi+\omega t)\hat{y}+\cos(\phi+\omega t)\hat{z}$, $\rho(\underset{\sim}{h})=n(\underset{\sim}{h})m_w$, find:

$\frac{\partial n}{\partial r}=\hat{r}\cdot(\frac{\partial n}{\partial y}\hat{y}+\frac{\partial n}{\partial z}\hat{z})=\frac{\partial n}{\partial y}\sin(\phi+\omega t)+\frac{\partial n}{\partial z}\cos(\phi+\omega t)$ and using 12.17A to derive $\frac{\partial n}{\partial y}$ and $\frac{\partial n}{\partial z}$ find:

$$\frac{\partial n}{\partial r}=n(\underset{\sim}{h})\{\frac{m_w}{KT}r\omega^2\sin(\phi+\omega t)-\frac{1}{T}\frac{\partial T}{\partial y}\}\sin(\phi+\omega t)+n(\underset{\sim}{h})\{\frac{m_w}{KT}r\omega^2\cos(\phi+\omega t)-\frac{1}{T}\frac{\partial T}{\partial z}\}\cos(\phi+\omega t)$$

And $\frac{\partial\ln(n)}{\partial r}=\frac{m_w}{KT}r\omega^2-\frac{1}{T}\frac{\partial T}{\partial r}=\frac{m_w}{KT}r\omega^2-\frac{\partial\ln T}{\partial r}$. Integrating the last expression yields:

$n(r)=n(o)\frac{T(o)}{T(r)}\exp\frac{m_w}{KT}r^2\omega^2$ and $P(r)=n(r)KT(r)=n(o)KT(o)\exp\frac{m_w}{KT}r^2\omega^2$.

12.22C $\quad n(r)=n(o)\frac{T(o)}{T(r)}\exp\frac{1}{2}\frac{m_w}{KT}r^2\omega^2$

$$P(r)=n(o)KT(o)\exp\frac{1}{2}\frac{m_w}{KT}r^2\omega^2$$

$$T(r)=\frac{n(o)}{n(r)}T(o)\exp\frac{1}{2}\frac{m_w}{KT}r^2\omega^2$$

THIS NEEDS TO BE SEVERELY REWRITTEN
10. Evaporation of Water at S.T.P.

Within the context of our model for a water clathrate fig. 12.1 and 12.5, how is it possible for water to evaporate at S.T.P. The following molecular model for evaporation is designed using the experimentally determined values of specific heat and heat of vaporization.
The specific heat of water is:

$C_P=\frac{1cal}{gm K}o$ and $\frac{1cal}{gm K}o=(4.185\cdot10^7)(3.0\cdot10^{-23})\frac{erg}{W K}o=1.26\cdot10^{-15}=9.13K(\frac{erg}{W K}o)=7.86\cdot10^{-4}(\frac{ev}{W K}o)$

where as above, W stands for water molecule. A water molecule is made up of 3

atoms so that the specific heat per atom is: $9.13K(\frac{erg}{W K}o)=3.04K(\frac{erg}{atom K}o)$. With a

$1K^O$ rise in temperature, the change in kinetic energy of 1 water molecule is

$1.5K=2.07 \cdot 10^{-16}$erg and $(1.26 - .207)10^{-15}=1.05 \cdot 10^{-15}erg=7.61K$ is expended as work to pull a water molecule a distance $\Delta \overline{R}$ from each one of its neighbors. With 6

neighbor water molecules, $1.75 \cdot 10^{-16}$erg$=1.27K$ is expended as work to pull a water molecule from 1 neighboring water molecule. The binding energy B.E. of 2 water

molecules is B.E.$=-\dfrac{m^2 H}{(2r_0+\overline{R})}$ and ΔB.E.$=1.75 \cdot 10^{-16}=[\dfrac{m^2 H}{(2r_0+\overline{R})}2] \Delta\overline{R}=1.2\Delta\overline{R}$ and

$\Delta\overline{R}=1.6 \cdot 10^{-16}$cm

The mass of 1 water molecule is 18amu and the mass of an average air molecule is $m_A=28.7$amu. Therefore an unbound water molecule at temperature equilibrium with the atmosphere, will rise in the atmosphere.

The density of air at sea level at $T=20^0$C is $\rho_A=1.18 \cdot 10^{-3}\frac{gm}{cm}3$ with number density

$n_A =2.47 \cdot 10^{19}\frac{1}{cm}3$. In general whether or not a water clathrate of mass M_{CL} in thermal equilibrium with a still atmosphere rises from ground level depends on

whether M_{CL} is greater than or less than $\rho_A V_{CL}$. Using 12.14, $M_{CL}=2.6 \cdot 10^{-14}$gm and

$\rho_A V_{CL}=3.1 \cdot 10^{-17}$gm and therefore M_{CL} whether bound or unbound to its neighboring clathrates, stays at ground level. Water vapor is however observed at altitudes over 20 miles high and does not achieve that altitude as a water clathrate. We investigate the conditions under which single water molecules escape from a water clathrate and then recombine with other single water molecules to form a hollow sphere whose surface is composed of water molecules bound together and whose interior is filled with single water molecules at water vapor pressure concomitant with the pressure and temperature at altitude. The density of the sphere ρ_S is less than the density of air ρ_A i.e. $\rho_S<\rho_A$ and consequently the spheres float upwards.

The heat of vaporization of water at $T=20^0$C is:

$2.45 \cdot 10^{10}\frac{erg}{gm}= 0.730 \cdot 10^{-12}\frac{erg}{W}=0.456\frac{ev}{W} =5.29 \cdot 10^{3}K\frac{erg}{W}$. It therefore takes

$0.730 \cdot 10^{-12}\frac{erg}{W}=0.456\frac{ev}{W}$ to remove one water molecule from a clathrate starting

with a bound water molecule at $T=20^0$C and ending with an unbound water molecule at $T=20^0$C. The temperature equivalence of 0.456ev is $T=3.53 \cdot 10^{3}(^0K)$.

Assuming the surface temperature of a body of water is greater then that of the air, the source of the energy to vaporize the surface waters during the night is the body of water below the surface driven by a boundary layer temperature distribution with $\frac{\Delta T}{\Delta z}$ >0, z positive downwards.

Let n_W be the number of individual water molecules that escape from the water

surface per second per cm^2. The power it takes to liberate n_W is $0.730 \cdot 10^{-12} n_W (\frac{erg}{sec\, cm^2})$

and $0.730 \cdot 10^{-12} n_W = K_W \frac{dT}{dz}$ where $K_W = 5.92 \cdot 10^4 \frac{erg}{sec\, cm\, K^0}$ is the thermal conduction

of water. Solving for n_W: $n_W \doteq 8.1 \cdot 10^{16} \frac{\Delta T}{\Delta z} (\frac{W}{sec\, cm^2})$. There are $3.5 \cdot 10^{22} (\frac{W}{cm^3})$ water

molecules per cm^3 and $3.5 \cdot 10^{22} z (\frac{W}{z\, cm^2})$ water molecules in volume zx1cmx1cm.

Also, $n_W = 3.5 \cdot 10^{22} \frac{dz}{dt} \doteq 3.5 \cdot 10^{22} \frac{\Delta z}{\Delta t}$ and $\frac{\Delta z}{\Delta t} = 2.3 \cdot 10^{-6} \frac{\Delta T}{\Delta z} (\frac{cm}{sec}) = 0.20 \frac{\Delta T}{\Delta z} (\frac{cm}{day})$. A

table of values for $\frac{\Delta T}{\Delta z} (\frac{K^0}{m})$, $\frac{\Delta z}{\Delta t} (\frac{cm}{day})$, $n_W (\frac{W}{sec\, cm^2})$ is given in table 12.1. Note the

units of $\frac{\Delta T}{\Delta z}$.

TABLE 12.1

$\frac{\Delta T}{\Delta z} (\frac{K^0}{m})$	$\frac{\Delta z}{\Delta t} (\frac{cm}{day})$	$n_W (\frac{W}{sec\, cm^2})$
10	$2.0 \cdot 10^{-2}$	$8.1 \cdot 10^{15}$
1	$2.0 \cdot 10^{-3}$	$8.1 \cdot 10^{14}$
0.1	$2.0 \cdot 10^{-4}$	$8.1 \cdot 10^{13}$
0.01	$2.0 \cdot 10^{-5}$	$8.1 \cdot 10^{12}$

Night Time Production of Water Vapor

The Maxwell-Boltzmann energy distribution is $N_E = 1.13 N \int_{\frac{E}{KT}}^{\infty} S^{\frac{1}{2}} e^{-S} dS$ where N is the

number of collisions of a given water molecule with adjacent water molecules and N_E

is the number of collisions with energies greater than energy E. With $E = 0.730 \cdot 10^{-12} \frac{erg}{W}$,

by direct computation, $\frac{N_E}{N}$ becomes: $\frac{N_E}{N} = 6.80 \cdot 10^{-8}$.

Setting $N_E = 1$ yields $N = 1.47 \cdot 10^7$ which means that for every $1.47 \cdot 10^7$ collisions
suffered by a given water molecule, one of those collisions will have an energy
greater than or equal to N_E. But with $N_E = 1$ and if the energy of a given molecule can
be treated as varying continuously with time, then on average, 1 collision out of

$1.47 \cdot 10^7$ will have energy $E = 0.730 \cdot 10^{-12} \frac{erg}{W}$ and no collisions out of $1.47 \cdot 10^7$ will
have energies greater then E.

Let $\overline{v}_{x>0}$ represent the average speed in a given direction for a water molecule in the

clathrate at $T = 293^0 K$ and let τ represent the time taken for a water molecule to

traverse $\bar{R}_W$ with speed $\bar{v}_{x>0}$ where $\bar{v}_{x>0}\tau=\bar{R}_W$. By direct computation at $T=293^\circ K$,

$\bar{v}_{x>0}=1.46\cdot10^4\frac{cm}{sec}$, (Ref. 12.1), and using $\bar{R}_W=2.0\cdot10^{-9}cm$, (See Sec. 6) τ is:

$\tau=\frac{1}{N_\tau}=1.37\cdot10^{-13}sec.$ and $5N_\tau=3.65\cdot10^{13}\frac{hits}{sec}$. Consequently the maximum number

of water molecules n_W that escape the surface of the water per second per $(2r_W)^2$ is:

$n_W=5\frac{N_\tau}{N}=\frac{3.65\cdot10^{13}}{1.47\cdot10^7}=2.48\cdot10^6\frac{escapes}{sec(2r_W)^2}=2.48\cdot10^6(1.6\cdot10^{15})=4.0\cdot10^{21}\frac{escapes}{sec(cm)^2}.$

n_W is however severely limited by the thermal conduction of water $K_W=5.92\cdot10^4\frac{erg}{sec\,cm\,K^0}$.
Using table 12.1, $n_W=51\frac{\Delta T}{\Delta z}\frac{escapes}{sec(2r_W)^2}$ and consequently, if water leaves the water

surface as individual water molecules, then $2.48\cdot10^6-51\frac{\Delta T}{\Delta z}$ water molecules

recombine with their clathrate and $51\frac{\Delta T}{\Delta z}\frac{escapes}{sec(2r_W)^2}$ escape the water surface per

second per $(2r_W)^2$. Those water molecules in the interior of the fluid with energies

greater than or equal to $E=0.456\frac{ev}{W}$ escape from their clathrate but then are
readsorbed by a neighboring clathrate and do not escape to the surface.

On a cloudless day the average value for the energy from the sun at sea level e_S is:

$e_S\doteq1\cdot10^6(\frac{erg}{sec\,cm2})$. Let $p\cdot e_S$, $0<p<1$, represent the amount of the sun's photon
energy per second used to break the chemical bonds holding individual water
molecules to clathrates. $p\cdot e_S$ liberates individual water molecules from clathrates at

$20^\circ C$ to individual water molecules at $20^\circ C$. Using $p\cdot e_S=0.73\cdot10^{-12}n_W(\frac{erg}{sec\,cm}2)$:

$n_W=1.4\cdot10^{18}p(\frac{W}{sec\,cm}2)=3.5\cdot10^{22}\frac{\Delta z}{\Delta t}$, with $\frac{\Delta z}{\Delta t}=4\cdot10^{-5}p(\frac{cm}{sec})=3.5p(\frac{cm}{day})$.

A table of values for p, $\frac{\Delta z}{\Delta t}(\frac{cm}{day})$, and n_W $(\frac{W}{sec\,cm}2)$ is given in table 12.2.

TABLE 12.2

| P	$\frac{\Delta z}{\Delta t}(\frac{cm}{day})$	n_W $(\frac{W}{sec\,cm}2)$
1	3.5	$1.4\cdot10^{18}$
0.1	0.35	$1.4\cdot10^{17}$
0.01	0.035	$1.4\cdot10^{16}$

Day Time Production of Water Vapor

The tabulated value for $\frac{\Delta z}{\Delta t}$ must be multiplied by $\frac{n}{24}$, where n is the number of daylight hours of sunshine in a 24 hour period in order to derive the measured value of $\frac{\Delta z}{\Delta t}$.

11. Surface Tension

Consider the meniscus of water in a glass tube of radius r. The meniscus is due to the attraction of the water molecules on the water surface in contact with the glass graduate, to the glass graduate. Figure 12.4 . The pressure at the bottom of the water column of height h is $P=\rho_w gh$ with force $-F_p\hat{z}=-\pi r^2 P\hat{z}=-\pi r^2 \rho_w gh\hat{z}$. The origin of the surface tension force is the attraction between the monomolecular layer of water molecules on the surface of the water in contact with the glass tube, to the glass tube. The total force due to surface tension in the $+\hat{z}$ direction is: $\underset{\sim}{F}_T\hat{z}=2\pi r\sigma_T\cos\theta$ where σ_T is the coefficient of surface tension. The forces are in equal and opposite directions so that $\underset{\sim}{F}_T\hat{z}=F_P$ and solving for h: $h=2\sigma_T\cos\theta(\rho_w gr)^{-1}$. Experimentally $\sigma_T=72.8\frac{dy}{cm}$ and h becomes; $h=0.149\frac{\cos\theta}{r}$. The maximum value of the force f_M that one surface water molecule exerts on one of its neighbors in the radial direction due to surface tension is: $f_M=2r_W\sigma_T$ and using 12.12, $f_M=1.8\cdot10^{-6}dy$.

FIGURE 12.4
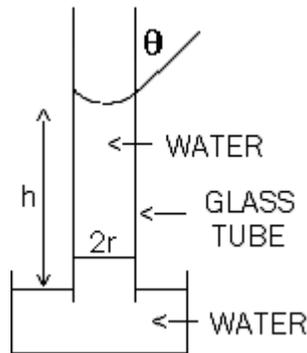
The work W_M to pull apart 2 water molecules held together by surface tension forces at $20^\circ C$ is $W_M\approx f_M\cdot\Delta s \approx f_M\cdot10^{-10}=1.8\cdot10^{-16}erg=1.1\cdot10^{-4}ev$ with a ΔT equivalence of, $\Delta T=0.85^\circ K$. i.e. raising the temperature by $0.85^\circ K$. destroys the surface tension. This is not observed and consequently the actual force f_A holding two water molecules together must be $f_A>f_M$. Note that assuming that $\Delta s \geq 10^{-9}cm$ yields $|\frac{\Delta\rho}{\rho}|\geq 15\%$ as the water climbs up the capillary tube. This is not observed.

12. Speed of Sound in Water and the Bulk Modulus of Water

The speed of sound in water is given by $C_S=\left[\dfrac{(C_p+3K)}{C_p}\dfrac{2r_wKT}{m\bar{R}_w(T)}\right]^{1/2}$. (Eq. 4.14)

Evaluating $\bar{R}_w(293)$ using the values derived above, $C_p=9K$, $r_w=1.25\cdot10^{-8}$cm, $m=3.01\cdot10^{-23}$gm,

And the experimentally determined value, $C_S=1.5\cdot10^5\frac{cm}{sec}$, yields: $\bar{R}_w(293)=2.0\cdot10^{-9}$cm.

The isothermal bulk modulus B_Θ is defined by $B_\Theta=-V(\dfrac{dP_e}{dV})_\Theta$ where P_e is the total external

pressure applied to a solid, liquid or gas sample where $B_\Theta=\dfrac{2r_o}{3R_o(T)}(\dfrac{C_p}{C_p+3K})\rho_oC_S^2$

(Reference 12.5). Applying the formula to 1 water molecule in the clathrate using the

values as above with $\rho_o=\dfrac{m}{8r_o}3=1.93\frac{gm}{cm}3$ yields: $B_\Theta=1.4\cdot10^{11}\frac{dy}{cm}2$ which is 6.7 times

the experimental value $B_{\Theta_{Ex}}=2.1\cdot10^{10}\frac{dy}{cm}2$. However, the experimental value $B_{\Theta_{Ex}}$

has been directly determined by compressing a water sample isothermally in an
enclosed pressure cell and measuring the change of length ΔL as a function of
applied pressure. The basic measurement $(-\Delta L)_{ex}$, is made by measuring the

electrical resistance in a Nirex wire between a point of attachment on the pressure
piston and a fixed point on the pressure casing: Once at external pressure $P_{ex}=0$
dy/cm2 and again at a final external pressure $P_{ex,f}$. Given the difficulty in making
consistent electric resistance measurements at high pressure, it is understandable
that human error may account for the experimentally determined

value, $B_{\Theta_{Ex}}=2.1\cdot10^{10}\frac{dy}{cm}2$. (Reference 12.2)

The basic measurement $(-\Delta L)_{ex}$, should be made using x-ray negatives, one taken at

external pressure $P_{ex}=0$ dy/cm2 and one taken at final external pressure $P_{ex,f}$.

13. Turbulent Flow in Water

We investigate the hypothesis that flowing water is composed of clathrates bound to
one another with each clathrate bound to 1 to 12 neighbor clathrates within a bundle
composed of N_{CL} clathrates where each clathrate contains N_W water molecules. See
12.13. The bundle of N_{CL} clathrates bound together is here called an S-clathrate or

super clathrate. Further, flow separation and turbulent flow of water in the wake of a solid object are hypothesized to be due to the collisions of S-clathrates with the solid object and consequent nonhomogenous breakup and recombination of S-clathrates into clathrates and S-clathrates where n is the number of clathrates plus S-clathrates per unit volume and $n=n(\underset{\sim}{j})$, $\underset{\sim}{j}=\underset{\sim}{x}+\underset{\sim}{\chi}(\underset{\sim}{x},t)$, $\underset{\sim}{\chi}(\underset{\sim}{x},0)=0$, $\underset{\sim}{x}\neq\underset{\sim}{x}(t)$ where $\underset{\sim}{x}$ is a stationary point in the water.

Consider a volume of water in a tank with a free surface initially at rest in inertial frame S. Using 4.6, the average pressure that 2 adjacent S-clathrates in still water exert on one another is:

12.15 $\qquad P_S \doteq \dfrac{2R_{SCL}n_{SCL}KT}{\overline{R}(h)}=\rho gh,\ \ h \geq 2R_{SCL}+\overline{R}(2R_{SCL})$

Where h is the depth of the water at the point where P_S is measured, n_{SCL} is the number of S-clathrates per unit volume in still water, R_{SCL} is the radius of a S-clathrate and $2R_{SCL}+\overline{R}(h)$ is the average distance between two adjacent S-clathrates.

Consider a solid cylinder of radius R_0 and length L_0 held stationary in flowing water. Figure 12.7 shows a cross section of the cylinder with the supports not shown. The top, sides and bottom of the tank are far enough away from the cylinder so that surface and wall affects are unimportant. Note that using the ideal gas law in place of 12.15 is incorrect: Assuming $P=nKT=\rho gh$ results in an incorrect sound speed and bulk modulus.

FIGURE 12.7
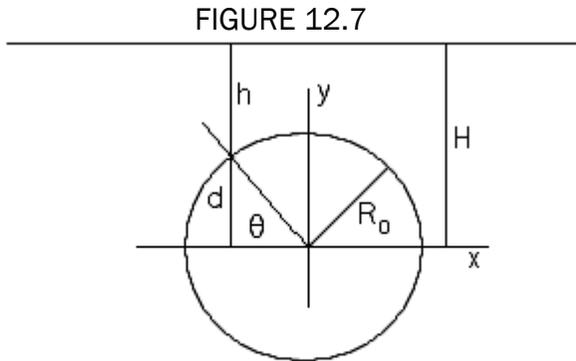
CYLINDER HELD STATIONARY IN FLOWING WATER

The water in the stream is composed of S-clathrates moving from left to right with speed U_X resulting in total pressure P_T normal to the surface of the cylinder on a point on the upstream side of the cylinder where:

$P_T=P_X\cos\theta+P_S$, $-\dfrac{\pi}{2}\leq\theta\leq\dfrac{\pi}{2}$, where $P_X\cdot\hat{x}=|\dfrac{\Delta m U_X}{\Delta t\,\Delta A}|\cdot\hat{x}=\dfrac{m|\Delta U_X|}{\Delta t\,\Delta A}\cdot\hat{x}$ and $|\Delta U_X|=U_X+U_X\cos2\theta=$

$U_X(1+\cos2\theta)$ and $\underset{\sim}{\Delta A}=-\Delta A\cdot\hat{x}$. $\underset{\sim}{P_X}\cdot\hat{x}$ becomes: $\underset{\sim}{P_X}\cdot\hat{x}=\dfrac{mU_X}{\Delta t\Delta A}(1+\cos2\theta)\cdot\hat{x}$ with

$m=\rho\Delta V=\rho U_X\Delta t\Delta A=m_{SCL}n_{SCL}U_X\Delta t\Delta A$ and $\underset{\sim}{P_X}\cdot\hat{x}=m_{SCL}n_{SCL}U_X^2(1+\cos2\theta)\cdot\hat{x}$. Also with

$(1+\cos2\theta)=2(1-\sin^2\theta)=2(1-[\dfrac{d}{R_0}]^2)$, $\underset{\sim}{P_X}\cdot\hat{x}$ becomes:

12.16 $\underset{\sim}{P_X}\cdot\hat{x}=m_{SCL}n_{SCL}U_X^2(1+\cos2\theta)\cdot\hat{x}=2m_{SCL}n_{SCL}U_X^2(1-[\dfrac{d}{R_0}]^2)\cdot\hat{x}$

For a given θ_0, suppose that U_X is large enough so that for $-\dfrac{\pi}{2}<-\theta_0\le\theta\le\theta_0<\dfrac{\pi}{2}$, collisions with the cylinder break the S-clathrates into clathrates plus S-clathrates so that before the collision all clathrates are S-clathrates with n_{SCL} S-clathrates per unit volume and after the collision n_{SCL} becomes $n(\underset{\sim}{j})>n_{SCL}$ for $0<t\le\tau_0$ for some τ_0 where $\underset{\sim}{j}$ at t=0 represents all points on the cylinder for $-\dfrac{\pi}{2}<-\theta_0\le\theta\le\theta_0<\dfrac{\pi}{2}$. After the collision $U_X=U(\underset{\sim}{j})_X$, however, $\overline{m}(\underset{\sim}{j})\cdot n(\underset{\sim}{j})=m_{SCL}n_{SCL}$ so that the functional form of $\underset{\sim}{P_X}$ before the collision equals the functional form of $\underset{\sim}{P_X}$ after the collision i.e.

$\underset{\sim}{P_X}\cdot\hat{x}=2m_{SCL}n_{SCL}U(\underset{\sim}{j})_X^2(1-[\dfrac{d}{R_0}]^2)\cdot\hat{x}=2\overline{m}(\underset{\sim}{j})\cdot n(\underset{\sim}{j})U(\underset{\sim}{j})_X^2(1-[\dfrac{d}{R_0}]^2)\cdot\hat{x}$ for $0<t\le\tau_0$.

$\underset{\sim}{P_S}$ however is dependent on $n(\underset{\sim}{j})$. After the breakup, $\underset{\sim}{P_S}$ becomes the turbulent pressure P_{tu} where $P_{tu}=\dfrac{2r_0(\underset{\sim}{j})n(\underset{\sim}{j})KT}{\overline{R}(\underset{\sim}{j})}$ and $2r_0(\underset{\sim}{j})+\overline{R}(\underset{\sim}{j})$ with $\overline{R}(\underset{\sim}{j})<<2r_0(\underset{\sim}{j})$ where

$2r_0(\underset{\sim}{j})+\overline{R}(\underset{\sim}{j})=\dfrac{1}{N_T}\sum d_{ij}$. d_{ij}=the center of mass to center of mass distance between the ith and j^{th} adjacent supra molecules (clathrates and S-clathrates and parts of S-clathrates) within a suitably small volume $\Delta V(\underset{\sim}{j})$ centered at $\underset{\sim}{j}$. The sum is taken over all adjacent pairs in $\Delta V(\underset{\sim}{j})$ and N_T is the total number of adjacent pairs in $\Delta V(\underset{\sim}{j})$.

Assuming that $\dfrac{r_0(\underset{\sim}{j})}{\overline{R}(\underset{\sim}{j})}=\dfrac{R_{SCL}}{\overline{R}(h)}$ for $H-R_0\le h\le H+R_0$, (See fig. 12.7), P_{tu} becomes:

12.17 $P_{tu}=\dfrac{2r_0(\underset{\sim}{j})n(\underset{\sim}{j})KT}{\overline{R}(\underset{\sim}{j})}=\dfrac{2R_{SCL}n(\underset{\sim}{j})KT}{\overline{R}(h)}=\dfrac{n(\underset{\sim}{j})}{n_{SCL}}P_S>P_S$ for $0<t\le\tau_0$

The S-clathrates strike the cylinder with fixed kinetic energy $\dfrac{1}{2}m_{SCL}U_X^2$.

Using 12.14: $m_{CL}=2.6\cdot10^{-14}$gm and $m_{SCL}=2.6\cdot10^{-14}N_{CL}$ and $r_{CL}=1.4\cdot10^{-5}$cm. With $-R_0\le d\le R_0$, the following scenario is anticipated.

(i) For small enough U_X, $0<U_X\le U_{Cr}$ for some U_{Cr}, the S-clathrates roll around the cylinder in creep flow. This necessitates a centripetal force $\underset{\sim}{f_C}=-m_{SCL}\dfrac{V_C^2}{R_0}\hat{R}_0$ with $\underset{\sim}{V_C}=V_C\cdot(\hat{R}_0\mathbf{X}\hat{z})$

where $\underset{\sim}{f_C}$ is provided by a chemical bonding force $-f_B\hat{R}_0$ between the S-clathrate

and the cylinder. In creep flow, $f_B \geq f_C$ where $f_B = m_{SCL}\dfrac{U_{Cr}^2}{R_0}$ for some given fixed N_{CL} where

U_{Cr} is the maximum flow rate at which the flow remains creep flow. Experimentally for $R_0 = 1cm$,

U_{Cr} is $U_{Cr} \approx 0.1\frac{cm}{sec}$ and f_B becomes: $f_B = 2.6\cdot10^{-16}N_{CL}(dy)$: Also $\frac{1}{2}m_{SCL}U_{Cr}^2 = 1.3\cdot10^{-14}N_{CL}U_{Cr}^2 \approx$

$1.3\cdot10^{-16}$erg.

(ii) For $U_X > U_{Cr}$ and consequent $m_{SCL}\dfrac{U_X^2}{R_0} > m_{SCL}\dfrac{U_{Cr}^2}{R_0} = f_B$ and $\frac{1}{2}m_{SCL}U_X^2 > \frac{1}{2}m_{SCL}U_{Cr}^2$, the

flow separates from the cylinder for $\frac{\pi}{2} \leq \theta \leq \frac{3}{2}\pi$.

(iii) The binding energy $Be_{CL,CL}$ holding 2 clathrates together is approximated using

$|BE_{CL,CL}| = \frac{3}{2}K(373-T) = \frac{3}{2}K(80) = 1.7\cdot10^{-14}erg=0.01$ev. In order to derive the
coefficient of viscosity of water, it is assumed that the binding energy $BE_{SCL,SCL}$
holding 2 S-clathrates together is $|BE_{SCL,SCL}| << |BE_{CL,CL}|$.

With $|BE_{CL,CL}| << |BE_{w,w}|$, let $|SBE_{SCL}|$ represent the absolute value of the self
binding energy of 1 super clathrate. i.e. $|SBE_{SCL}|$ is the energy necessary to pull
apart 1 S-clathrate into its N_{CL} constituent clathrates and $|BE_{w,w}|$ represents the
energy to pull 2 bound water molecules apart. i.e. $(H_2O)_2 \longrightarrow 2(H_2O)$.

A S-clathrate collides with the cylinder with large enough U_X so that $\frac{1}{2}m_{SCL}U_X^2 =$
$\frac{1}{2}m_{CL}N_{CL}U_X^2 \geq |SBE_{SCL}|$. For large enough N_{CL}, each clathrate is bonded to 6
clathrates in the interior of the clathrate and 5 clathrates on the surface of the
clathrate and $|SBE_{SCL}|$ may be approximated by $|SBE_{SCL}| = 6|BE_{CL,CL}|N_{CL}$ and

$\frac{1}{2}m_{CL}N_{CL}U_X^2 \geq |SBE_{SCL}|$ becomes $\frac{1}{2}m_{CL}U_X^2 \geq 6|BE_{CL,CL}| = 1.0\cdot10^{-13}erg=0.06$ev. Solving
for U_X yields:

12.18 $U_X \geq 0.29\frac{cm}{sec}$

Note the independence of U_X from N_{CL}.

On striking the cylinder with $U_X \geq 0.29\frac{cm}{sec}$, the S-clathrate breaks into N_{CL} individual

clathrates each with mass $m_{CL} = 2.4\cdot10^{-12}$gm. The N_{CL} clathrates separate from the

cylinder and in those parts of the flow where $\dfrac{n(j)}{\underset{\sim}{n}_{SCL}} > 1$ and using 12.17 it follows that

$P_{tu} > P_S$. The increased pressure and the random nature of the direction of the vector
velocities of the released water clathrates, destroys the laminar flow in the lee of the
cylinder and is the physical cause of turbulence.

In the lee of the cylinder, there is a mixture of $N_{CL}=1$ clathrates and N_{SCL} super clathrates. In those regions where $N_{CL}=1$ clathrates surround N_{SCL} super clathrates, there is a pressure gradient inwards towards the super clathrates and a vortex forms that dies away as the $N_{CL}=1$ clathrates reform into N_{SCL} super clathrates. The absence of laminar flow and the creation of vortices, is characteristic of turbulent flow.

In order to break up an individual clathrate into its $N_w=8.7\cdot10^8$ water molecules (See

12.14) requires $6|BE_{w,w}|\cdot N_w =6f_{w,w}\cdot\Delta s\cdot N_w \doteq 6(10^{-3})(10^{-10})N_w =6\cdot10^{-13}N_w(erg)=0.37N_w(ev)$ where $f_{w,w}$ is the average force necessary to pull apart two isolated water molecules and Δs is the distance necessary to pull apart two-isolated water molecules.

The kinetic energy of a clathrate moving with speed U_X is: $\frac{1}{2}m_w\cdot N_w\, U_X^2=1.5\cdot10^{-23}\cdot N_wU_X^2$

And setting $1.5\cdot10^{-23}\cdot N_wU_X^2=6\cdot10^{-13}N_w$ yields:

12.19 $U_X=2\cdot10^5\frac{cm}{sec}=1.3\cdot C_S$ C_S is the speed of sound in water

U_X is the minimum speed at which a clathrate must be moving as it strikes a stationary solid object in order that all N_w water molecules of which the clathrate is composed, are freed from one another.

14. Shock Wave in Earth's Atmosphere

Leading any axis-symmetric solid object traveling faster than or equal to the local speed of sound (Mach 1) is a high pressure, high-density surface of revolution called a shock wave. In Figure 12.8, a solid cube 2 cm on a side is moving from right to left at Mach 2.0. All measurements are made by an observer O in an inertial frame at rest with respect to the ground.

Figure 12.8- Shock Wave
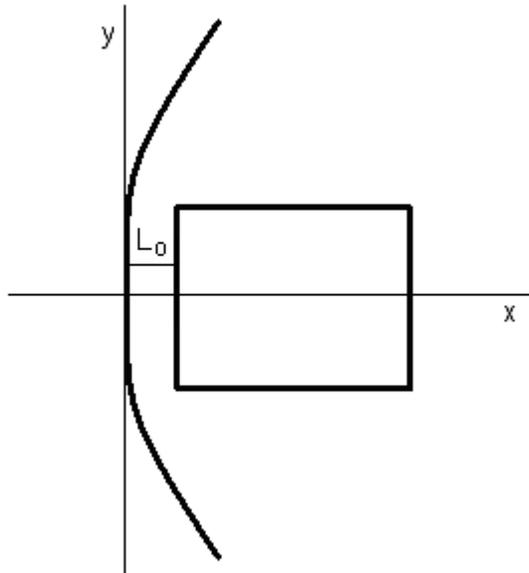
The shock wave acts as a shield in front of the solid object and the majority of incident air molecules reflect off of its surface and a minority penetrate the shock. Figure 12.9 represents the solid moving into still air at arbitrary Mach number M but before the formation of the shock. The air molecules at S.T.P. in A are idealized to be in stationary serried ranks separated by the average distance λ_0 between the molecules. Note that assuming that the molecules in serried ranks are separated by the mean free path, yields an air density ~10^3 times smaller than the measured air density.

The average velocity of the molecules in A is taken to be $0\frac{cm}{sec}$ except for the molecules directly in contact with the solid. For M>>1 this is a good approximation, but for M$\lesssim$1, a bad approximation. The struck molecules in A travel the distance λ_0 to the left in time t_0 with average velocity $-2MC_0\hat{x}$; Strike the molecules in the 2nd rank and exchange velocities. In time t_0, the solid object travels distance $\frac{\lambda_0}{2}$ with velocity $-MC_0\hat{x}$. Considering A through H, it is apparent that $n_1 \doteq 2n_0$ where n_0 is the molecule number density in A and n_1 is the molecule number density in H. Also $\rho_1 \doteq 2\rho_0$ where ρ_0 is the mass density in A and ρ_1 is the mass density in H. The percussive wave generated by the moving object travels with speed $2MC_0$ as measured by observer O. However the average speed of the molecules is MC_0, i.e.

$$[(2MC_0)\cdot 8 + 0\cdot 8]16^{-1} = MC_0$$

The adiabatic sound speed in A is $C_0 = (\gamma \frac{P_0}{\rho_0})^{\frac{1}{2}} = (\gamma \frac{n_0 KT_0}{\rho_0})^{\frac{1}{2}}$ and the adiabatic sound speed in H as measured w.r.t. the moving object is, $C_1 = (\gamma \frac{P_1}{\rho_1})^{\frac{1}{2}} = (\gamma \frac{n_1 KT_1}{\rho_1})^{\frac{1}{2}} = (\gamma \frac{n_0 KT_0 T_1}{\rho_0 T_0})^{\frac{1}{2}} = C_0(\frac{T_1}{T_0})^{\frac{1}{2}}$.

Figure 12.9 Water Droplets Not Shown

A

B

C

D

....

E

F

G

....

H
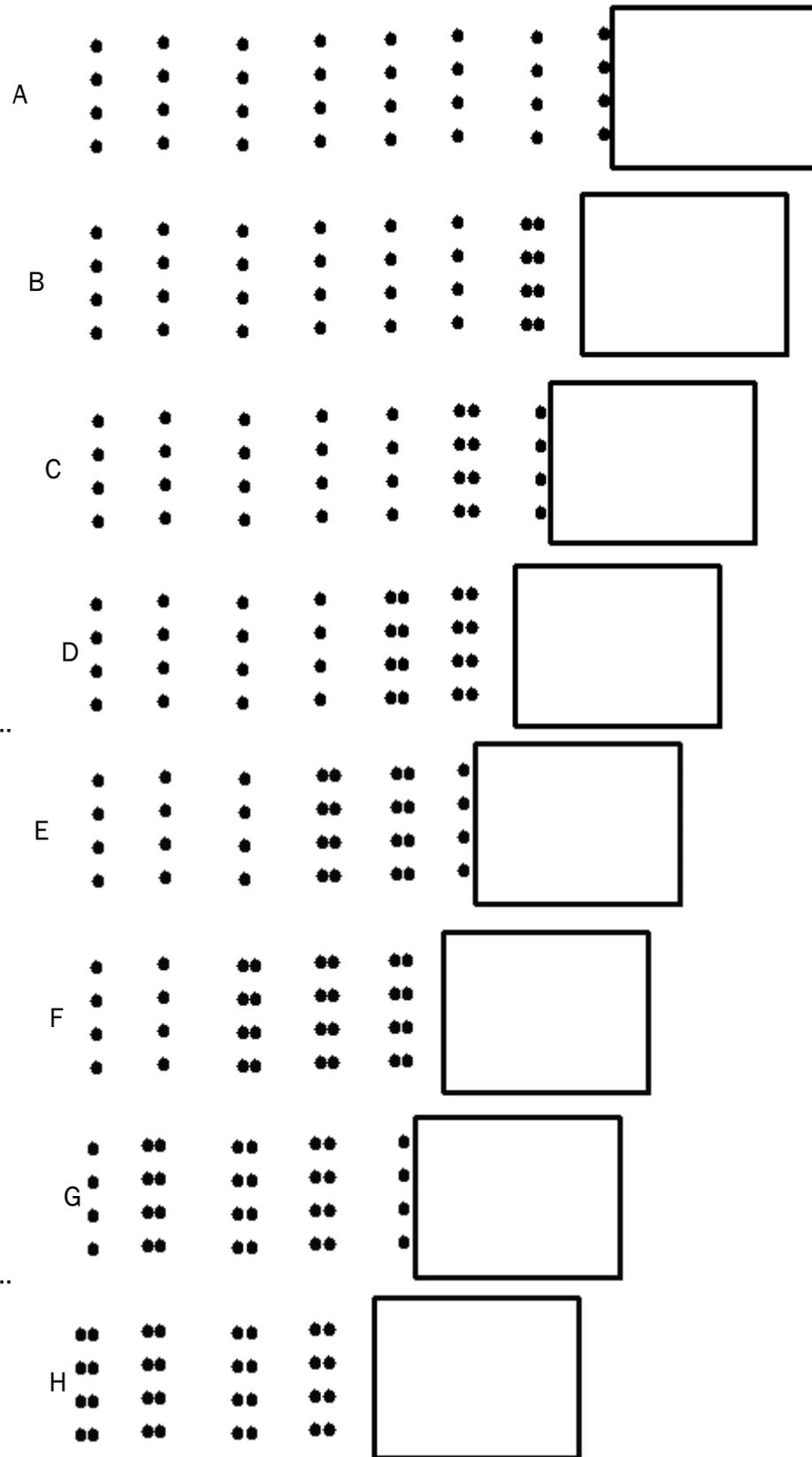
For a shock to form with $C_S = M \cdot C_o = C_o(\frac{T_1}{T_0})^{\frac{1}{2}}$ yields: $T_1 = M^2 T_0$. This is not observed and consequently a shock wave does not form in air that is in accord with the ideal gas law and $\frac{n_0}{\rho_0} = \frac{n_1}{\rho_1} = \frac{1}{\overline{m}_0}$.

The density at fig. 12.9 H in front of the moving object ρ_f is $\rho_f = 2\rho_0$ where ρ_0 is the background air density before the passage of the moving object, fig. 12.9 A. The leading edge separating $\rho = \rho_0$ from $\rho = 2\rho_0 = \rho_f$ is traveling with speed $2MC_o$: The moving object is traveling with speed MC_o.

The movement of the solid object causes the breakup of water droplets directly in front of the moving object resulting in an increase in the number of molecules per unit volume from n_o to $n_1 = n_o + n_{rw}$ where n_{rw} is the number of newly released water molecules per unit volume (Not Shown). As the water droplets break up, the number density increases from n_o to n_1 but the mass density ρ_o, at the instant of the creation of n_{rw} directly in front of the moving object, remains the same, fig. 12.9 A (Water droplets not shown).

As the water droplets are pushed away from the moving object, fig. 12.9 H, the number density increases from n_1 to $n_f = 2n_1$ and the mass density increases from ρ_0 to $\rho_f = 2\rho_0$.

The phase speed of sound C_2 at fig. 12.9 H directly in front of the moving object is the speed of sound with respect to the air plus the speed of the air with respect to the ground. With no water droplets, $n_w = 0$, C_2 is:

$C_2 = (\gamma \frac{2n_o KT_0}{2\rho_0})^{\frac{1}{2}} + MC_o = (\gamma \frac{n_o KT_0}{\rho_0})^{\frac{1}{2}} + MC_o = (1+M)C_o$. With the advent of the breakup of water droplets, the speed of sound C_f at fig. 12.9 H directly in front of the moving object is:

$C_f = (\gamma \frac{n_f KT_0}{\rho_f})^{\frac{1}{2}} + MC_o = (\gamma \frac{2n_1 KT_0}{2\rho_0})^{\frac{1}{2}} + MC_o = (\gamma \frac{\overline{m}_f KT_0}{\overline{m}_0})^{\frac{1}{2}} + MC_o = (\gamma \frac{n_o(1+\frac{n_{rw}}{n_o})KT_0}{\rho_0})^{\frac{1}{2}} + MC_o =$

$[(1+\frac{n_{rw}}{n_o})^{\frac{1}{2}} + M]C_o$. For future use:

12.20 $\qquad C_2 = (1+M)C_o$

$\qquad\qquad C_f = [(1+\frac{n_{rw}}{n_o})^{\frac{1}{2}} + M]C_o$

The time interval Δt that it takes the trailing edge of the nascent shock beginning at $x = L_0$ at $t = 0$ with wave length λ_0 (Fig. 12.8) to over take the leading edge of the nascent shock (i.e. The leading edge separating $\rho = \rho_0$ from $\rho = 2\rho_0 = \rho_f$, traveling with speed $2MC_o$) is: $\Delta t = (L_0 - \lambda_0)(C_2)^{-1} = L_0(C_f)^{-1} = (L_0 - \lambda_0)[(1+M)C_o]^{-1} = L_0([(1+\frac{n_{rw}}{n_o})^{\frac{1}{2}} + M]C_o)^{-1}$

Solving the last two expressions for $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M$ yields: $(1+\frac{n_{rw}}{n_0})^{\frac{1}{2}}+M=(1+M)(1-\frac{\lambda_0}{L_0})^{-1}$.

Δt and $\frac{n_{rw}}{n_0}$ become:

12.21 $\qquad \Delta t=(1-\frac{\lambda_0}{L_0})\frac{L_0}{(M+1)C_0}$

$$\frac{n_{rw}}{n_0}=\left\{\frac{1+\frac{\lambda_0}{L_0}\cdot M}{1-\frac{\lambda_0}{L_0}}\right\}^2-1$$

Note that using $MC_0=10(N_{rw})^{\frac{1}{2}}\frac{cm}{sec}$ (See sec 10), it follows that: $\lim\limits_{M\to O} N_{rw}=0$ and

$\lim\limits_{M\to O}\frac{n_{rw}}{n_0}=0$ and using 12.20, $\lim\limits_{M\to O}C_f=C_2$. Using $\Delta t=(L_0-\lambda_0)(C_2)^{-1}=L_0(C_f)^{-1}$:

$\lim\limits_{M\to O}L_0(C_f)^{-1}=L_0(C_2)^{-1}=(L_0-\lambda_0)(C_2)^{-1}$ and consequently, $\lim\limits_{M\to O}\lambda_0=0$. Observationally $\lim\limits_{M\to O}L_0=0$ and using 12.21, $\lim\limits_{M\to O}\Delta t=0$. Summing up:

12.22 $\quad \lim\limits_{M\to O}\frac{n_{rw}}{n_0}=0,\ \lim\limits_{M\to O}\lambda_0=0,\ \lim\limits_{M\to O}L_0=0$ and $\lim\limits_{M\to O}\Delta t=0$

If the total volume of floating water droplets is 1% of the total volume of air through which the solid object passes and using $MC_0=10(N_{rw})^{\frac{1}{2}}\frac{cm}{sec}$ (See sec 10), $n_{rw}=7.8\cdot10^8\cdot N_{rw}$ and $\Delta P=n_{rw}KT$ as developed for table 2.5 and 12.6 yields N_{rw} (The number of water molecules released per floating droplet after 1 collision with the moving object) and n_{rw} (The number of water molecules released per cm^3 after 1 collision with the moving object) and ΔP (The change in pressure as measured from an inertial frame at rest with respect to the center of mass of the moving air. The change in pressure is due to the release of water molecules from droplets thereby increasing n_f). C_0 as experimentally determined at sea level for $T=20^oC$ is, $C_0=3.4\cdot10^4(\frac{cm}{sec})=760$mph.

Using $MC_0=2C_0$, a table of values of N_{rw}, n_{rw} and ΔP computed for Mach2 is presented

TABLE 12.7, Sea Level, Mach2

N_{rw}	$2C_0$(mph)	$n_{rw}(\frac{1}{cm^3})$	$\Delta P(\frac{dy}{cm^2})$	$\Delta P(\frac{lb}{ft^2})$
$4.6\cdot10^7$	1520	$3.6\cdot10^{16}$	$1.5\cdot10^3$	0.54
$4.6\cdot10^7N_C$	1520	$3.6\cdot10^{16}N_C$	$1.5\cdot10^3N_C$	$0.54N_C$
$7.8\cdot10^8$	1520	$6.1\cdot10^{17}$	$2.55\cdot10^4$	9.2

in table 12.7. If the total number of collisions is N_C, then the N_{rw}, n_{rw} and ΔP columns are multiplied by N_C where $0 \le N_C \le 17$.

The change in pressure ΔP_{M2} due to the collision of air molecules with the moving object at Mach2 is: $\Delta P_{M2} = \rho(Mach2)^2 = (2\rho_0)(6.8 \cdot 10^4)^2 = 1.1 \cdot 10^7 \frac{dy}{cm}2$.

Using 12.21, Δt and $\frac{n_{rw}}{n_0}$ are evaluated and compiled in table 12.8 for a solid of cross section 2cmX2cm and $L_0 \approx 0.6$cm traveling at Mach 2 as in fig. 12.8. At $T = 20^0C$ at sea level, $n_0 = 2.5 \cdot 10^{19}$ and for the droplet under consideration, fig 12.6, $0 \le \frac{n_{rw}}{n_0} \le 2.4 \cdot 10^{-2}$

Table 12.8, Sea Level, Mach2

$\frac{n_{rw}}{n_0}$	$\frac{\lambda_0}{L_0}$	Δt(sec)
0	0	$5.9 \cdot 10^{-6}$
$1.0 \cdot 10^{-3}$	$1.67 \cdot 10^{-4}$	$5.9 \cdot 10^{-6} (1 - 1.67 \cdot 10^{-4})$
$1.0 \cdot 10^{-2}$	$1.67 \cdot 10^{-3}$	$5.9 \cdot 10^{-6}(1 - 1.67 \cdot 10^{-3})$
$2.0 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$5.9 \cdot 10^{-6}(1 - 3.3 \cdot 10^{-3})$
$2.4 \cdot 10^{-2}$	$6.6 \cdot 10^{-3}$	$5.9 \cdot 10^{-6}(1 - 6.6 \cdot 10^{-3})$

As $t^- \to \Delta t = t_0 - 0$, the trailing edge overtakes the leading edge and the energy density, mass density and pressure between the leading and trailing edge increase forming a shock. Observationally, the shock is traveling with group speed MC_0 at approximately constant distance L_0 in front of the moving object. This means that water droplets in front of the shock are being broken up by collision with the molecules making up the shock and the shock is entering an atmosphere where the speed of the shock is

$C_S = MC_0 = (1 + \frac{n_{w,s}}{n_0})^{\frac{1}{2}} C_0$ and $(M^2 - 1)n_0 = n_{w,s}$ where $n_{w,s}$ is the number of water molecules per cm^3 that are released from water droplets by collision with the shock and reflect in front of the shock.

How is it that the speed of the shock remains constant at MC_0? If $n_{w,s} > (M^2 - 1)n_0$ the shock starts to travel away from the moving object, loses energy and ceases to exist and if $n_{w,s} < (M^2 - 1)n_0$, the moving object overtakes the shock and again the shock ceases to exist. If $n_{w,s} = (M^2 - 1)n_0$ the shock remains at a ~fixed distance in front of the moving object and energy is constantly pumped into it by the moving object.

The shock acts as a shield and depending on the mass density and thickness of the shock, a certain percentage P of the incident air molecules in front of the shock reflect off the shock and 100−P percent go through the shock. Among those that reflect off of the shock are the water droplets that are broken up and release $n_{w,s}$ water molecules per cm^3 in front of the shock.

Of the 100−P percent that go through the shock, n(100−P) percent reflect off the moving body and make up the molecules that are pumped back into the shock and (1−n)(100−P) move around and behind the moving body where 0<n<1.

15,   Reference

12.1.   Milton Van Dyke, "An Album of Fluid Motion", The Parabolic Press, Stanford California, 1982